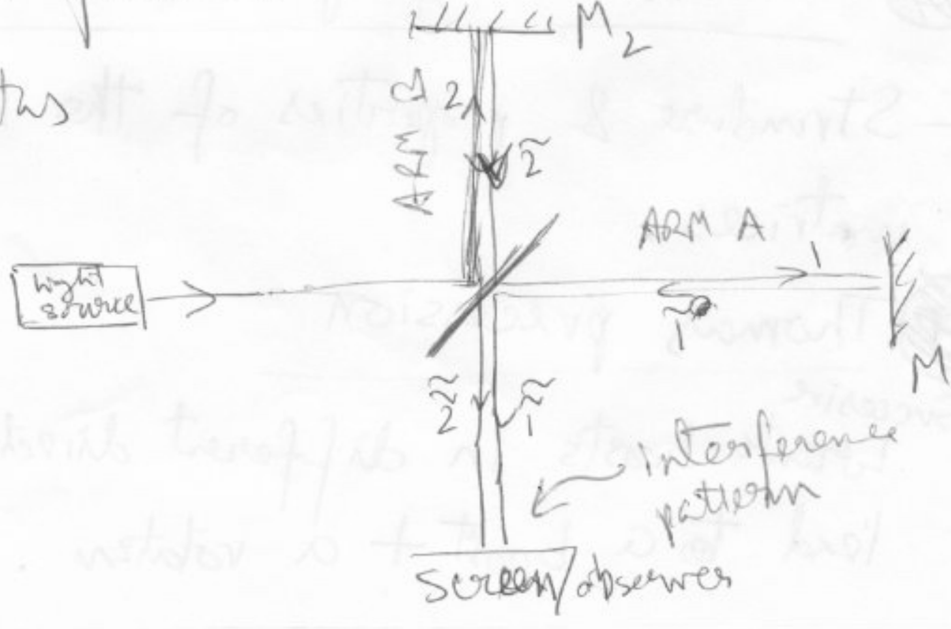
(6a) * THE MICHELSON-MORLEY EXPERIMENT

* ~~The Michelson-Morley Experiment.~~

- In the 19th century, light was thought to be vibrations of an all-pervasive medium, called ETHER.
- If the earth moves w.r.t. ETHER, light should have different speeds in different directions.
- Michelson-Morley apparatus attempted to measure this effect, and found a NULL result.
- Correct conclusion/interpretation: there is no ether. Light speed is the same in all frames.
- Apparatus



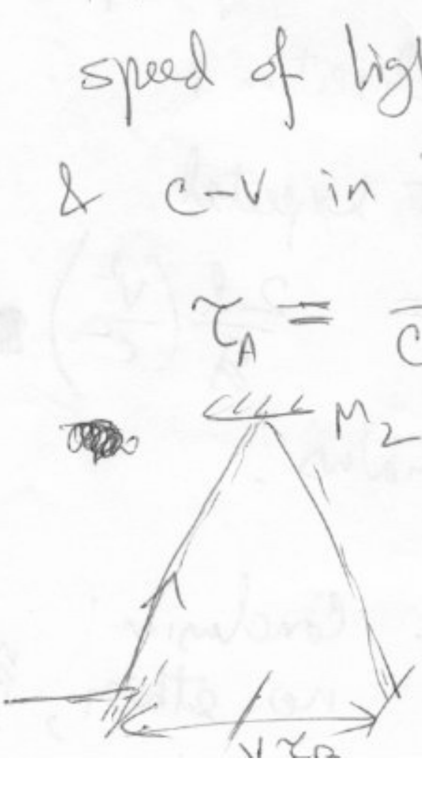
* Idea: if light beams $\tilde{\tilde{1}}$ and $\tilde{\tilde{2}}$ are exactly in phase, constructive interference. If exactly out of phase, destructive interference. In practice, interference fringes due to slight misalignment (not exact rays):

If length of one arm is changed, fringes shifted.

Change by one wavelength/cycle $\longrightarrow$ shift by one fringe.

* Ether interpretation. Imagine earth's motion thru ether along arm A direction. Then speed of light is $c+v$ in one direction & $c-v$ in the other direction. (Now known to be incorrect)

$$\tau_A = \frac{l}{c+v} + \frac{l}{c-v} = \frac{2l}{c}\left(\frac{1}{1-v^2/c^2}\right)$$



As seen from ether, the B path length is $2\sqrt{l^2+\left(\frac{v\tau_B}{2}\right)^2}$

$$\Rightarrow \tau_B = \frac{2\sqrt{l^2+(v\tau_B/2)^2}}{c}$$

(6c)

$$\Rightarrow \tau_B = \frac{2l}{c} \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \tau_A - \tau_B = \frac{2l}{c}\left(\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\simeq \frac{l}{c}\left(\frac{v^2}{c^2}\right)$$ using binomial series, $v << c$

$\Rightarrow$ expected # of fringe shifts caused by time difference

If distance diff. $= \lambda$! shift by one fring.

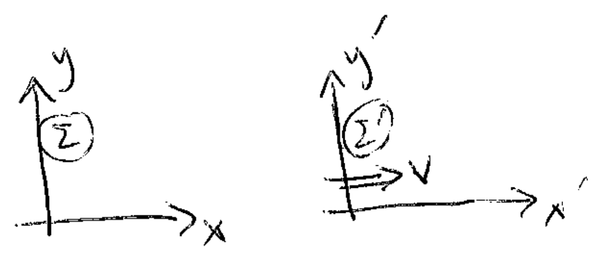
$$\simeq \frac{c(\tau_A - \tau_B)}{\lambda} = \frac{c}{\lambda} \cdot \frac{l}{c}\left(\frac{v^2}{c^2}\right) = \frac{l}{\lambda}\left(\frac{v^2}{c^2}\right)$$

Compare with rotated apparatus: B in the direction of earth's motion: expect fringe shift in opposite direction.

Thus fringe shift expected between two orientations $= \frac{2l}{\lambda}\left(\frac{v^2}{c^2}\right)$

$\simeq 0.4$ in their apparatus.

RESULT: Not observed. Conclusion: no ether, no preferred frame

* 

Standard config;
or standard boost;

Σ' moves in common x, x' direction, with velocity v relative to Σ. Watches synchronized $(t = t' = 0)$ at instant when Σ, Σ' are coincident.

* LT for standard boost:

$$x' = \gamma_v (x - vt) \qquad y' = y \qquad z' = z$$
$$t' = \gamma_v (t - vx/c^2)$$

<u>OR</u>

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma_v & -\frac{v}{c}\gamma_v & 0 & 0 \\ -\frac{v}{c}\gamma_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

* Inverse LT: Σ moves with velocity $-v$ relative to Σ'. So

$$x = \gamma_v (x' + vt') \qquad y = y' \qquad z = z'$$
$$t = \gamma_v (t' + vx'/c^2)$$

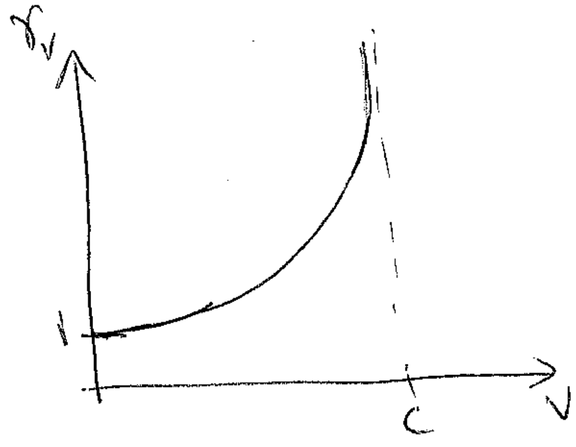
4a

→ can obtain inverse LT by algebraically solving LT.

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma_v & +\frac{v}{c}\gamma_v & 0 & 0 \\ +\frac{v}{c}\gamma_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$$

→ can obtain inverse LT matrix by INVERTING the LT matrix. Tedious exercise in matrix inversion.

* $\gamma_v = \dfrac{1}{\sqrt{1 - v^2/c^2}}$ Lorentz factor



For $v \ll c$, $\gamma_v \simeq 1$, so LT reduces to Galilean Transformations.

For v <u>not</u> small, various non-intuitive effects are predicted by LT's.

Overview of topics

[A] Lorentz Transfo, BOOSTS & ROTATIONS, matrix representation, rapidity

[B] Time dilation

If stationary clock in Σ measures time interval τ_0 (e.g., difference betn tick at t and tock at $t+\tau_0$)

measured from Σ', the interval is $\tau' = \gamma_v \tau_0$

(Seen from Σ', events don't happen at same location.)

If stationary clock in Σ' measures time interval τ_0', then measured from Σ, the interval is $\tau = \gamma_v \tau_0'$

Whose clock runs slower? Depends on: in which frame clock is stationary.

"Moving clocks run slower".

4c

C Length contraction

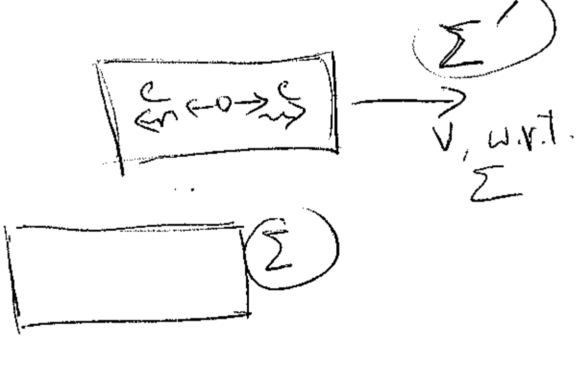
~~Length~~ Moving rods appear shorter.

Provided: end measurements done simultaneously in each frame.

D Simultaneity is Relative

Two events simultaneous in Σ': Does not imply they are simultaneous in Σ.

Ex.



In Σ', ~~reach~~ light reaching left & right walls are simultaneous events.

From Σ, ~~Times seem~~ events are not simultaneous.

E Relativistic momentum

$$p = \gamma_v m v = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

~~Some~~ Older texts call $\gamma_v m$ the "relativistic mass"

Then m is the "rest mass".

Nowadays: MASS = rest mass

Note momentum $\to \infty$ as $v \to c$

$\Rightarrow$ nothing travels faster than light.

F Relativistic Energy

$E = \gamma_v mc^2$ for object with speed v, mass m.

Object at rest has "rest energy": $E = mc^2$

due to its mass.

$\to$ Mass can be converted to energy

(nuclear fusion, nuclear fission)

4e

* Kinetic energy $= \gamma_v mc^2 - mc^2$

$$= (\gamma_v - 1)mc^2 \xrightarrow[v \ll c]{} \approx \frac{1}{2}mv^2$$

* Also, using $E = \gamma_v mc^2$ & $p = \gamma_v mv$,

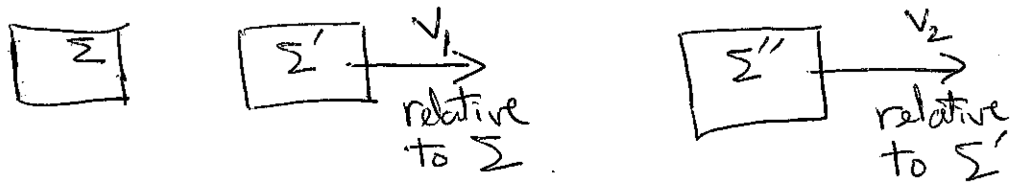
can show $E^2 = p^2c^2 + m^2c^4$

* Massless objects (photons): $E = pc$

$E = hf = \frac{hc}{\lambda}$ $\qquad f =$ frequency

$p = \frac{hf}{c} = \frac{h}{\lambda}$ $\qquad \lambda = c/f =$ wavelength

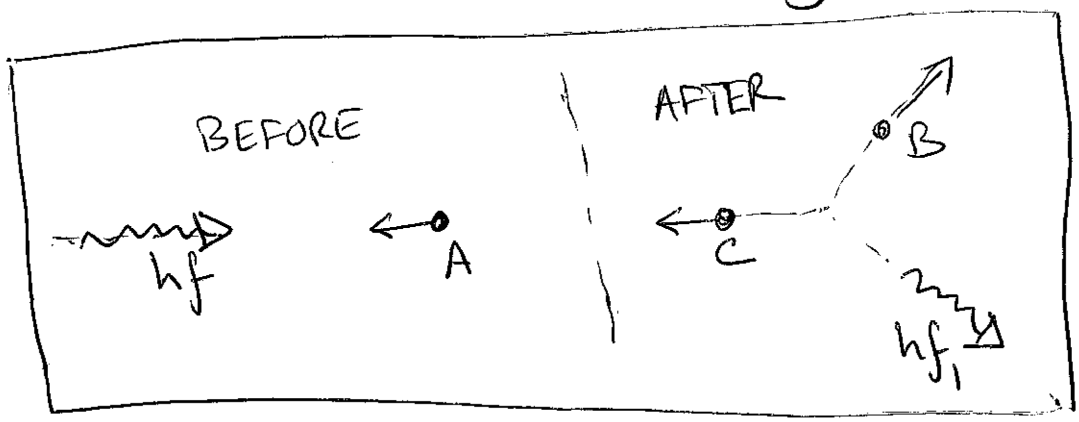
G Relativistic addition of velocities :



Σ″, relative to Σ, has velocity $\dfrac{v_1+v_2}{1+\frac{v_1 v_2}{c^2}}$

H Relativistic collisions & decay :

Ex.1



Total relativistic energy before = total relativistic energy after

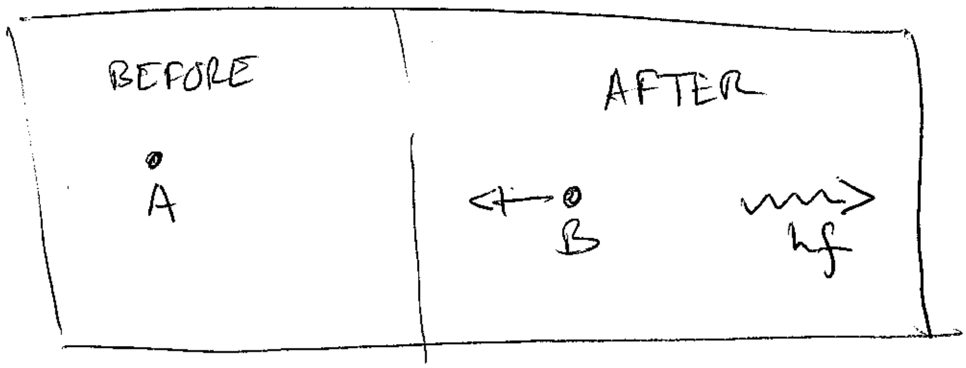
Total relativistic momentum before

= total relativistic momentum after

(1+3 equations)

Ex. 2

DECAY



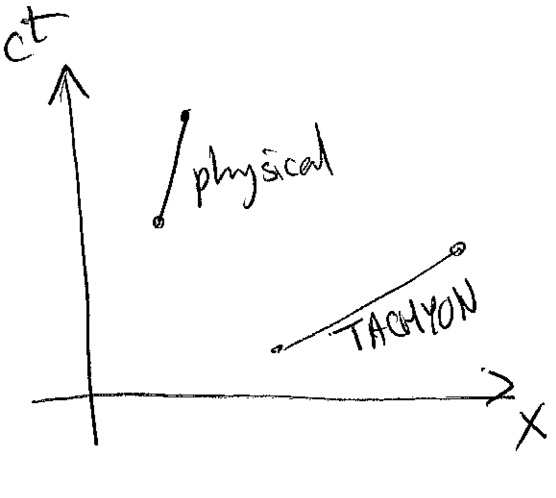
I Relatinistic Doppler Effect

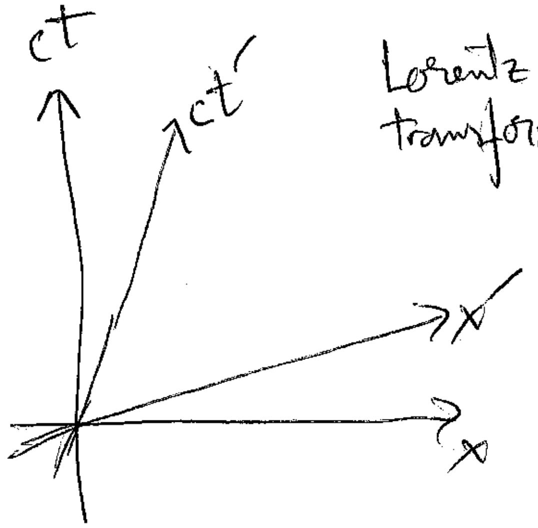
hf relative to Σ

hf' relative to Σ'

$f' \neq f$

J SPACETIME DIAGRAMS





Lorentz transformation

$\boxed{K}$ 4-vectors & 4-tensors

(ct, x, y, z) of an event is a 4-vector,
vector in Minkowski space.

$\left(\frac{E}{c}, p_x, p_y, p_z\right)$ of a particle is another 4-vector

(Density, current density) is a 4-vector.

All 4-vectors transform the same way.

Mechanics, reformulated in terms of 4 vectors.

$F^{\mu\nu} \rightarrow$ electromagnetic field tensor
with 2 indices.

If expressed in terms of 4-vectors and 4-tensors, physical laws are Lorentz-invariant.

4i

L Electrodynamics in tensor notation

Electric & magnetic fields, together, form an antisymmetric 4-tensor $F^{\mu\nu}$.

Scalar, vector potentials for 4-vector
$$A^\mu = \left(\frac{\phi}{c}, \vec{A}\right)$$

All of Maxwell's equations, E&M

$$\boxed{\begin{aligned} &\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0, \quad \vec{\nabla}\cdot\vec{B} = 0, \quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \quad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \\ &\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0, \quad \vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) \\ &\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla}\times\vec{A} \end{aligned}}$$

reformulated concisely:

$$\boxed{\begin{aligned} &\partial_\mu J^\mu = 0, \quad \partial_\mu F^{\mu\nu} = J^\nu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \\ &K^\mu = q F^{\mu\alpha} v_\alpha \end{aligned}}$$

→ Explicitly Lorentz-invariant, i.e., consistent with LT

$\boxed{M}$ The Lorentz group and the Poincaré group

* Structure & properties of the LT matrices

→ ~~...~~ two successive LT's ~~.~~ makes an LT

→ LT's consist of rotations & boosts

etc.

$\boxed{N}$ Wigner rotation / Thomas precession

Successive boosts in different directions lead to a boost + a rotation.