* LT in matrix form; RAPIDITY

Three common forms for LT for standard boost:

① $x' = \gamma_v(x - vt)$, $y' = y$, $z' = z$, $t' = \gamma_v\left(t - \frac{v}{c^2}x\right)$

② $$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma_v & -\gamma_v \frac{v}{c} & 0 & 0 \\ -\gamma_v \frac{v}{c} & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Common to write $$\begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

③ $$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi & 0 & 0 \\ -\sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$\phi = \tanh^{-1}\left(\frac{v}{c}\right)$ is the RAPIDITY

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* If only standard boosts, can omit y, z, y', z'.

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma_v & -\gamma_v \frac{v}{c} \\ -\gamma_v \frac{v}{c} & \gamma_v \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} \equiv \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

* Inverse transform? Should be obtained by replacing $v \longrightarrow -v$, or $\phi \to -\phi$

If $L(v) = \begin{pmatrix} \gamma_v & -\gamma_v \frac{v}{c} \\ -\gamma_v \frac{v}{c} & \gamma_v \end{pmatrix}$, then

$$L^{-1}(v) = \begin{pmatrix} \gamma_v & \gamma_v \frac{v}{c} \\ \gamma_v \frac{v}{c} & \gamma_v \end{pmatrix} = L(-v)$$

$\boxed{\text{Exercise: show by multiplying the two matrices: obtain unit matrix}}$

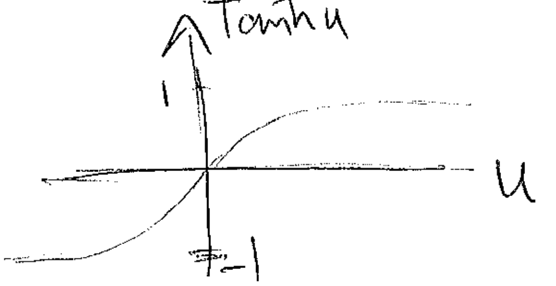
If $\Lambda(\phi) = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$, then

$$\Lambda^{-1}(\phi) = \Lambda(-\phi) = \begin{pmatrix} \cosh\phi & \sinh\phi \\ \sinh\phi & \cosh\phi \end{pmatrix}$$

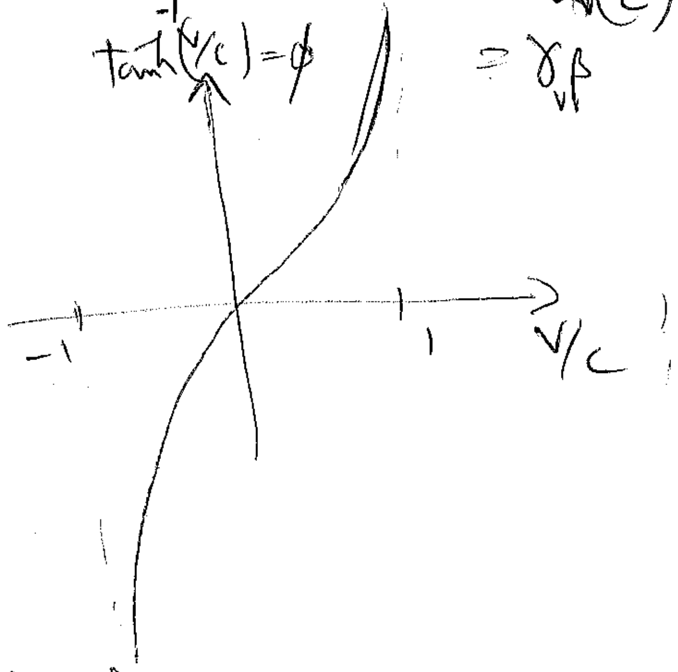
$\boxed{\text{Exercise: show by multiplying.}}$

* Rapidity: $\tanh\phi = \frac{v}{c} = \beta \Rightarrow \cosh\phi = \gamma_v$

$\sinh\phi = \gamma_v\left(\frac{v}{c}\right) = \gamma_v\beta$



$\Rightarrow$



ϕ is a continuous, unbounded variable ($-\infty$ to ∞ values). Not an angle.

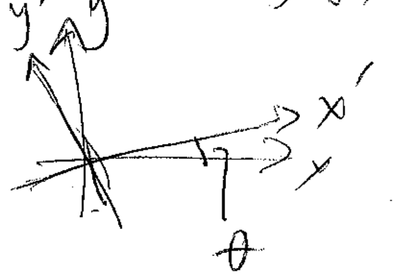
As $v \to c$, $\phi \to \infty$.

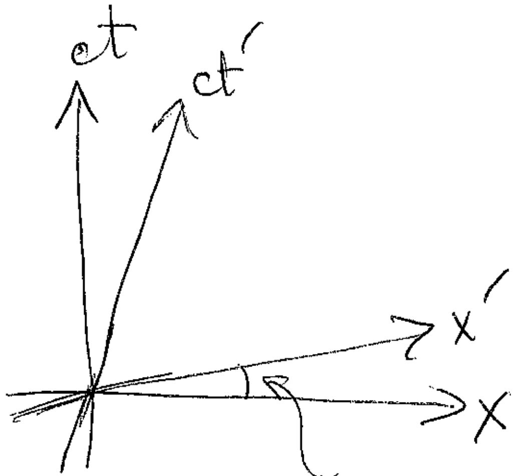
For small $\frac{v}{c}$, $\phi \approx \frac{v}{c}$

10d

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

Reminiscent of Rotations: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$





$\tan^{-1}\left(\frac{v}{c}\right) = \tan^{-1}(\tanh\phi)$

* LT has same form for <u>intervals</u>!

$$\Delta x' = \gamma_v(\Delta x - v\Delta t)$$

$$\Delta t' = \gamma_v\left(\Delta t - \frac{v}{c^2}\Delta x\right)$$

or

$$\begin{pmatrix} c\Delta t' \\ \Delta x' \end{pmatrix} = \begin{pmatrix} \gamma_v & -\gamma_v \frac{v}{c} \\ -\gamma_v \frac{v}{c} & \gamma_v \end{pmatrix} \begin{pmatrix} c\Delta t \\ \Delta x \end{pmatrix}$$

TIME DILATION

Consider Σ, Σ' ~~is~~ with standard boost between them. Consider two events.

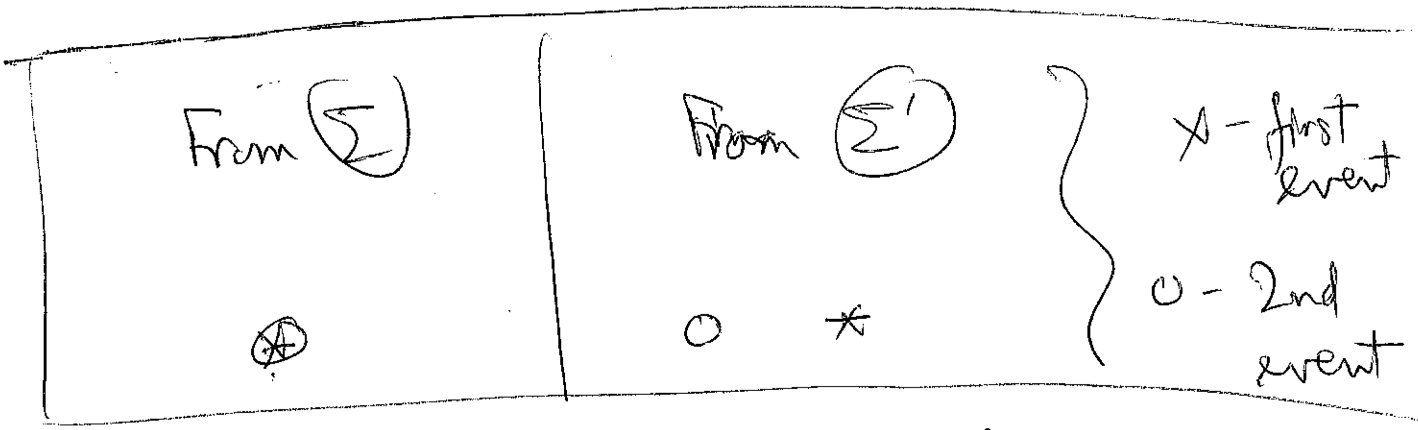
$$c^2(\Delta t)^2 - (\Delta x)^2 = c^2(\Delta t')^2 - (\Delta x')^2$$

If they happen at the same location in Σ? (if they are EQUILOCAL in Σ)?

Ex: ① clock at rest in Σ, two ticks

② Human at rest in Σ, two heartbeats

Then $\Delta x = 0$, and $|\Delta x'| = v|\Delta t'|$



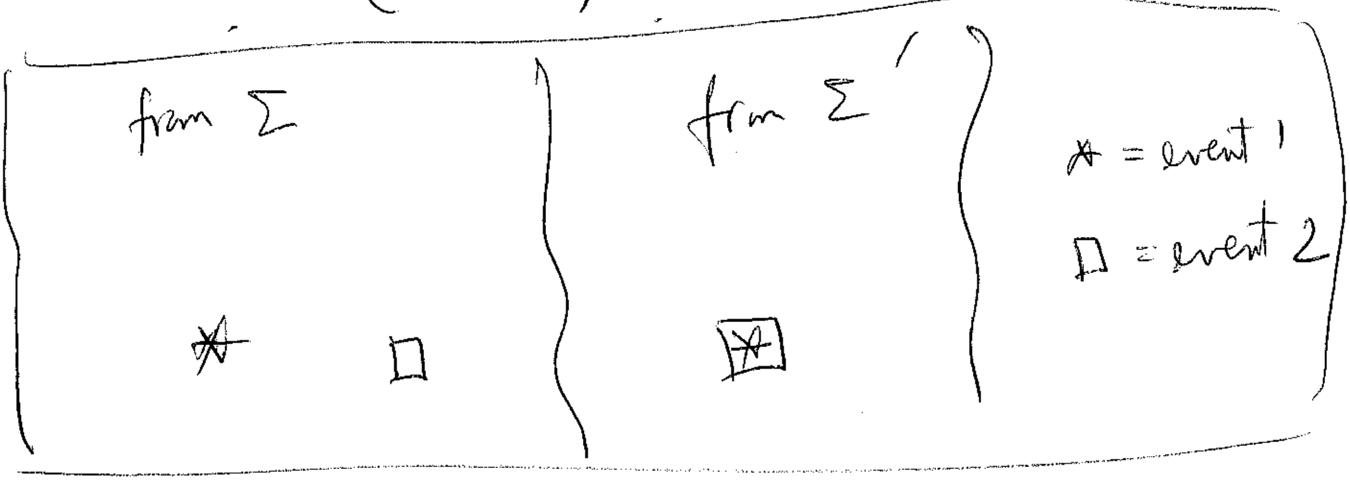
$$c^2(\Delta t)^2 = c^2(\Delta t')^2 - v^2(\Delta t')^2$$

$$\Rightarrow (\Delta t')^2 = \frac{c^2}{c^2 - v^2}(\Delta t)^2 = \gamma_v^2 (\Delta t)^2$$

$$\Rightarrow \quad \Delta t' = \gamma_v \Delta t \quad \text{if} \quad \Delta x = 0$$

(11b)

* What about two event equilocal in Σ'? $(\Delta x' = 0)$



spacetime interval $(\Delta s)^2$ is invariant:

$$c^2(\Delta t)^2 - (\Delta x)^2 = c^2(\Delta t')^2 - (\Delta x')^2 \Rightarrow$$

$$c^2(\Delta t)^2 - v^2(\Delta t)^2 = c^2(\Delta t')^2$$

$$\Rightarrow (\Delta t)^2 = \frac{(\Delta t')^2}{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow \Delta t = \gamma_v \Delta t' \quad \text{when} \quad \Delta x' = 0$$

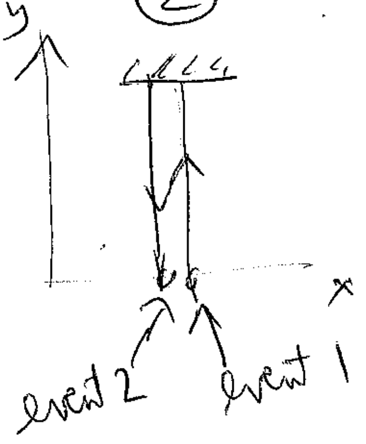
Contradictory? No:

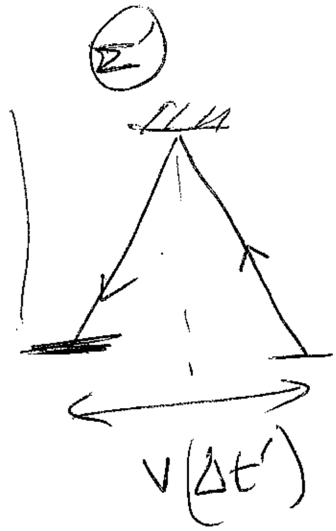
$\Delta t' = \gamma_v \Delta t$ when $\Delta x = 0$

$\Delta t = \gamma_v \Delta t'$ when $\Delta x' = 0$

Time dilation is RECIPROCAL.

* Time dilation from mirror expt.





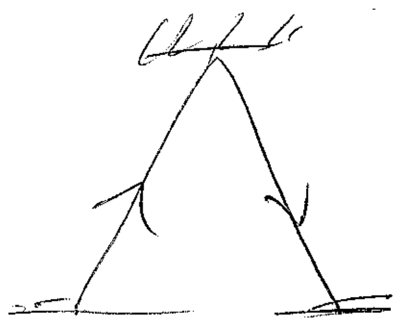
$$\Delta t = \frac{2h}{c} \qquad \Delta t' = \frac{2\sqrt{h^2 + \left(\frac{1}{2} v \Delta t'\right)^2}}{c}$$

$$\Delta t' = \frac{2h}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma_v \Delta t$$

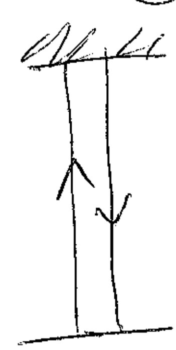
when $\Delta x = 0$

Reciprocal:

From Σ



From Σ'



Show $\Delta t = \gamma_v \Delta t'$ when $\Delta x' = 0$

11d

* PROPER TIME INTERVAL BETWEEN TWO EVENTS

= the time interval measured in the frame ~~for~~ which the two events are equilocal

* Alternate defn: If $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta \vec{r})^2$ is positive, then $\frac{\Delta s}{c}$ is the proper time interval.

→ The two defns are equivalent due to invariance of Δs.

* The proper age of an object / person = age measured from frame moving with the object/person.

* Proper lifetime of a radioactive p'cle = lifetime measured from a frame moving with the particle.

Radioactive particles in motion $\Rightarrow$ provide the most prominent experimental verification of time dilation.

* Time dilation :-

time interval measured in <u>other</u> frame

$= \gamma_v \times$ proper time interval

Other frame $\equiv$ frame where events are at different locations.

11f

* Time dilation from LT equations

Write LT eqs $\left[x' = \gamma_v (x - vt),\ t' = \gamma_v \left(t - \frac{v}{c^2} x\right) \right]$

in terms of intervals (TWO Events)

$$\Delta x' = \gamma_v (\Delta x - v \Delta t)$$
$$\Delta t' = \gamma_v \left(\Delta t - \frac{v}{c^2} \Delta x\right)$$

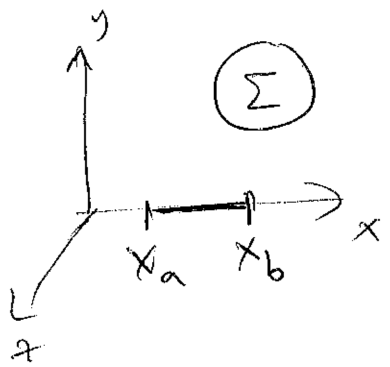
<u>Show</u>: If $\Delta x = 0$, then $\Delta t' = \gamma_v \Delta t$

If $\Delta x' = 0$, then $\Delta t = \gamma_v \Delta t'$ (a bit more work)

* LENGTH CONTRACTION

* ~~Using LT~~

Consider stationary rod in Σ.



Consider events (x_a, t_a), (x_b, t_b) (Σ-observer), $t_a = t_b$

Two ends measured at same time (But doesn't matter, as x_a, x_b don't change)

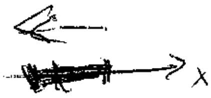
Length of rod from Σ: $L_0 = x_b - x_a$ IRRESPECTIVE of t_a, t_b

L_0 = "proper length" = length in rest frame of rod.

Length as seen from Σ'?

Ends should be measured at same time

$\longrightarrow$ same time for Σ'.



(x'_a, t'_a) and (x'_b, t'_b)

Now t'_a, t'_b does matter, $\rightarrow$ need $t'_a = t'_b$

Use inverse LT:

$$x_a = \gamma_v (x'_a + v t'_a)$$
$$x_b = \gamma_v (x'_b + v t'_b)$$

with $t'_a = t'_b$

(12b)

Subtract $\quad X_b - X_a = \gamma_v \underbrace{(X'_b - X'_a)}$

$$L_{(\Sigma')} = \frac{L_0}{\gamma_v}$$

↑ Length in Σ', because $t'_a = t'_b$

$$\text{Length in other frame} = \frac{\text{proper Length}}{\gamma_v}$$

PROPER LENGTH = length in frame for which object is stationary

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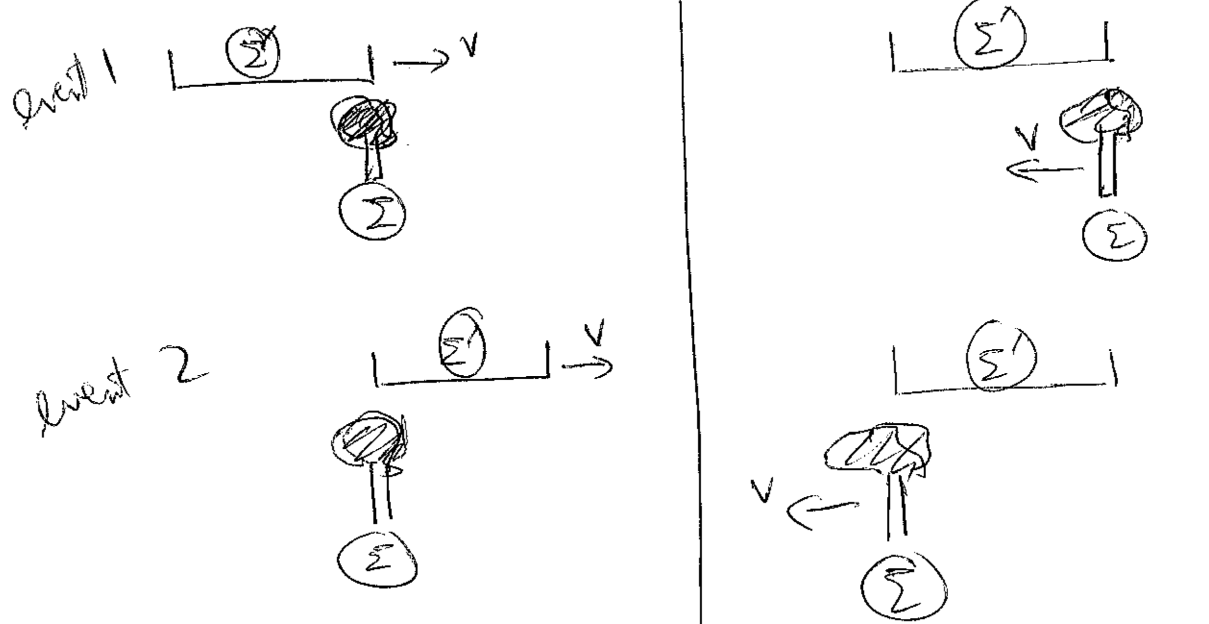
* Using time dilation

Consider — sled as frame Σ',
ground observer as frame Σ. Length of sled?

from Σ	from Σ'

event 1

event 2



$L = v\Delta t \qquad\qquad L' = v\Delta t'$

Proper time interval ? Δt (events equilocal in Σ) So $\Delta t' = \gamma_v \Delta t$

$$\Rightarrow \quad L = v\Delta t = \frac{v\Delta t'}{\gamma_v} = \frac{L'}{\gamma_v}$$

L' is proper length; sled is stationary in Σ'.

Length contraction obtained from time dilation

LOSS of SIMULTANEITY

Two distant events being simultaneous in Σ doesn't mean they are simultaneous in Σ'.

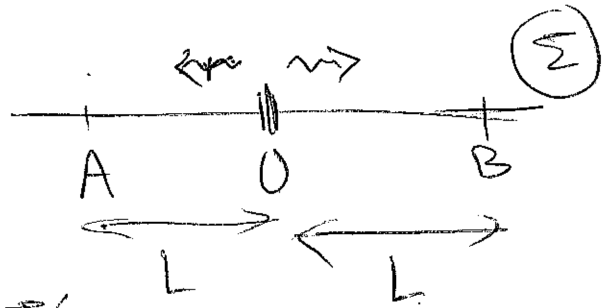
$$(x_a, t_a) \ \& \ (x_b, t_b)$$

$$t_b' - t_a' = \gamma_v\left(t_b - \frac{v}{c^2}x_b\right) - \gamma_v\left(t_a - \frac{v}{c^2}x_a\right)$$

$$= \gamma_v(t_b - t_a) - \frac{v}{c^2}\gamma_v(x_b - x_a)$$

$\Rightarrow t_a = t_b$ does not imply $t_a' = t_b'$

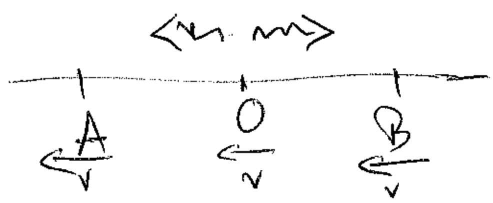
* Demonstration with Light-pulses



Pulse reaching A & pulse reaching B are simultaneous events in Σ.

$\tau_A = \tau_B = \frac{L}{c}$

In Σ'



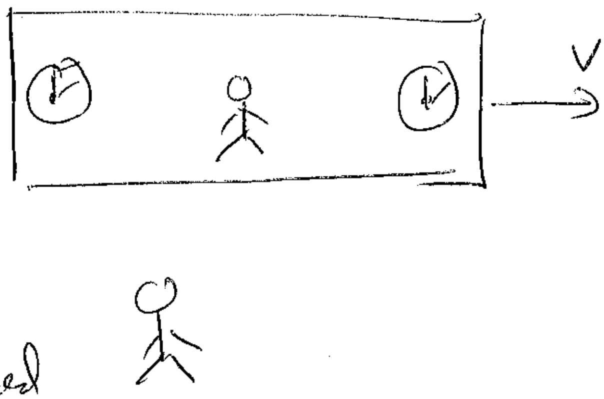
Pulse reaches A before pulse reaches B.

Exercise: find τ_A', τ_B'

(13b)

Example:

"REAR CLOCK AHEAD"



Clocks synchronized on train frame, at different locations, will not appear synchronized on ground frame.

Clock at back of Wagon appears to be "ahead", by Lv/c^2.

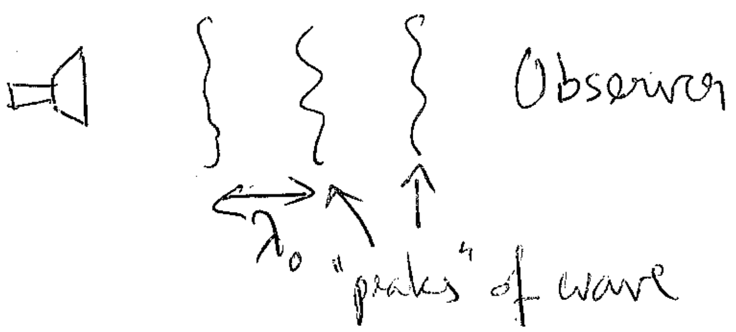
* Doppler effect for EM waves (Relativistic Doppler effect)

Review

Doppler effect for sound : related to motion w.r.t. medium.

Sound wave velocity : W relative to medium.

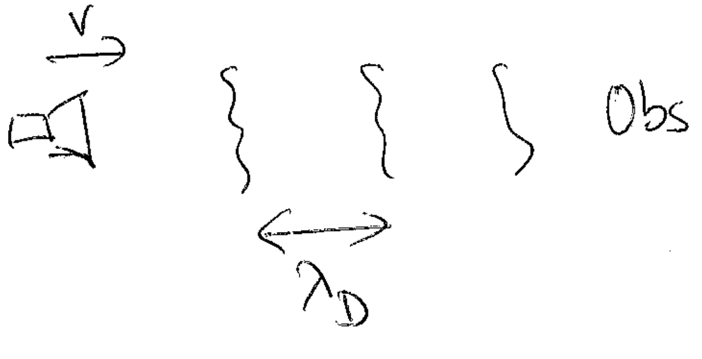
① No motion



Frequency
$$f_0 = \frac{W}{\lambda_0}$$

Time interval between Two peaks $= \frac{1}{f_0} = \frac{\lambda_0}{W}$

② Moving source



Distance betn successive peaks

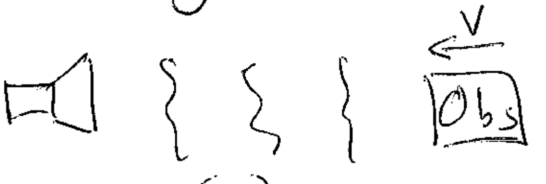
$$\lambda_D = \lambda - V\left(\frac{1}{f_0}\right)$$

$$= \frac{W}{f_0} - \frac{V}{f_0}$$

$$\Rightarrow f_D = \frac{W}{\lambda_D} = \frac{W}{(W-V)/f_0} = f_0 \frac{1}{1-V/W}$$ ← Doppler-shifted freq.

(146)

(2) Moving observer



$\lambda = \frac{W}{f_0}$

Speed of soundwave relative to observer $= W + V$

$$f_D = \frac{W+V}{\lambda_0} = \frac{W+V}{W/f_0} = f_0\left(1+\frac{V}{W}\right)$$

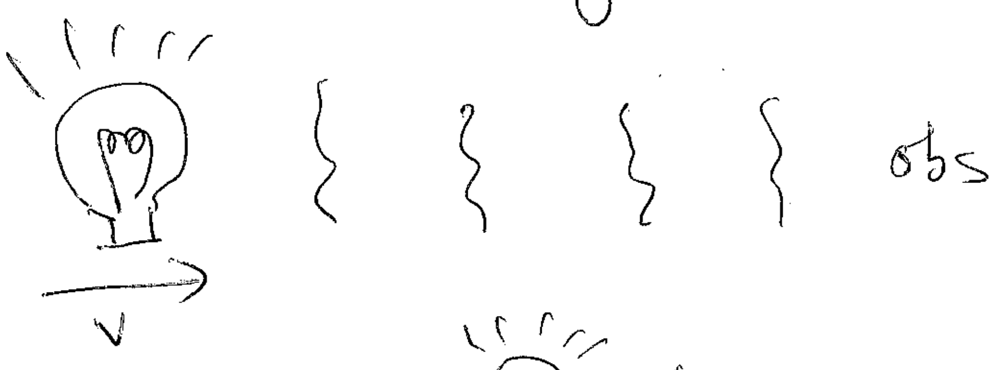
Doppler shift for sound:

$$f_D = \frac{f_0}{1-V/W} \quad \text{or} \quad f_0\left(1+\frac{V}{W}\right)$$

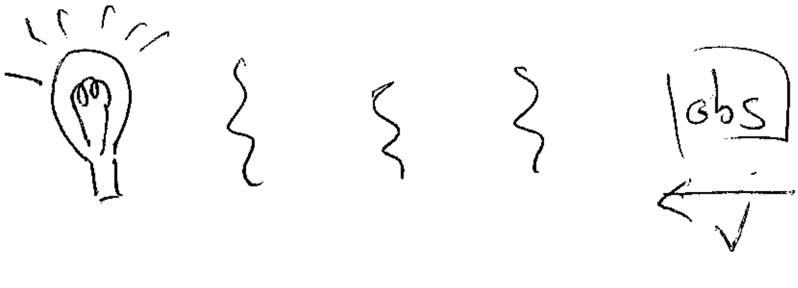
moving source   moving observer.

* Now relativistic (light wave)

No medium. No distinction between moving source & moving observer.



Equivalent:



Two effects combine:

(1) Time between successive peak emission:

proper time $\tau_0 = \frac{1}{f_0}$ (according to source)

According to observer: $\tau = \gamma_v \tau_0 = \frac{\gamma_v}{f_0}$

(2) From observer's frame:

Source travels distance $v\tau$ in one period

$$\lambda_D = \lambda - v\tau \quad \text{(distance between peaks)}$$

$$= c\tau - v\tau = (c-v)\tau$$

$$f_D = \frac{c}{\lambda_D} = \frac{c}{(c-v)\tau} = \frac{1}{\left(1-\frac{v}{c}\right)}\frac{f_0}{\gamma_v}$$

$$\boxed{f_D = \sqrt{\frac{1+v/c}{1-v/c}}\, f_0} \qquad \boxed{f_D = \left(\frac{1}{\gamma_v}\right)\left(\frac{1}{1-\frac{v}{c}}\right) f_0}$$

effect 1 effect 2

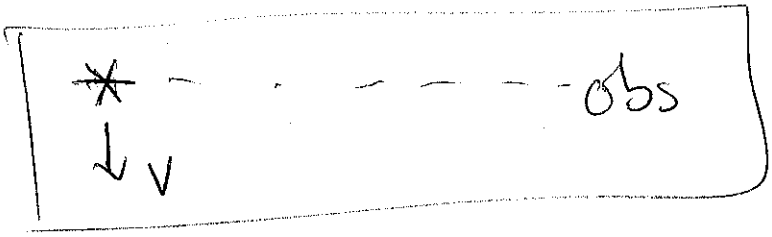
* Important in astrophysics/cosmology

Source moving toward us → increase of observed frequency → "blue shift" of spectral lines

Source moving away → "red shift" of spectral lines

14d

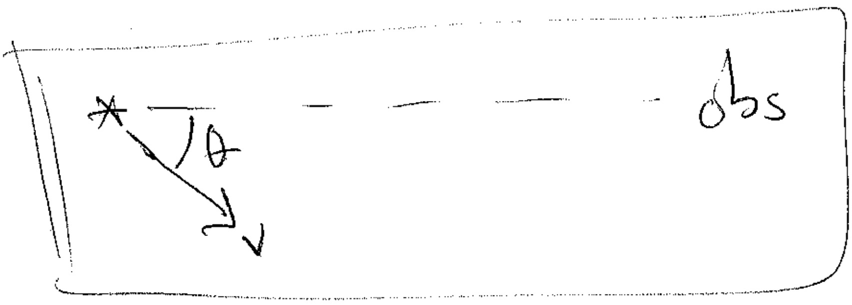
* TRANSVERSE DOPPLER EFFECT



Only time dilation effect

$$f_D = \frac{f_0}{\gamma_v} = f_0\sqrt{1-\frac{v^2}{c^2}}$$

LONGITUDINAL + TRANSVERSE



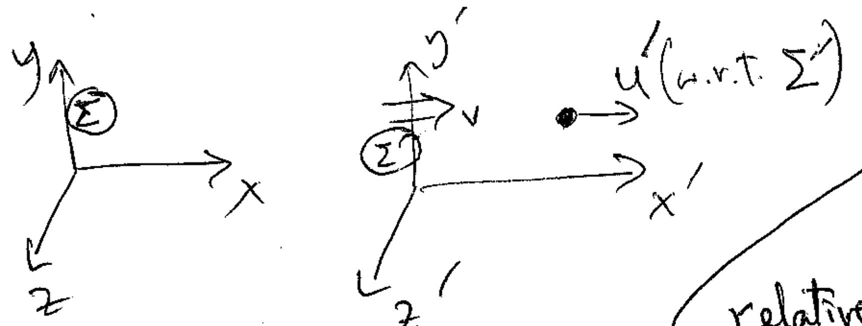
time dilation factor γ_v

+

effective wavelength reduced, but only due to longitudinal component.

$$f_D = f_0 \frac{\sqrt{1-v^2/c^2}}{1-\frac{v}{c}\cos\theta}$$

* Relativistic addition of velocities (longitudinal)



Speed of object relative to Σ is NOT $u'+v$

Rapidities add simply, velocities don't.

$u' = \dfrac{\Delta x'}{\Delta t'}$ we want to find $u = \dfrac{\Delta x}{\Delta t}$

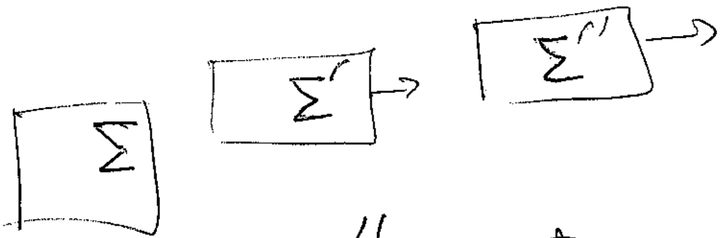
$$u = \frac{\Delta x}{\Delta t} = \frac{\gamma_v(\Delta x' + v\,\Delta t')}{\gamma_v(\Delta t' + \frac{v}{c^2}\Delta x')}$$

Question: which EVENTS are we using LT for?

$$= \frac{\frac{\Delta x'}{\Delta t'} + v}{1 + \frac{v}{c^2}\frac{\Delta v'}{\Delta t'}} = \frac{u' + v}{1 + \frac{v u'}{c^2}} \neq u' + v$$

<u>Note</u> if velocities are not constant, would need $u' = \lim_{\Delta t' \to 0} \dfrac{\Delta x'}{\Delta t'}$ ..... etc.

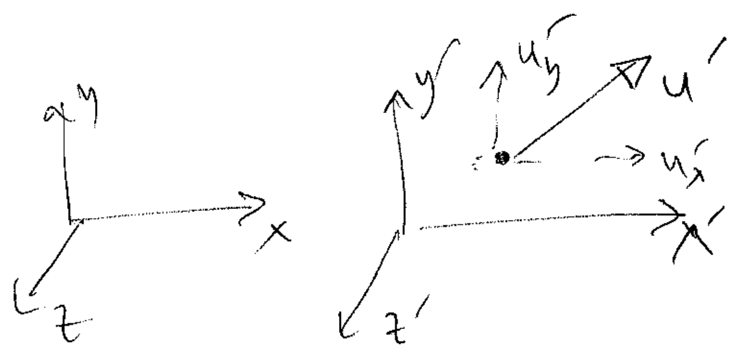
<u>Another setup</u>



Σ' moves at v_1 w.r.t. to Σ, Σ'' moves at v_2 w.r.t. Σ'. → What's the speed of frame Σ'' w.r.t. Σ?

Answer: Σ'' moves at speed $\dfrac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$ with respect to Σ. Assuming v_1, v_2 are in SAME DIRECTION

Addition of velocities: transverse component



$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$

$$u_y = \frac{\Delta y}{\Delta t} = \frac{\Delta y'}{\gamma_v\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)}$$

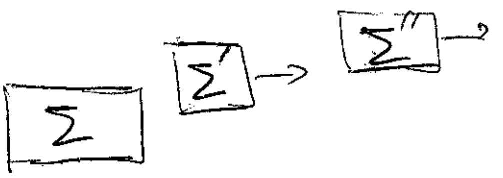
NOTE γ_v not γ_u or $\gamma_{u'}$

$$= \frac{\Delta y' / \Delta t'}{\gamma_v\left(1 + \frac{v}{c^2}\frac{\Delta x'}{\Delta t'}\right)} = \frac{u_y'}{\gamma_v\left(1 + \frac{v u_x'}{c^2}\right)}$$

$$\boxed{u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} \quad u_y = \frac{u_y'}{\gamma\left(1 + \frac{u_x' v}{c^2}\right)}, \quad u_z = \frac{u_z'}{\gamma\left(1 + \frac{u_x' v}{c^2}\right)}}$$

Contrast with Galilean: $u_x = u_x' + v, \; u_y = u_y', \; u_z = u_z'$

* Relativistic addition formula ensures that all speeds remain below c:



Σ' moves at $0.9c$ w.r.t. Σ

Σ'' moves at $0.9c$ w.r.t. Σ'

Speed of Σ'' relative to Σ?

NOT $\boxed{0.9c + 0.9c = 1.8c}$ X

instead $$\frac{0.9c + 0.9c}{1 + \frac{(0.9c)(0.9c)}{c^2}} = \frac{1.8c}{1 + (0.9)^2} = \frac{1.8c}{1.81} < c$$

---

* Yet Another setting

A $\overrightarrow{|V_A|}$ B $\overleftarrow{|V_B|}$

w.r.t. GROUND

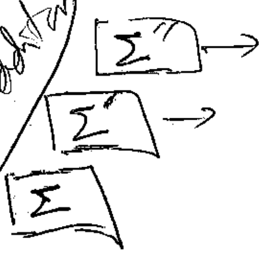
$|V_A|$, $|V_B|$ are speeds, velocities are $+|V_A|$ & $-|V_B|$

Speed of A relative to B? (or B relative to A)

$$= \frac{|V_A| + |V_B|}{1 + \frac{|V_A V_B|}{c^2}}$$

Again, this is $< c$ even though $|V_A| + |V_B| > c$

(24) Rapidity addition



Velocity of Σ' w.r.t. S $\quad v_1$

Rapidity of Σ' w.r.t. S $\quad \phi_1$

$$\tanh \phi_1 = v_1/c$$

Show: $\phi_1 = \frac{1}{2}\ln\left(\frac{c+v_1}{c-v_1}\right)$

Vel. of Σ'' relative to Σ' is v_2

Rapidity of Σ'' w.r.t. Σ' is ϕ_2, $\quad \tanh\phi_2 = \frac{v_2}{c}$

Velocity of Σ'' w.r.t Σ is v

Rapidity of Σ'' w.r.t. Σ is ϕ $\left[\tanh\phi = \frac{v}{c}\right]$

We've learned $\quad v = \dfrac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \quad \cdots$ (1)

and, using matrix represent^n of LT, we find $\quad \phi = \phi_1 + \phi_2 \quad \cdots$ (2)

Exercise: Derive (1) from (2)

using $\tanh(\phi_1 + \phi_2) =$

$$= \frac{\tanh\phi_1 + \tanh\phi_2}{1 + \tanh\phi_1 \tanh\phi_2}$$

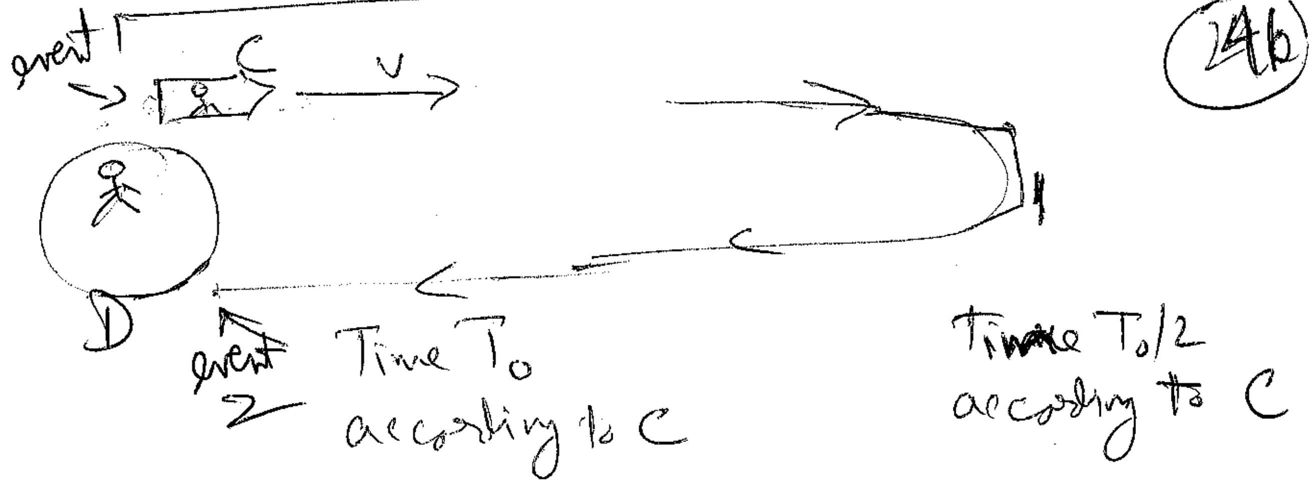
★ Subtracting velocities is confusing.

My suggestion: use <u>SPEEDS</u> $|v_1|$ & $|v_2|$

If to be added, $\dfrac{|v_1| + |v_2|}{1 + \frac{|v_1 v_2|}{c^2}}$

If to be subtracted, $\dfrac{|v_1| - |v_2|}{1 - \frac{|v_1 v_2|}{c^2}}$

* TWIN PARADOX



Who is older, when they meet again?

Both C & D see events at same position (own position).

If C's time is "proper" $\longrightarrow$

D's age = C's age $\times \gamma$

If D's time is "proper" $\longrightarrow$

C's age = D's age $\times \gamma$

$\longrightarrow$ "paradox"

(24c)

<u>Resolution</u>: perspectives not equivalent.

C underwent <u>acceleration</u>.

C was not in a <u>single</u> inertial frame.

Correct answer: calculate piece by piece,

event 1 — C → event 3 — $T_0/2$

D*

C's time is "proper".

For D: C's journey takes $\gamma_v\left(\frac{T_0}{2}\right)$

← C

D

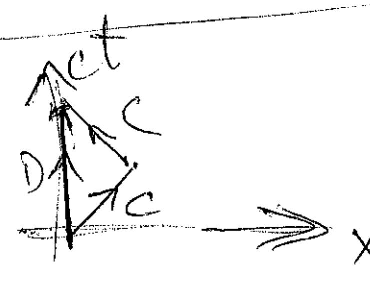
C's time is proper.

For D, C's journey takes $\gamma_v\left(\frac{T_0}{2}\right)$

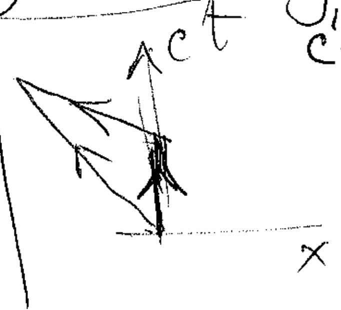
C ages by T_0

D ages by $\gamma_v\left(\frac{T_0}{2}\right) + \gamma_v\left(\frac{T_0}{2}\right) = \gamma_v T_0$

D is older when they meet again.



D's frame shown with $ct \perp x$

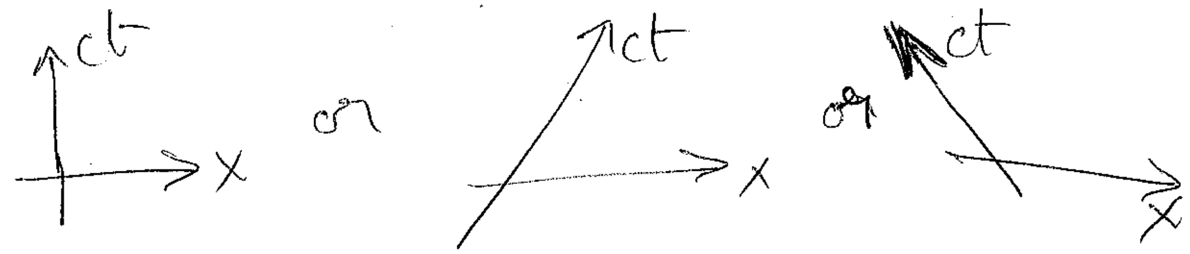


C's outgoing frame has $ct \perp x$

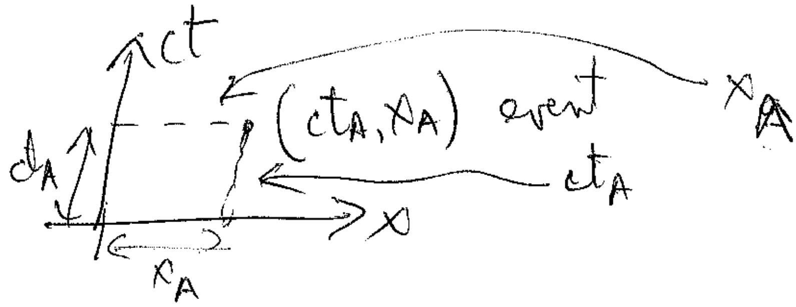
* MINKOWSKI (SPACETIME) DIAGRAMS

Representing The class of events (= one time coordinate + one [two] space coordinates) on two [3]-dimensional drawings.

* Choose two non-parallel directions as x and ct directions. (Don't have to be ⊥)

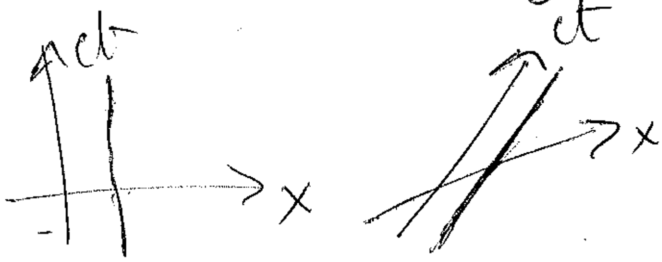


*



choice of axis ⟺ choice of frame.

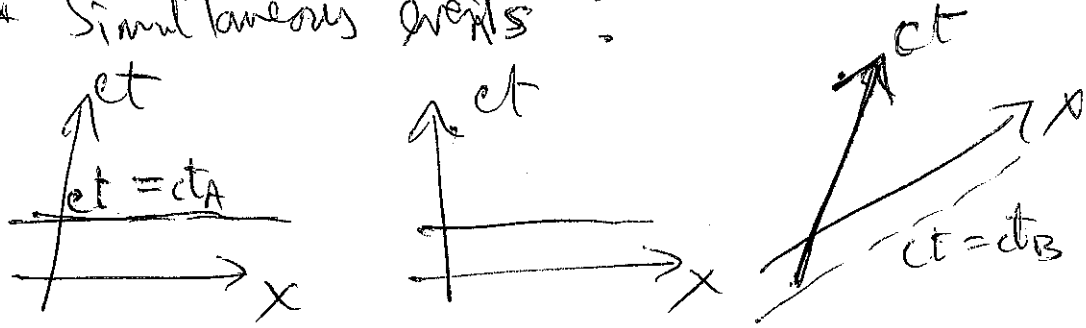
* Events corresponding to stationary object:



PARALLEL TO ct-axis.
(x-coordinate = constant)

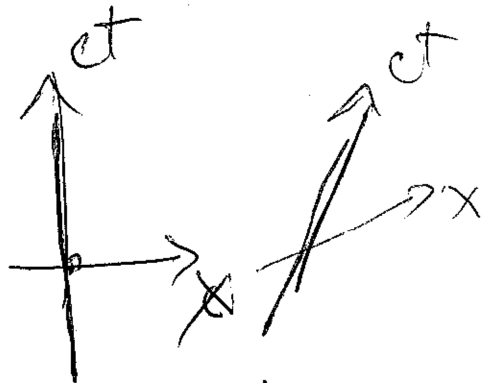
M.D 2

* Simultaneous events :

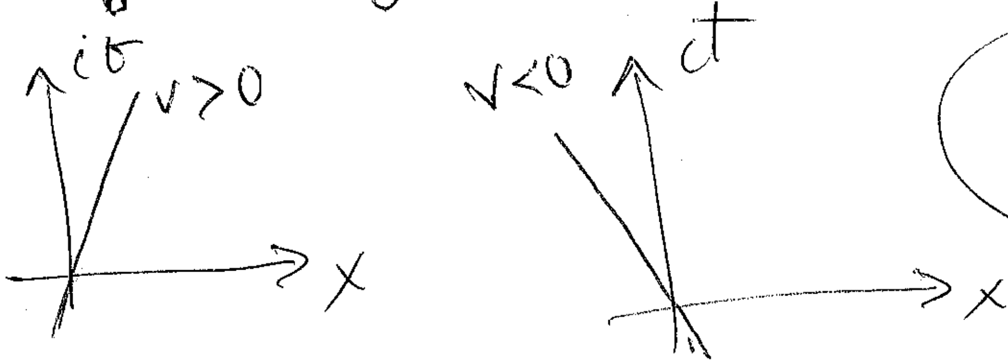


line Parallel to x-axis

* Object sitting at origin :



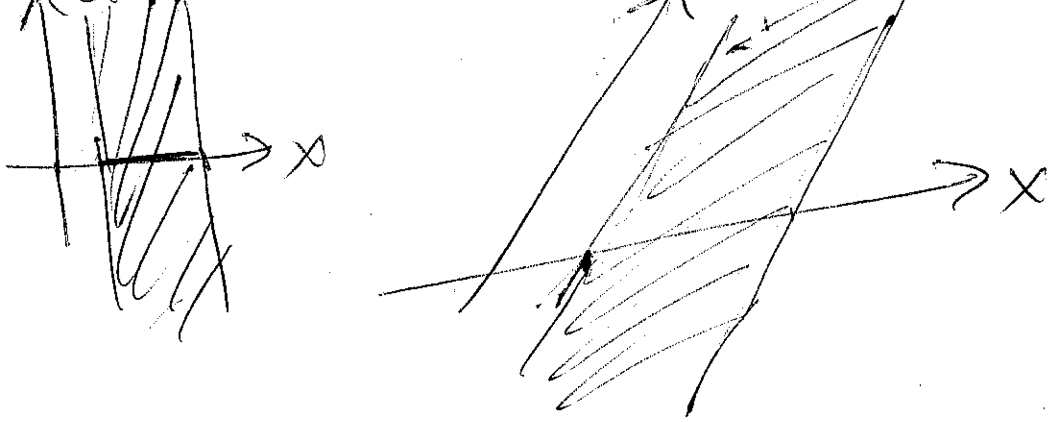
* Object moving with velocity ... $\frac{dx}{dt} = v$



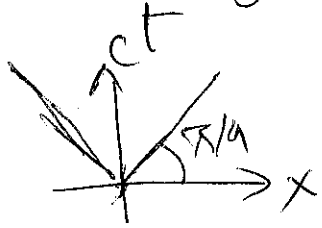
WORLD LINE

If $ct \perp x$, speed $= \frac{1}{\text{slope}}$

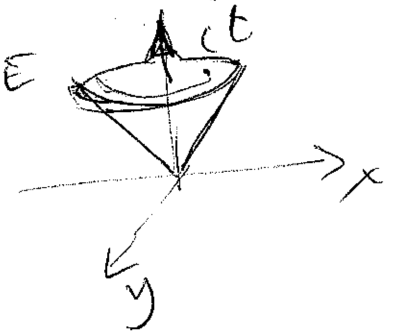
* Stick, stationary.



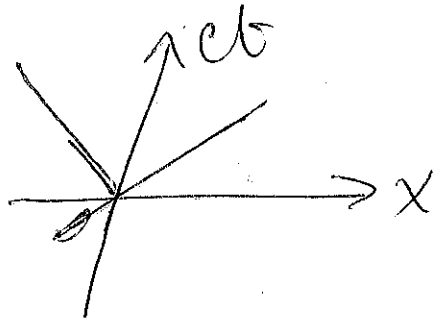
* A light pulse starting at $(ct, x) = 0$:



A LIGHT CONE

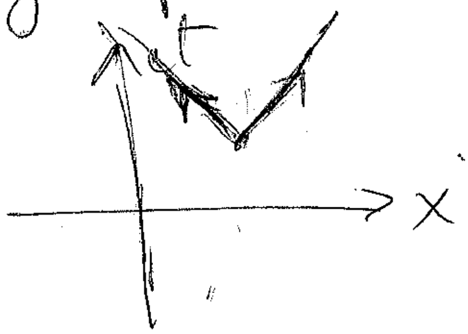


If $ct \neq x$,
world-line of a photon still BISECTS ct & x directions.

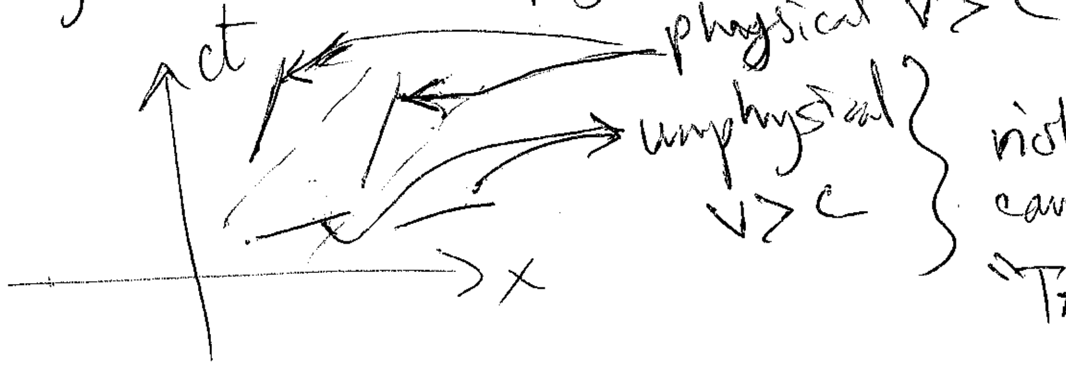


$\boxed{\text{SHOW!}}$

Light pulse starting at x_0 at time t_0:



Physical & un-physical world-lines:



M.D4

9

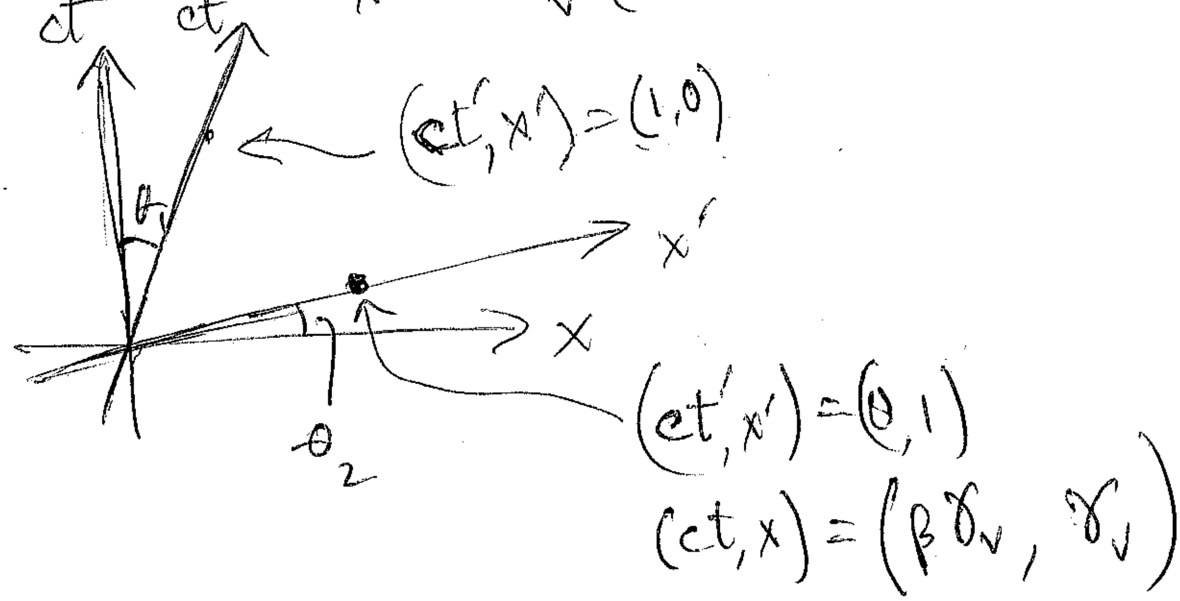
Two frames Σ, Σ', coincide at $t = t' = 0$, Σ' has ~~rel~~ relative velocity v.

* (ct, x) & (ct', x') coincide
$= (0,0)$ $= (0,0)$

* ct' axis: when $(ct', x') = (1, 0)$,

we get $ct = \gamma_v\left(ct' + \frac{v x'}{c}\right) = \gamma_v$

$x = \gamma_v\,(x' + v t') = \gamma_v \frac{v}{c} = \beta_v \gamma_v$



$$\boxed{\theta_1 = \theta_2 = \tan^{-1}\beta = \tan^{-1}\frac{v}{c}}$$

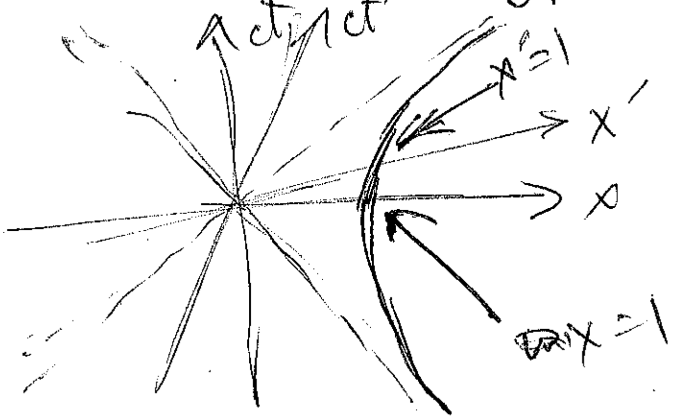
One ct' unit is $\sqrt{\gamma_v^2 + \beta^2\gamma^2}$ in ct-units

$$\frac{\text{One } ct' \text{ unit}}{\text{One } ct \text{ unit}} = \gamma_v\sqrt{1+\beta^2} = \sqrt{\frac{1+\beta^2}{1-\beta^2}} = \sqrt{\frac{c^2+v^2}{c^2-v^2}}$$

(M.D.T)

* Similarly, $\dfrac{\text{one } x'\text{-unit}}{\text{one } x\text{-unit}} = \sqrt{\dfrac{1+v^2/c^2}{1-v^2/c^2}}$

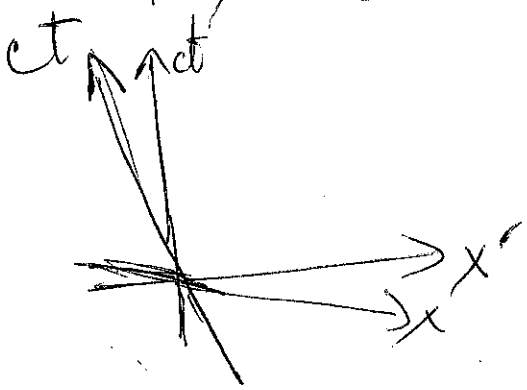
* <u>Calibration hyperbola:</u> $c^2t^2 - x^2 = c^2t'^2 - x'^2$



Same magnitudes of x, x', x'' in different frames fall on a hyperbola $c^2t^2 - x^2 = -a^2$

Same magnitudes of ct, ct', ct'' in different frames fall on a hyperbola: $c^2t^2 - x^2 = +b^2$

* If (ct', x') chosen perpendicular,

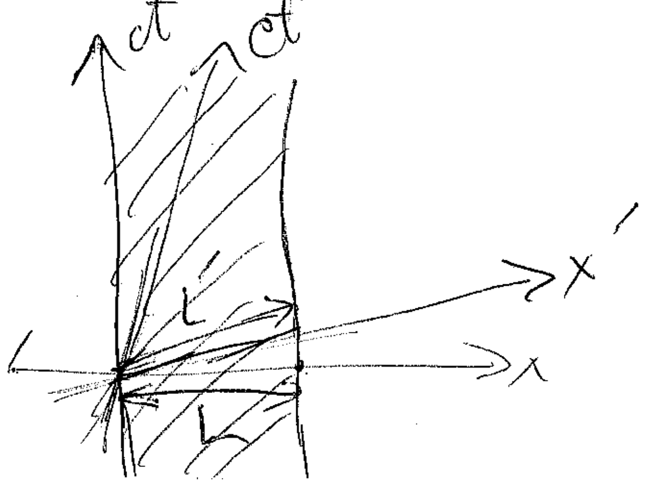


* $\boxed{\text{Length contraction}}$

Rod stationary in Σ

Length L (in x-units) in Σ

Length L' (in x'-units) in Σ'



M.D.C

$$L' = \frac{L}{\cos\theta} \quad \text{in } x\text{-units}$$

$$= \frac{L}{\cos\theta} \Big/ \sqrt{\frac{1+\beta^2}{1-\beta^2}} \quad \text{in } x' \text{ units}$$

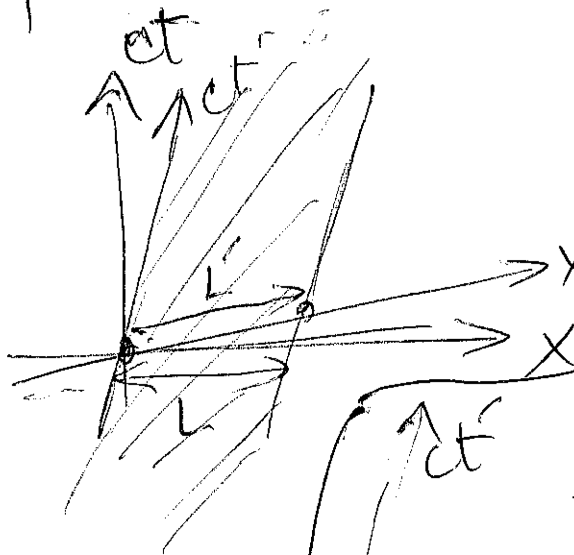
$$= \frac{L}{\cos\theta}\sqrt{\frac{1-\beta^2}{1+\beta^2}} \qquad \left\{ \begin{aligned} \frac{1}{\cos\theta} &= \sqrt{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} \\ &= \sqrt{1+\tan^2\theta} \\ &= \sqrt{1+\beta^2} \end{aligned} \right.$$

$$= L\sqrt{1+\beta^2}\,\frac{\sqrt{1-\beta^2}}{\sqrt{1+\beta^2}}$$

$$= L\sqrt{1-\beta^2} = \frac{L}{\gamma_v}$$

Length contraction formula!

If stick is at rest in Σ' frame,



Show that

$$L = \frac{L'}{\gamma}$$

EXERCISE