Due on Friday, October 2nd, 8PM.

To be scanned and uploaded as a SINGLE PDF FILE.

The questions indicated as [**SELF**] are for self-study and additional exercise. They will not be marked. (No need to include them in the work that you scan in.) You are however strongly advised to work through them.

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1. (a) [**2+2 pts.**] Look up and report the definitions of
   - the hermitian conjugate of a matrix;
   - a hermitian or self-adjoint matrix.
   (Try any textbook on quantum mechanics or linear algebra, wikipedia,....)

   (b) [**4×2 pts.**] Find out whether the following matrices are hermitian or not. This will involve calculating the hermitian conjugate of each matrix and comparing with the matrix itself.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 1 & i \\ 1 & 1 & 0 \\ -i & 0 & 5 \end{pmatrix}$$

   (c) [**SELF**] Look up Pauli matrices. One of the above matrices is a Pauli matrix. Which one, and which Pauli matrix? What do Pauli matrices represent?

2. The equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) \;+\; V(x)\psi(x) \;=\; E\psi(x)$$

   is the (time-independent) Schroedinger equation for a single particle in a potential $V(x)$. Solutions of this equation are stationary states of a particle in the potential $V(x)$. Note that E is a constant (independent of x) giving the energy of the stationary state.

   (a) [**2 pts.**] Write down the time-independent Schroedinger equation for the case of no potential, $V(x) = 0$.

   (b) [**6 pts.**] Show that the function $e^{-x^2/2\sigma^2}$ is not a solution of this equation for $V = 0$. You can do this by substituting $\psi(x) = e^{-x^2/2\sigma^2}$ on both sides of the equation, and then showing that the two sides cannot be equal at all x.

(c) **[5 pts.]** Show that the function $\sin(kx)$ is a solution of this equation with $V = 0$, for a specific value of the energy E. Express this value of E in terms of k, $\hbar$ and m.

(d) **[SELF]** Is $\cos(kx)$ also a solution? Corresponding to the same E or different E?

(e) **[5 pts.]** Is the function e^{ikx} also a solution? Corresponding to the same E or different E?

(f) **[SELF]** If $f_1(x)$ and $f_2(x)$ are both solutions, corresponding to the same value of E, then show that any linear combination $c_1 f_1(x) + c_2 f_2(x)$ is also a solution.
Does this work if $f_1(x)$ and $f_2(x)$ are both solutions, but correspond to different values E_1 and E_2?

(g) **[SELF]** Can e^{ikx} be expressed as a linear combination of $\sin(kx)$ and $\cos(kx)$?

3. (a) **[2 pts.]** Now write down the time-independent Schroedinger equation for the case of a harmonic potential $V(x) = \frac{1}{2}m\omega^2 x^2$.

(b) **[7 pts.]** Show that the gaussian function $e^{-x^2/2\sigma^2}$ is a solution of this equation, for specific values of σ and E.
Indicate clearly the constant values of σ and E for which the gaussian function is a solution. (i.e., express σ and E as functions of $\hbar$, m and ω, such that $e^{-x^2/2\sigma^2}$ is a solution)

(c) **[3 pts.]** Sketch plots of $e^{-x^2/2\sigma^2}$ as a function of x, for $\sigma = 1$ and $\sigma = 2$. Both curves should be on the same graph. Which property of the gaussian curve does σ represent?

(d) **[3 pts.]** Does σ increase or decrease with increasing strength of the trapping potential (increasing ω)? Explain why your answer makes sense physically.

(e) **[3 pts.]** Look up the expression for the energy eigenvalues (energy levels) for the quantum harmonic oscillator. (E.g., on the wikipedia page on "quantum harmonic oscillator".) Explain how your answer for E matches one of the energy eigenvalues.

(f) **[2 pts.]** Now imagine that the harmonic potential is not centered at zero, but at a, i.e., $V(x) = \frac{1}{2}m\omega^2(x-a)^2$. Using physical arguments, write down the corresponding gaussian solution to the time-independent Schroedinger equation.