Due on Friday, October 9th.

The questions indicated as [**SELF**] are for self-study and additional exercise: no need to submit your work on these questions.

− − − − − − − − − − − − − − ⋆ − − − − − − − − − − − − −

1. Consider a two-state system, so that the Hilbert space is two-dimensional, i.e., the Hamiltonian is a 2×2 matrix and the wavefunction is a vector with two complex components. Let's denote the wavefunction as

$$\psi(t) \;=\; \begin{pmatrix} u(t) \\ v(t) \end{pmatrix}$$

We will consider a relatively simple (diagonal) Hamiltonian:

$$H \;=\; \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix}$$

   (a) [**6 pts.**] Using the time-dependent Schroedinger equation, write down differential equations for the functions $u(t)$ and $v(t)$.

   (b) [**7 pts.**] Solve the differential equations, so that $u(t)$ and $v(t)$ are expressed in terms of the initial values $u(0)$ and $v(0)$.

   (c) [**5 pts.**] Show that the norm of the wavefunction, $|u(t)|^2 + |v(t)|^2$, is independent of time.

   (d) [**3 pts.**] Provide an expression for $\psi(t)$, in the case that the initial state is

$$\psi(0) \;=\; \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

2. Electromagnetic waves as particles:

   (a) Visible light has a wavelength in the range of 400-700 nm.

      i. [**SELF**] What is the frequency range of visible light?

      ii. [**4 pts.**] What is the energy range of a photon of visible light?

   (b) [**4 pts.**] A kitchen microwave operates at frequency 2.5 GHz and at power 750 Watts. How many photons does the microwave emit per second?

(c) **[SELF]** How many microwave photons [from the previous question] are required to heat up a 0.2-liter glass of water by 10°C? The heat capacity of water is roughly 4185 J/(kg·K), and a liter of water weighs roughly 1kg.

(d) **[SELF]** A low-power laser (10^{-3} Watts) gives out light of wavelength 633 nm. How many photons does it emit per second?

3. Consider the wavefunction of a single particle confined to the positive half of a one dimensional line, i.e., the wavefunction vanishes in the region $x < 0$. The wavefunction is known to be

$$\psi(x) = \begin{cases} Ne^{-x/\lambda} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

(a) **[8 pts.]** Calculate the constant N so that the wavefunction is normalized. Keep your answer general: do not assume N to be real.

(In case normalization has not yet been covered in class: please look it up. E.g., you could try googling "wavefunction normalization".)

(b) **[10 pts.]** Calculate the probablity of finding the particle inside the region $x \in [\lambda, 2\lambda]$. Reminder: $|\psi(x)|^2$ is the 'probability density'.

(c) **[2+1 pts.]** Sketch a plot of the probability density, $|\psi(x)|^2$, as a function of position x. Show the region on this plot whose area represents the probability you have calculated above.
(Hand-drawn sketch, please, no computer printouts.)

(d) **[SELF]** The given wavefunction is not acceptable as a stationary solution of the time-indepenedent SE. Why?

4. **[SELF]** We need to learn "bra-ket" notation. Please read through Nash's notes, sections 1 to 4. It will be assumed that you have learned linear algebra previously; it won't be repeated in class.