Due Wednesday November 18th, 2020.

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1. **The unix/linux command line**

   (a) Explain what the following commands do. (Hint: look up 'piping'.)

      (i) `ps -f | grep firef`

      (ii) `grep includegraphics manuscript.tex | wc`

   (b) You can look at file permissions using "ls -l". You find out that one of your files has permissions

   `rwxr-xr--`

   Explain what this means.

   (c) What command would I use to change permissions on a file so that only I can read it?

2. **Newton's method for finding a minimum**

   (a) Consider the single-variable function

   $$f(x) = 1 + x^{4/3}$$

   which has a single minimum at $x = 0$. Write down the iteration equation for Newton' method.

   The iterations do not converge. By sketching the derivative $f'(x)$, explain graphically why.

   (b) We want to find the location of the minimum of the function of 2 variables

   $$f(x, y) = x^2 + xy + y^2 + 3x$$

   using Newton's method. Set up the iteration equations. You will have to invert a 2×2 matrix.

   Because the function is quadratic, the iteration should converge in a single step. Starting from any initial point (you can choose), show that the iteration converges in a single step.

(c) I have a multidimensional minimization problem and calculating the inverse of the Hessian ($\mathbb{H}^{-1}$) is too hard for me. As a very crude approximation, I replace $\mathbb{H}^{-1}$ by $\lambda\mathbb{I}$, where λ is a small number and $\mathbb{I}$ is the unit matrix. The resulting algorithm is then equivalent to a widely used algorithm. Which one?

3. **Statistical Mechanics**

Note: Some of the following is worked out in the student projects linked to on the webpage, under "Ising model".

Remember $Z = \sum_\alpha e^{-\beta E_\alpha}$, where E_α is the energy of the configuration α. At thermal equilibrium, each configuration appears with probability $P_\alpha = \frac{1}{Z} e^{-\beta E_\alpha}$.

(a) Show that the system energy and its square has expectation values

$$\langle E \rangle = \frac{1}{Z}\frac{\partial Z}{\partial \beta} = \frac{\partial}{\partial \beta} \ln Z \qquad \text{and} \quad \langle E^2 \rangle = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2}$$

(b) Hence show that the specific heat

$$C_v = \frac{\partial \langle E \rangle}{\partial T}$$

can be written as $C_v = \beta^2 \Big(\langle E^2 \rangle - \langle E \rangle^2 \Big)$. I am probably setting the Boltzmann constant to unity here.

(c) Perform the corresponding derivation for the magnetic susceptibility. You might have to remember that the configuration energy contains a term $-BM_\alpha$, where B is the magnetic field (often written as H) and M_α is the total magnetization of configuration α.