MP468C — Computational Physics 2 — Lab 03

1. Use Monte Carlo integration to compute the volume and centre of mass of the 3-dimensional homogeneous body (torus section) defined by

$$x^2 + \left(\sqrt{y^2 + z^2} - 5\right)^2 < 4, \quad y < 1, \quad z > 2.$$

The volume V and center of mass $\vec{R}$ are given by the integrals

$$V = \int\int\int dx\, dy\, dz\,, \qquad \vec{R} = \frac{1}{V}\int\int\int \vec{r}\, dx\, dy\, dz\,.$$

Hint: The volume can be computed in a similar way to how we computed π, by noting that the body is enclosed by the rectangular box

$$-2 < x < 2, \quad -7 < y < 1, \quad 2 < z < 7.$$

Continued on next page....

2. Generate a Markov chain of real numbers distributed according to the exponential distribution

$$P(x) = Ne^{-|x|}$$

where N is a normalization factor. The support is the entire real line, not just the positive half.

Given a value x_i, choose a proposal for the next value **uniformly** from the interval $(x_i - \delta, x_i + \delta)$. Then accept or reject this proposal according to the Metropolis criterion.

Values near each other in the Markov chain tend to be correlated. It is therefore common not to save every value but every m-th value. I suggest m around 100.

Try $\delta \sim 10^{-1}$, $\delta \sim 10^{0}$, $\delta \sim 10^{1}$.
Record the acceptance rate (efficiency) in each case.

I experimented a bit and I think that, which features of the distribution you obtain more easily, depends on the value of δ. Describe and explain your findings regarding the effect of δ.

Question: To check for exponential behavior, might it be helpful to plot one of the axes on a logarithmic scale? Which axis (horizontal or vertical), and why?