MP468C — Computational Physics 2 — Lab 07

1. Consider the function

$$F(w) = \sum_{j=1}^{20} \Big[wH(j/20) - G(j/20) \Big]^2$$

with

$$H(x) = x^2 \qquad \text{and} \qquad G(x) = x^3 + 0.1\sin(30x)\,.$$

   (a) The function $F(w)$ is known to have a minimum between $w = -5$ and $w = +7$. Find this minimum using the method of golden sections.

   (b) Set up the iteration rule for using Newton's algorithm to find the minimum.

   Because $F(w)$ is a quadratic function of w, your iteration should converge to minimum in a single iteration. Check if this happens.

2. Consider the function:

$$f(x, y) \;=\; \frac{1}{2}(x-1)^4 + 5(y-2)^2$$

Since the function is explicitly given, you can calculate the gradient, $\nabla f = (f_x . f_y)$.

Write a program that attempts to find the minimum of this function using the gradient descent algorithm, i.e., according to the update rule

$$\vec{r}_{i+1} \;=\; \vec{r}_i - \eta \nabla f(\vec{r}_i).$$

Start from the point $(-1, -2)$, and use the value $\eta = 0.01$.

Plot the trajectory you obtain, i.e., the points (x_i, y_i) you obtain during the iteration, on the x-y plane.

How many iterations do you need to reach the minimum to within $(\Delta x, \Delta y) = (0.01, 0.01)$?

3. Consider the ordinary differential equation

$$x\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + 5y \;=\; x^2$$

defined on the interval $x \in [-1, 1]$, satisfying boundary conditions

$$y(-1) = -3, \quad y(1) = 3.$$

(a) Turn this equation into a set of linear equations, by discretizing the interval $x \in [-1, 1]$ into N equal pieces. (This will be on paper.)

How many equations do you have, and how many variables? Answer very very carefully, and ponder till you are absolutely sure of your answer.

(b) For arbitrary N, write a program that creates the matrix and the constant vector corresponding to the linear equation above. Solve this by explicit matrix inversion, for $N = 10$, $N = 50$ and $N = 100$. Plot your approximate solutions for $y(x)$, $x \in [0, 1]$.