“Dynamical properties of spins in coupled quantum dots with exchange interaction and anisotropic terms”

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Motivation

We investigate the effect of anisotropic terms due to spin-orbit to the quantum evolution of the system, and how those terms affect the operation of a "*Half Swap*" gate, which is the foundation block for the construction of the CNOT Gate

The exchange interaction between two localized electronic spins is sufficient for rigorous construction of two-bit quantum gate.
D. Loss, D. DiVincenzo, 1998, Phys. Rev. A., **57** , 120

The spin-orbit interaction between both spins introduces new correction terms to the equation describing the system.
D. Loss, D. DiVincenzo, 1998, Phys. Rev. A., **57** , 120

“ *Half* Swap” gate

$$U_{swap}\left|\uparrow,\downarrow\right\rangle = \left|\downarrow,\uparrow\right\rangle; \quad U_{\sqrt{Swap}}\left|\uparrow,\downarrow\right\rangle = \tfrac{1}{2}\Big((1+i)\left|\uparrow,\downarrow\right\rangle + (1-i)\left|\downarrow,\uparrow\right\rangle\Big)$$

The construction of the XOR gate requires the “*half swap*” gate.

D. Loss, D. DiVincenzo, 1998, Phys. Rev. A., **57** , 120

$$U_{XOR} = e^{i(\pi/2)S_1^z} e^{i(\pi/2)S_2^z} U_{\sqrt{sw}} e^{i\pi S_1^z} U_{\sqrt{sw}},$$

The “*Half swap*” is the resulting quantum gate of the evolution of *H(t)* = $J(t)S_1 \cdot S_2$ and imposing the condition

$$x(t) = \int_0^t J(\tau)d\tau = \pi/2$$

In the computational basis, $\left|1\right\rangle = \left|\uparrow,\uparrow\right\rangle, \left|2\right\rangle = \left|\uparrow,\downarrow\right\rangle, \left|3\right\rangle = \left|\downarrow,\uparrow\right\rangle, \left|4\right\rangle = \left|\downarrow,\downarrow\right\rangle$

$$U_{\sqrt{swap}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad U_{swap} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

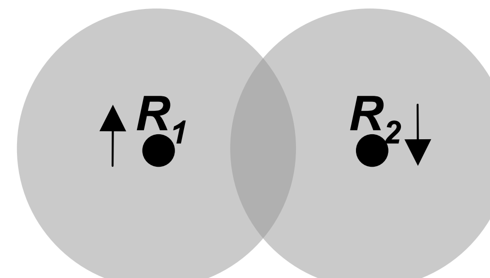
Physical system

Consider two quantum dots with one electronic spin (S_1, S_2) each.

Dzialoshinski, J. Chem. Solids **4**, 241 (1958); T. Moriya, Phys. Rev **120**, 91 (1960).

The Hamiltonian that describes their interactions on the spin-space is:

$$H = P^2 \big/ 2m + V(r) + \frac{\hbar}{2m^2c^2}\left[\nabla V(r) \times p\right] \cdot S$$



$$H(t) = J(t)\left\{S_1 \cdot S_2 + \beta(t) \cdot S_1 \times S_2 + S_1 \cdot \overline{\overline{\Gamma}}(t) \cdot S_2\right\}$$

The dynamical terms *J(t)*, *β(t)* and *Γ(t)* are superexchange variables related to the tunneling (*t*) and spin-orbit exchange (*C*) amplitudes by

$$J \rightarrow t^2, \quad B \rightarrow tC, \quad \overline{\overline{\Gamma}} \rightarrow C \cdot C$$

For simplicity let us consider the anisotropic vector ***β(t)*** only in the z direction. ***Γ****(t)* will be excluded from our calculation because it is of second order in the spin-orbit coupling.

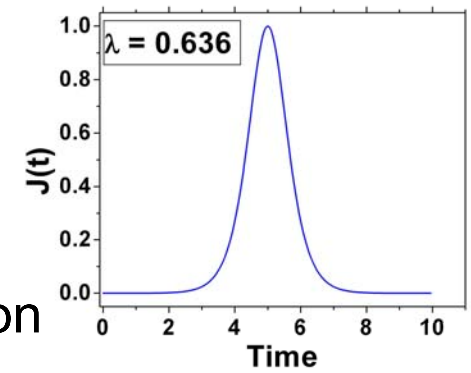
The pulse must cause a transition in the states of the system

The form of the pulse *J(t)* to be used is,

G. Burkard, D. Loss, Phys. Rev. Lett. **88**, 047903 (2002).

N. E. Bonesteel *et al*, Phys. Rev. Lett. **87**, 207901 (2001).

$$J(t) = J_0 Sech^2\left[2\lambda t\right]$$



where λ is the control parameter, for the gate operation

We study two forms of the anisotropic vector

$$\beta_z(t) = \beta_0$$

(constant)

$$\beta_z(t) = \beta_0 J(t)$$

(Time dependent)

We use the computational basis for two particles with no double occupancies:

$$|1\rangle = |\uparrow,\uparrow\rangle, |2\rangle = |\uparrow,\downarrow\rangle, |3\rangle = |\downarrow,\uparrow\rangle, |4\rangle = |\downarrow,\downarrow\rangle$$

The wave function in this basis is expressed as:

$$|\Psi(t)\rangle = a_0(t)|1\rangle + b_0(t)|2\rangle + c_0(t)|3\rangle + d_0(t)|4\rangle,$$

where the amplitudes evolve according to time dependent Schrödinger equation. We use as initial condition, $|\Psi(t=0)\rangle = |2\rangle$

And will utilize the following $J_0 = 1; \quad \eta = 1$

Dynamical Properties: Magnetization and concurrence

The magnetization per site, is an useful property for evaluating the *"half swap"* gate:

$$\sigma_z(t) \equiv \left\langle \Psi(t) \middle| \sigma_{1z} \middle| \Psi(t) \right\rangle$$

Where σ_{1z} is the Pauli matrix for particle 1

T. A. Kaplan, C. Piermarocchi, Phys. Rev B, **70**, 161311

For the "*Half swap*" gate the condition is: $\sigma_z(t) = 0$

A necessary property for quantum computation is the entanglement of the system.

A pure state of a pair of quantum systems, such as our two quantum dots, is called entangled if it is unfactorizable, as is the case, for example, for the triplet and singlet states.

The entanglement of the system will be determined by using the expressions of concurrence.

The concurrence as presented originally is a measure of entanglement.

The formula presented for concurrence makes use of the “spin flip” transformation

W. K. Wooters, Phys. Rev. Lett. **80**, 2245 (1998).

$$C = \left| \left\langle \Psi(t) \middle| \hat{\Psi}(t) \right\rangle \right| \longrightarrow \left| \hat{\Psi}(t) \right\rangle = \left(\sigma_{1y} \otimes \sigma_{1y} \right)$$

“Spin flip” transformation

We determine magnetization and concurrence in terms of the time dependent amplitudes for the states $|2\rangle$ and $|3\rangle$.

$$\left.\begin{array}{l} \sigma_z(t) = |b_0(t)|^2 - |c_0(t)|^2 \\ C(t) = \sqrt{4|b_0(t)|^2 |c_0(t)|^2} \end{array}\right\} \Rightarrow C(t) = \sqrt{1-\sigma^2(t)} \qquad (3)$$

Results:

Time-independent anisotropic corrections: $\beta(t)=\beta_0$

Solving the time dependent Schrödinger, equation for a constant β_0, we can obtain the amplitudes of the states $|2\rangle$ and $|3\rangle$

$$b(t)=\frac{1-i\beta_0}{\sqrt{2+2\beta_0}}\exp\left[\frac{-i\eta}{4}\left(-1+2\sqrt{1+\beta_0^2}\right)x(t)\right]$$

$$c(t)=-\frac{1-i\beta_0}{\sqrt{2+2\beta_0}}\exp\left[\frac{i\eta}{4}\left(1+2\sqrt{1+\beta_0^2}\right)x(t)\right]$$

Where $x(t)$ is area of the pulse $J(t)$

$$x(t)=\int_0^t J(\tau)d\tau=\pi/2$$

Where $b(t)$ and $c(t)$ are amplitude of the wave function expressed in the singlet-triplet base

$$b_0(t)=\frac{1-i\beta_0}{\sqrt{2+2\beta_0}}\left[b(t)-c(t)\right];\quad c_0(t)=\frac{1}{\sqrt{2}}\left[b(t)+c(t)\right]$$

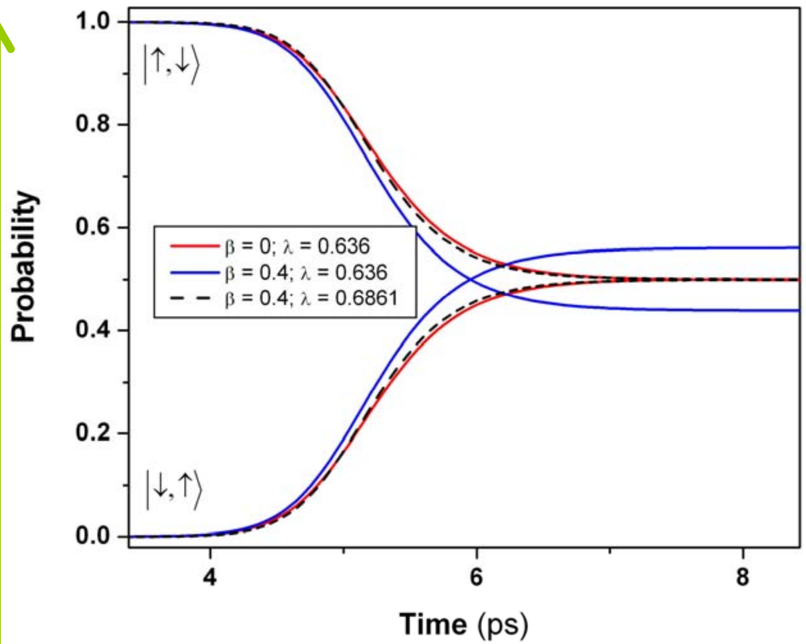
Employing these, the magnetization and concurrence are given by

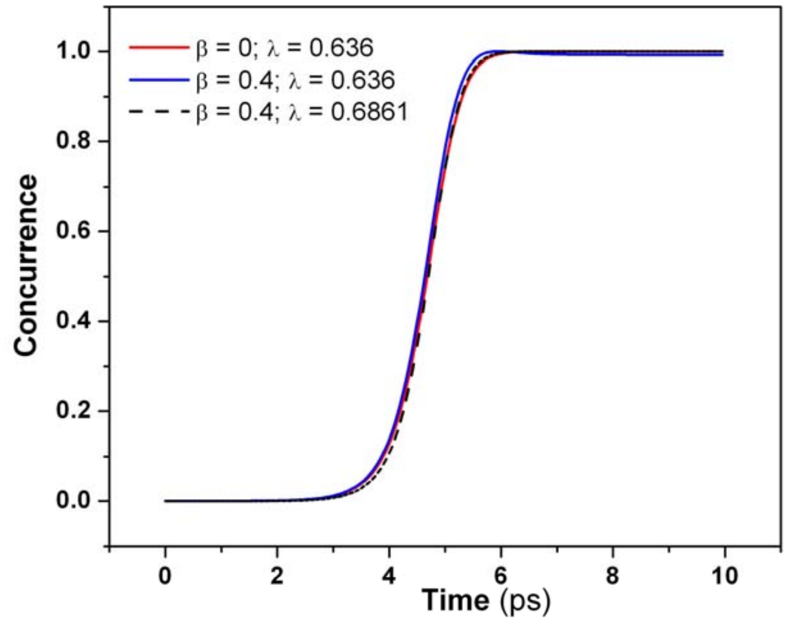
$$\sigma_z(t) = \cos\left[\sqrt{1+\beta_0^2}\,x(t)\right]; \quad C(t) = sen\left[\sqrt{1+\beta_0^2}\,x(t)\right]$$

Imposing the condition $\sigma_z=0$ (C=1), for anisotropic corrections to produce the correct gate operation is

$$x(t) = \frac{(2n+1)}{\sqrt{1+\beta_0^2}}\frac{\pi}{2} \qquad (4)$$

The anisotropic corrections causes a change in the overall area *x(t)* of the pulse *J(t)*.





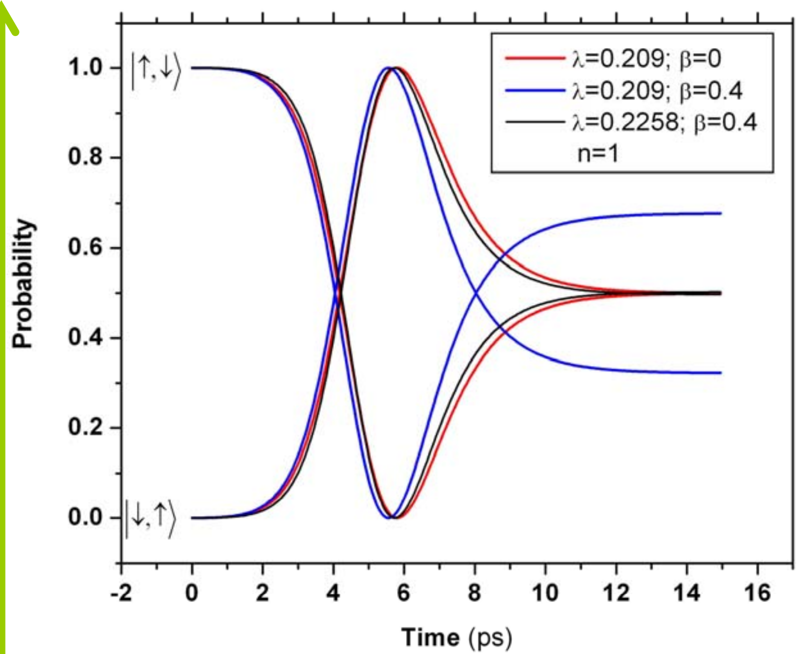
$$\left(\lambda = 0.636;\quad \beta_0 = 0\right) \Rightarrow x(t) = \pi/2\left(\text{"}Half\ swap\text{"}\right)$$

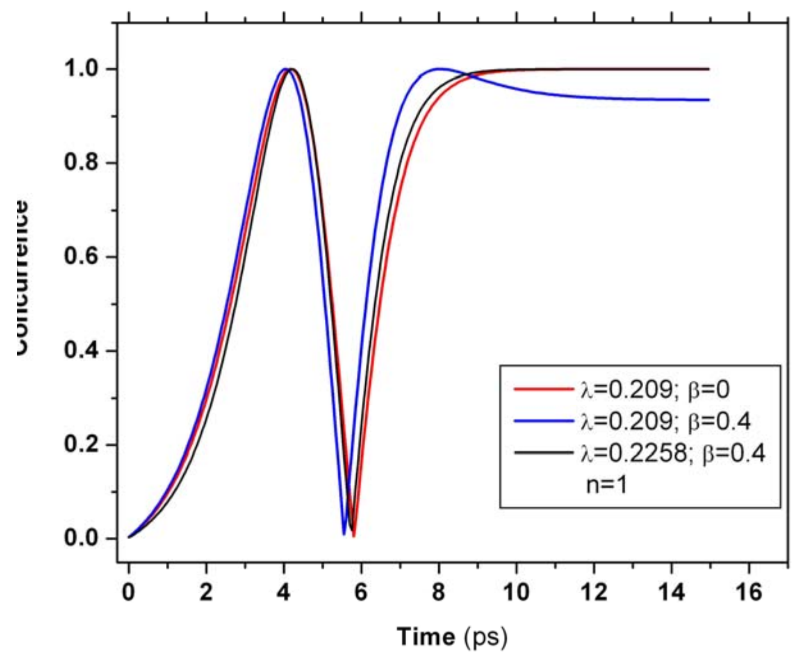
$$\left(\lambda = 0.636;\quad \beta_0 = 0.4\right) \Rightarrow \text{error in gate operation}$$

$$\left(\lambda = 0.6861;\quad \beta_0 = 0.4\right)$$

Adding anisotropic terms changes the value of *λ*, which proposes that ***β****(t)* could be used as a control parameter.

We notice that the concurrence between states is a dynamical variable that depends on the anisotropic vector.





The condition for *x(t)* allows us to obtain different conditions for different values of *n*.

$$\left(\lambda = 0.209; \quad \beta_0 = 0\right) \Rightarrow x(t) = \pi/2\left(\text{"}Half\ swap\text{"}\right)$$

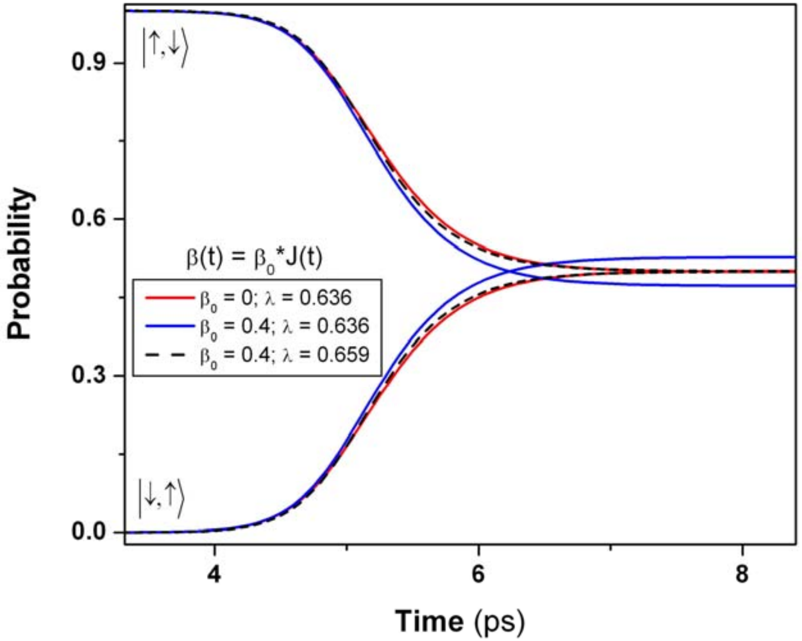
$$\left(\lambda = 0.209; \quad \beta_0 = 0.4\right) \Rightarrow \text{error in gate operation}$$

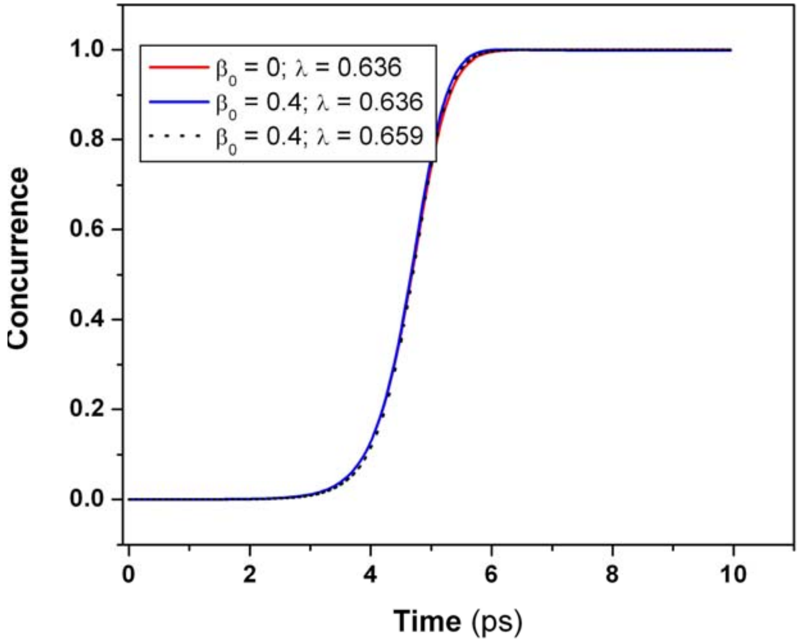
$$\left(\lambda = 0.2558; \quad \beta_0 = 0.4\right) \Rightarrow x(t) = \pi/2\left(\text{"}Half\ swap\text{"}\right)$$

Time-dependent anisotropic corrections

The following results are purely numerical.

$$\beta_z(t) = \beta_0 J(t)$$





$$\left(\lambda = 0.636; \quad \beta_0 = 0\right) \Rightarrow x(t) = \pi/2\left(''Half\ swap''\right)$$

$$\left(\lambda = 0.636; \quad \beta_0 = 0.4\right) \Rightarrow \text{error in gate operation}$$

$$\left(\lambda = 0.659; \quad \beta_0 = 0.4\right)$$

The curves show similar behavior as those presented for the case with constant *β(t).*

The value obtained for *λ* in this case is around 4% lower, then the constant case.

Analytical result using time dependent perturbation theory will be presented elsewhere to justify this behavior

Conclusions

We have studied the effect that the anisotropic term due to spin-orbit interaction has on the operation of a *"half swap"* gate.

We have presented results showing that by choosing correctly the form of the pulse *J(t)* we are able to eliminate the error caused by the anisotropic correcting to the *"half swap"* gate. In the gate operation in both constant and time dependent *β.*

Results of time dependent Concurrence for the states have been presented. An expression for *C(t)* has been found, as a function of the magnetization per site *σ(t).*

We would like to thank DGAPA-UNAM project 114403, CONACYT project 43673 and CICESE for their financial support in this project.