# Quantum dynamics of a photodetection process

Andrew A. Ignatenko
(andi_blr@mail.ru)

B. I. Stepanov Institute of physics

National Academy of Sciences of Belarus

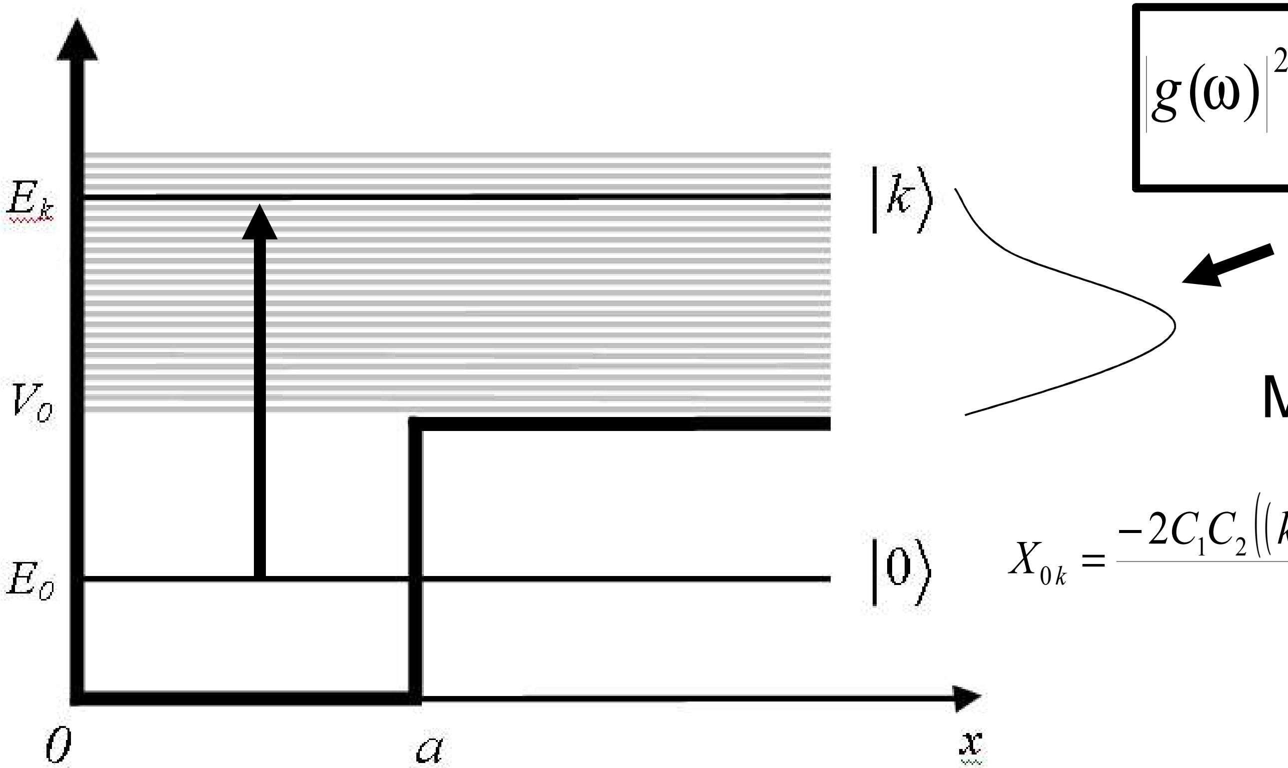
# Motivation

- A central problem of quantum measurement theory

- “Single-photon to single electron” transformation is a main process underlying in modern photodetection technique

- Fundamental interest in investigation of matter-light interaction

# Simple Model



Ek – exited energy level
E0 – ground level
a – width of potential well
V0– well height

## Interaction constant

$$\boxed{|g(\omega)|^2 = \frac{1}{\hbar^2}\frac{\hbar\omega_0}{2\varepsilon_0 V}|eX_{0k}|^2}$$

## Matrix element

$$X_{0k} = \frac{-2C_1C_2\left((k_1a)(k_1'a)-\xi_0^2\sin(k_1a)\sin(k_1'a)\right)}{a^2\left(k_1^2-k_1'^2\right)^2}$$

## Wave vectors

$$k_1' = \sqrt{\frac{2mE}{\hbar^2}}, k_2' = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}, k_1 = \sqrt{\frac{2mE_1}{\hbar^2}}, k_2' = \sqrt{\frac{2m(V_0-E_1)}{\hbar^2}}$$

Wave function of free photoelectron

$$\Psi_k(x)=\sqrt{\frac{2}{L-a}}\sin\left(k_2'+\delta\right),\quad \delta=arctg\left(\frac{k_2'}{k_1'}tg\left(k_1'a\right)\right)-k_2'a$$

$C_0\left(t\right)$ is a probability of finding an electron in the well

$$\frac{dC_0\left(t\right)}{dt}=\int_0^{\infty}\int_{\omega}\left|g\left(\omega\right)\right|^2\rho\left(\omega\right)e^{-i\omega\left(t-\tau\right)}C_0\left(t\right)d\tau d\omega$$

$$\rho\left(\omega\right)=\frac{\hbar\xi_0^2\left(L-a\right)}{2\pi V_0k_2'a^2}$$

Laplace transformation of $C_0\left(t\right)$

$$C_0^{[s]}=\frac{\hbar}{V_0}\frac{1}{s_\nu+ir_\omega-g_n^2R\left(s_\nu\right)}$$

$$s_\nu=\frac{s\hbar}{V_0}$$

# Wave packet of free photoelectron

$$\boxed{\Psi_{ph}(x,t)=\sum_k C_k(t)\Psi_k(x)}$$

$$C_k(t)=-ig_k\int_0^t e^{-i\omega_k(t-\tau)}C_0(\tau)\,d\tau$$

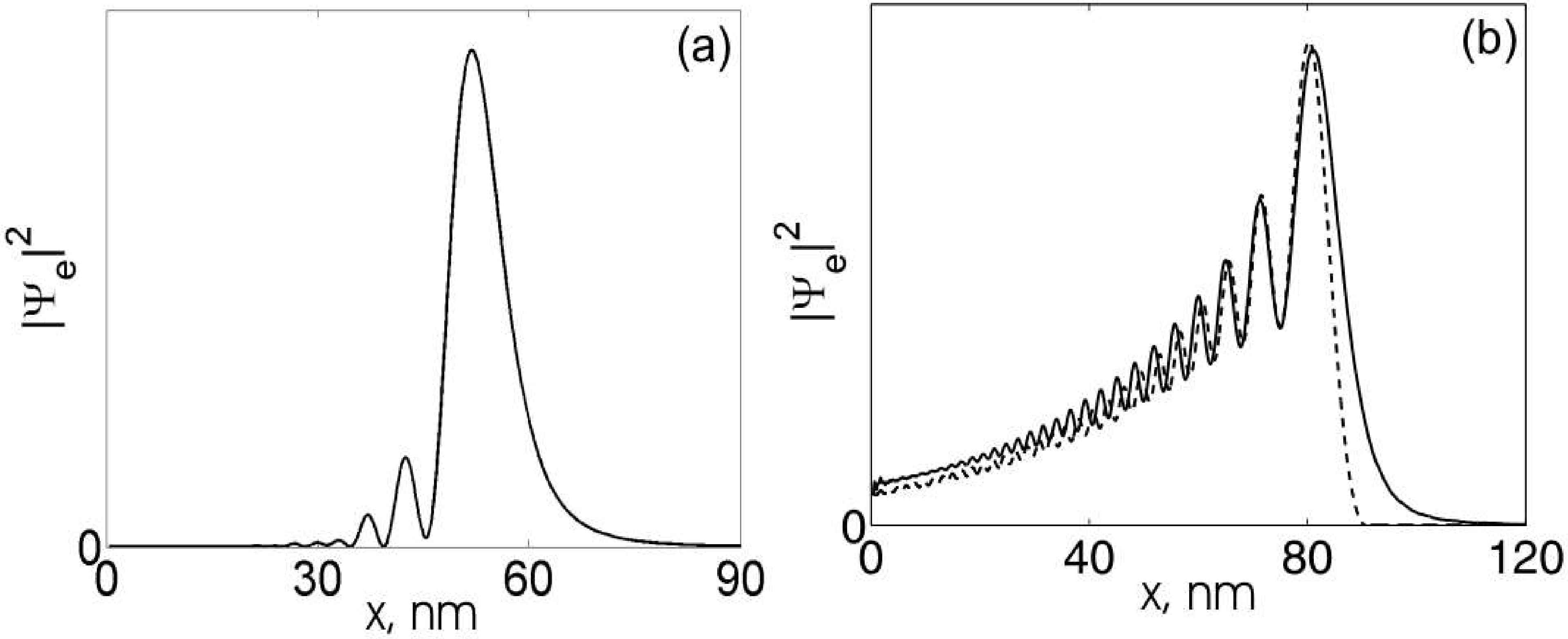
probability of finding the electron at the k-th level of continuum

## Approximation of wave packet using Frenel integrals

$$\boxed{\widetilde{\Psi}(x,t)=A(t)e^{-\gamma t_x}S\left(\gamma t_x b(t)\right)}$$

$$\underbrace{S(z)=\int_0^z \sin\left(\pi t^2/2\right)\,dt}_{\text{Frenel Integral}}\qquad t_x=t-x/v\qquad \underbrace{\gamma=2\pi\left|g(\omega)\right|^2\rho(\omega)}_{\text{decay}}\qquad \underbrace{A(t)=e^{-\alpha t}\qquad b(t)=b_1e^{-\gamma b_2 t}}_{\text{These functions are responsible for decreasing the amplitude of the packet and widening of the structure accordingly}}$$

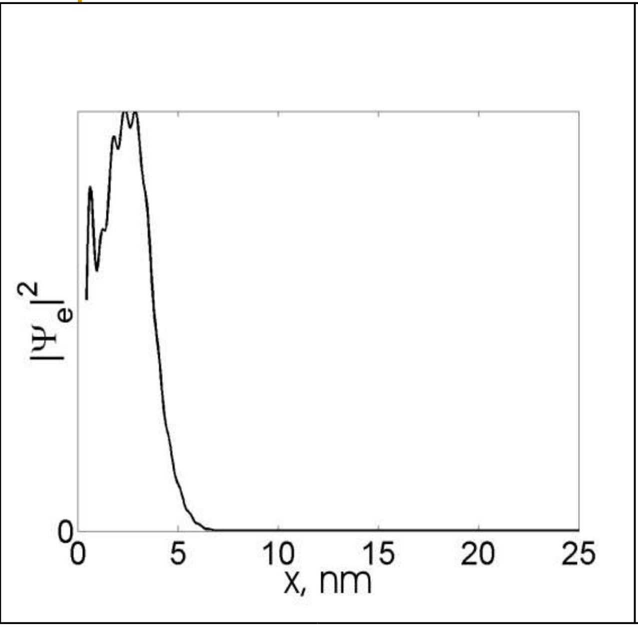
# Wavepacket of the free photoelectron and its approximation

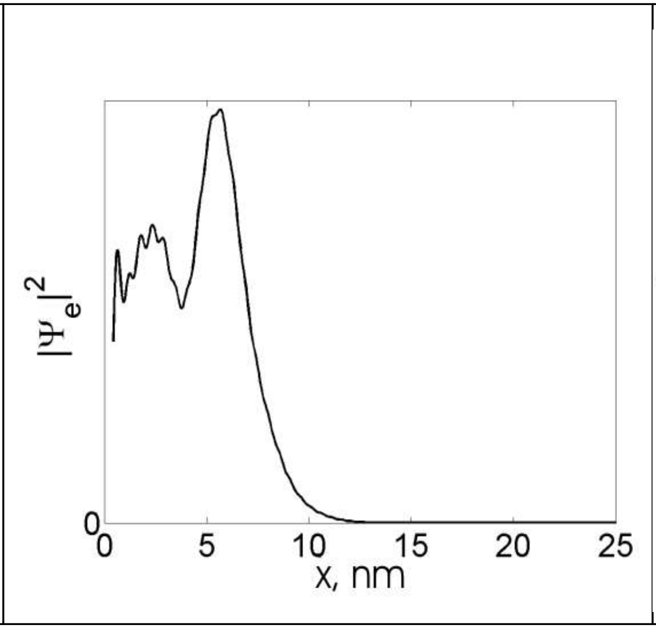


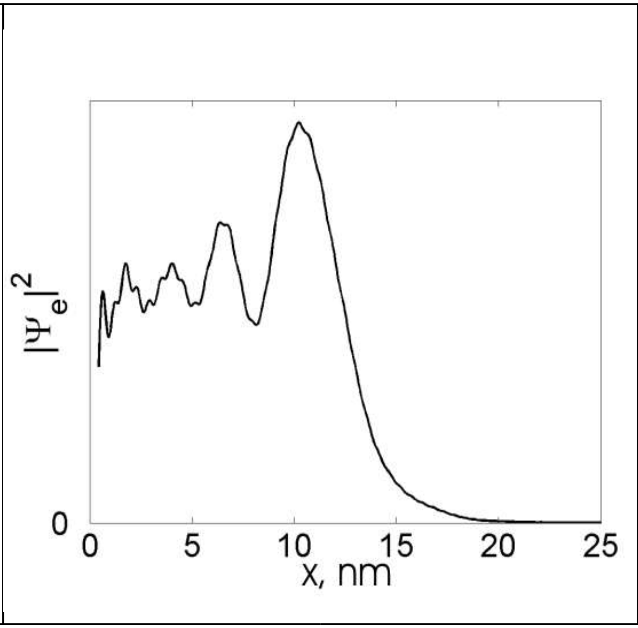
3 eV Photon energy 4 eV

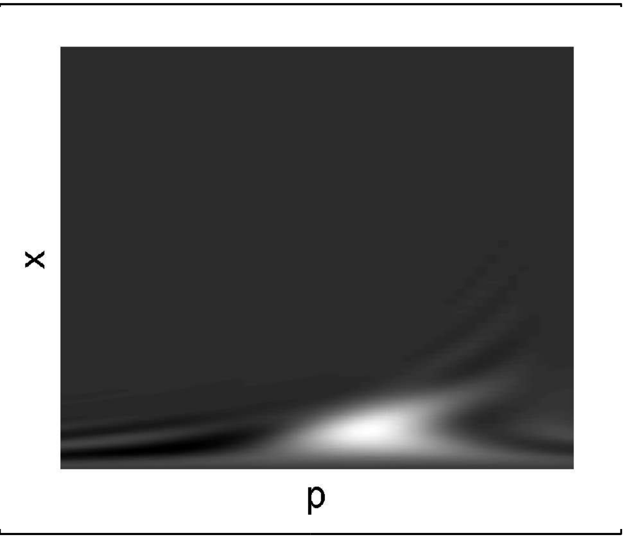
Threshold energy is 2.7 eV

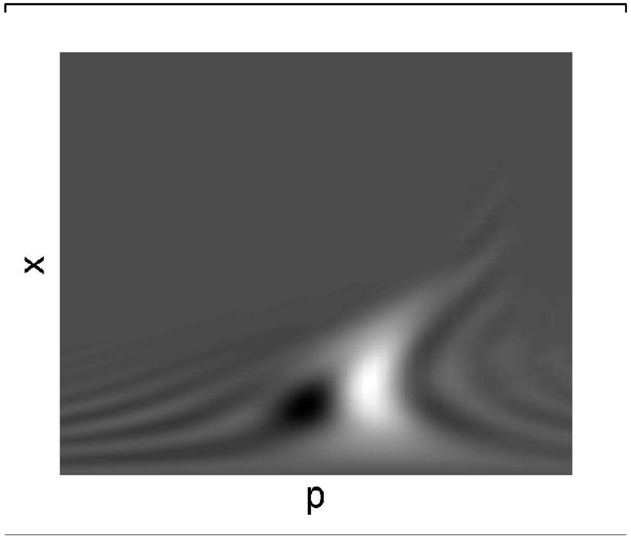
# Wigner Function of the free electron

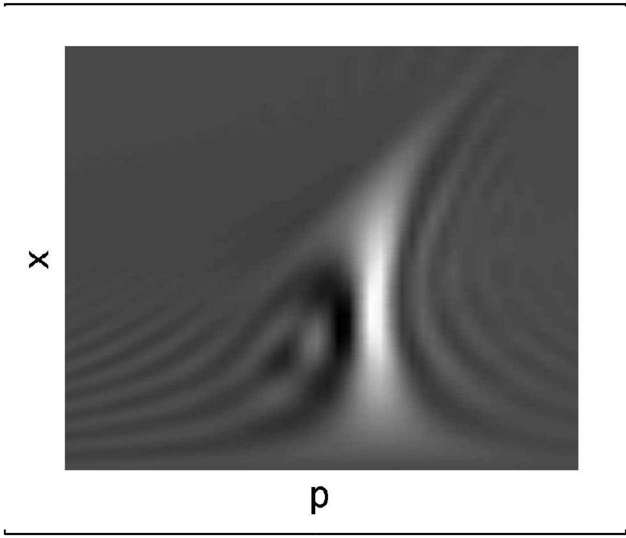












# 3D wave packet

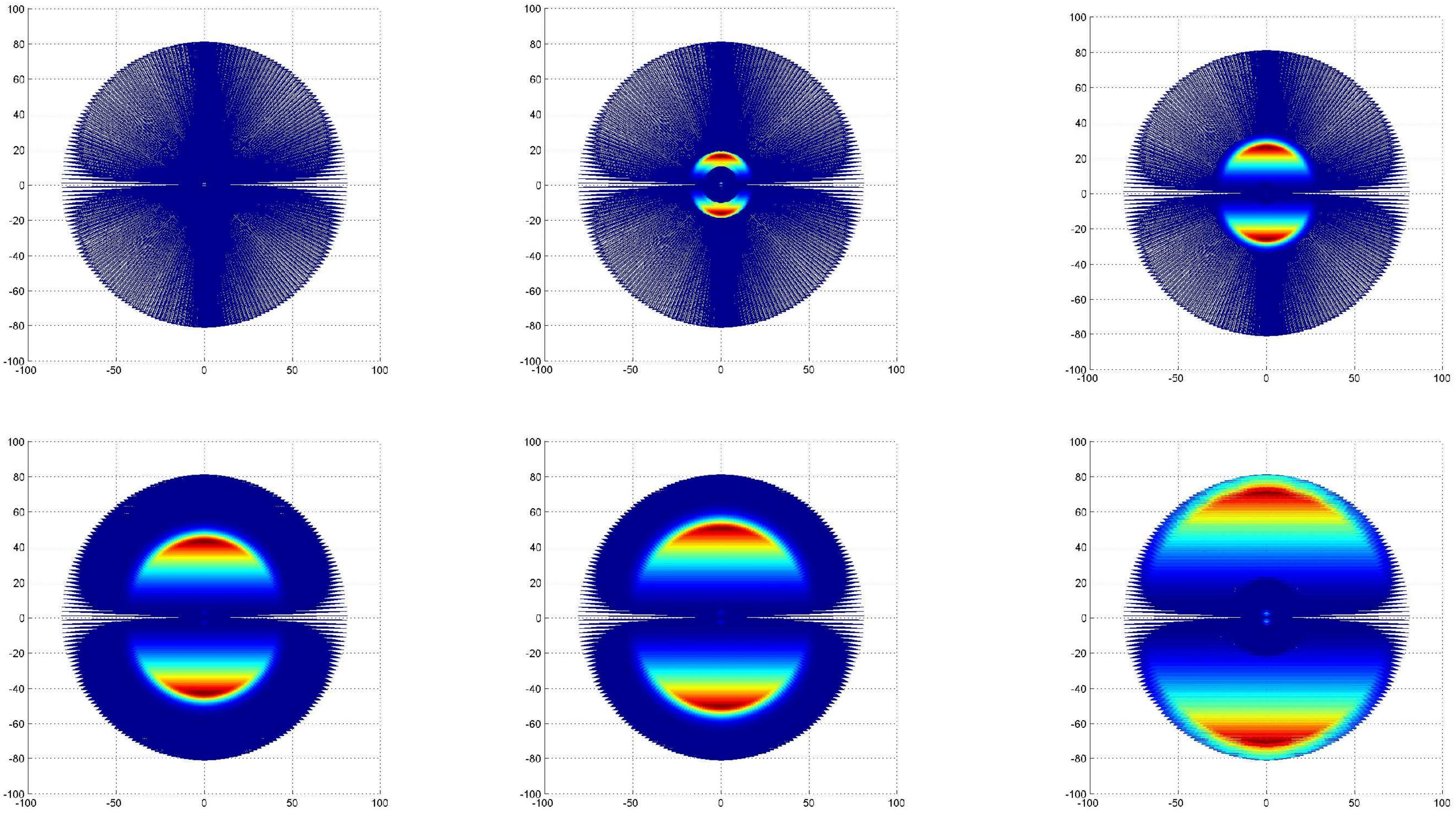
$$\Psi_{3D} = R(r)\,Y_{lm}(\theta,\varphi)$$

$$R(r) = \frac{C}{\sqrt{r}}\left\{J_{3/2}\left(k_2{}'r\right) - ctg\left(k_2{}'L\right)J_{-3/2}\left(k_2{}'r\right)\right\}$$

| l, m | 0,0 | 1,0 | 1, ±1 |
|---|---|---|---|
| $Y_{lm}$ | $\frac{1}{\sqrt{4\pi}}$ | $i\sqrt{\frac{3}{\pi}}\cos\theta$ | $\mp i\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\varphi}$ |

l is equal to 0 for initial state of the electron

# 3D – model of the wavepacket



# Conclusion

- Temporal dynamics of the free electron wavepacket has been investigated and finite time of its creation was found
- The oscillations of the trailing edge of the wavepacket were found
- Wigner function showed that photoelectron is in a squeezed state
- 3D-model of wavepacket has been built