Preparation of a GHZ state

Experimental realization

- A first atom in $|e_1\rangle$ performs a $\pi/2$ pulse

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|e_1, 0\rangle + |g_1, 1\rangle\right)$$

- A second atom in $1/\sqrt{2}(|i_2\rangle + |g_2\rangle)$ performs a QPG gate without affecting the field state (QND)

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|e_1, 0\rangle \otimes \frac{1}{\sqrt{2}}(|i_2\rangle + |g_2\rangle) + |g_1, 1\rangle \otimes \frac{1}{\sqrt{2}}(|i_2\rangle - |g_2\rangle)\right)$$

An atom-field-atom GHZ state

- A third atom in $|g_3\rangle$ can perform a π pulse in order to read the field state

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\left(|e_1, g_3\rangle \otimes \frac{1}{\sqrt{2}}(|i_2\rangle + |g_2\rangle) + |g_1, e_3\rangle \otimes \frac{1}{\sqrt{2}}(|i_2\rangle - |g_2\rangle)\right)$$

Detection of the GHZ state

2nd Ramsey pulse for atom 2 at frequency ν_0

It corresponds to the QND detection conditions:

$$\frac{1}{\sqrt{2}}(|i_2\rangle + |g_2\rangle) \rightarrow |i_2\rangle$$

$$\frac{1}{\sqrt{2}}(|i_2\rangle - |g_2\rangle) \rightarrow |g_2\rangle$$

Detection of the GHZ state

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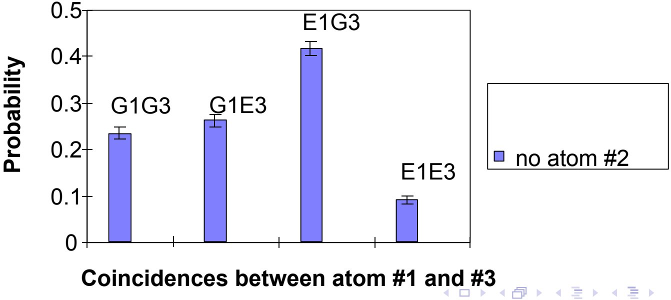
Results



Detection of the GHZ state

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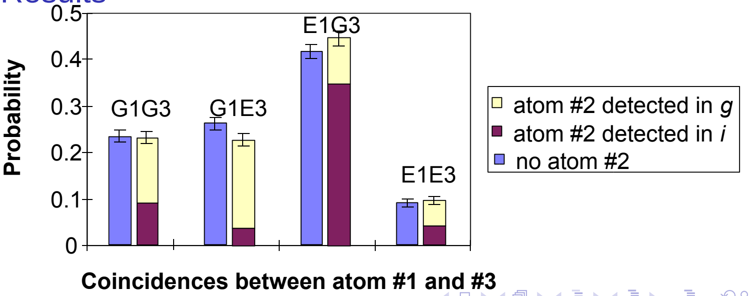
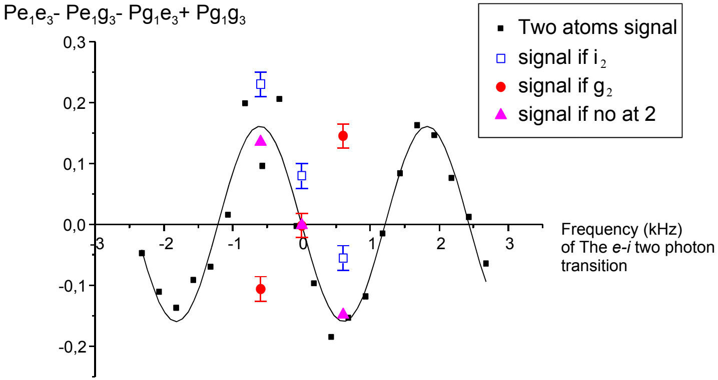
Results



Correlation in another basis

EPR correlation for atoms 1 and 3

Measurement of $\langle \sigma_{x,1}\sigma_{\phi,3} \rangle$ as in previous experiments.



QPG action of atom 2 changes the sign of the correlation of atoms 1 and 3.

General conclusion

- Simple quatum gates demonstrated
- most complicated algorithm uses up to 4 qubits and entangles 3 of them
- complete measurement of the density matrix not realised (see ion trap experiment)
  - early experiment
  - data acquisition time too long due to the very low probability for detecting coincidences (20h for the 3 atom experiment)

Decoherence in Quantum Mechanics
A cavity QED approach

Gilles NOGUES
Laboratoire Kastler Brossel
Ecole normale supérieure, Paris

A century of quantum mechanics

- And yet an intriguing theory which defies our « classical intuition »
- Many paradoxes as soon as one tries to interpret its results
  - Entanglement: after interaction 2 subsystems, although separated in space, cannot be considered as independant
  - Superposition principle: Any linear combination of physical state is a physical state

The Schrödinger cat

- A gedanken experiment

A cat in a box with an excited atom

If the atom desexcite, a setup kills the cat

What is the state of the system after a half-lifetime of the atom?

- Three related questions
  - Why do we need the box? ⟶ Environment
  - Why do we found the cat dead or alive? ⟶ Preferred basis
  - Why can't we predict the outcome of a single realization? ⟶ ?

Outline

- A cavity QED experiment
  - How to create and destroy and restore a coherent superposition
- The decoherence in quantum mechanics
  - Effect of the size of the system and of the environment
- Monitoring the decoherence
  - What can be an optical Schrödinger cat?
  - Observing the decoherence
- Beyond decoherence
  - Pushing the limits?

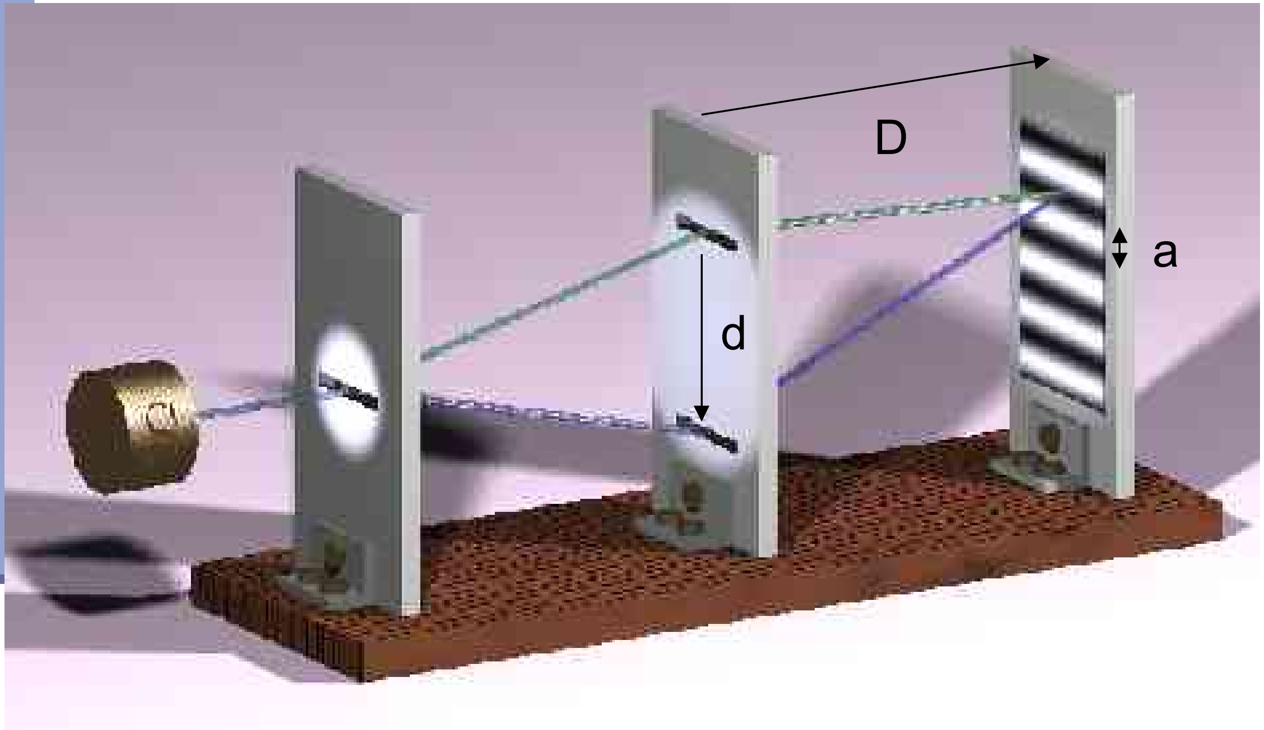
A cavity QED experiment on complementarity

How to wash out fringes in an interferometer by gaining a « which-path » information?

How to restore the interference by manipulating the information

The "strangeness" of the quantum

- Superposition principle and quantum interferences
  - The sum of quantum states is yet another possible state
  - A system "suspended" between two different classical realities



$$|\Psi\rangle = |\Psi_1\rangle + |\Psi_2\rangle$$

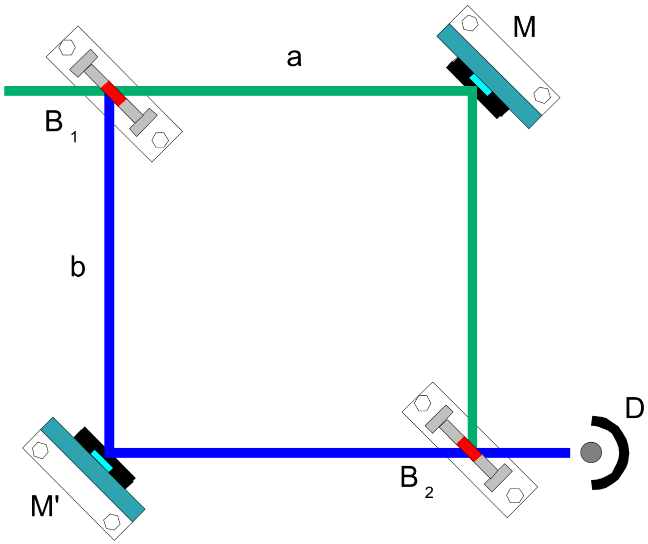
$$I = I_0 + 2\,\mathrm{Re}\langle\Psi_1|\Psi_2\rangle$$

$$a = \lambda\frac{D}{d}$$

- Feynman: Young's slits experiment contains all the mysteries of the quantum

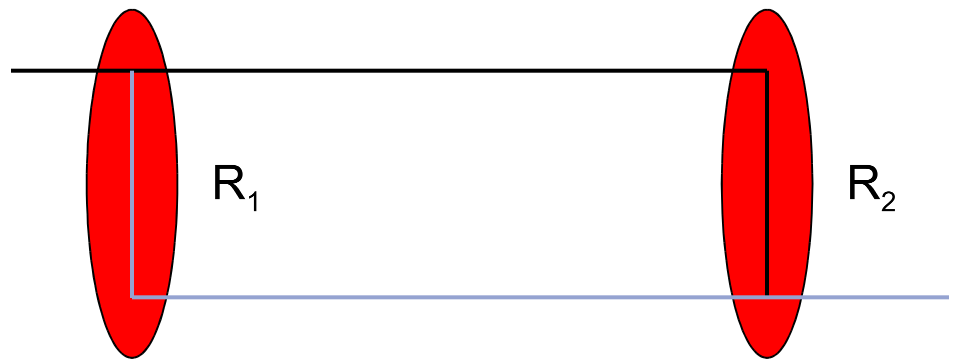
Interferometers for photons and atoms

- Mach-Zender

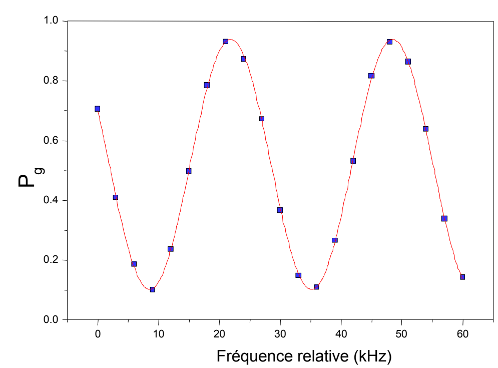


- 2 optical path separated by beamsplitters

- Ramsey:



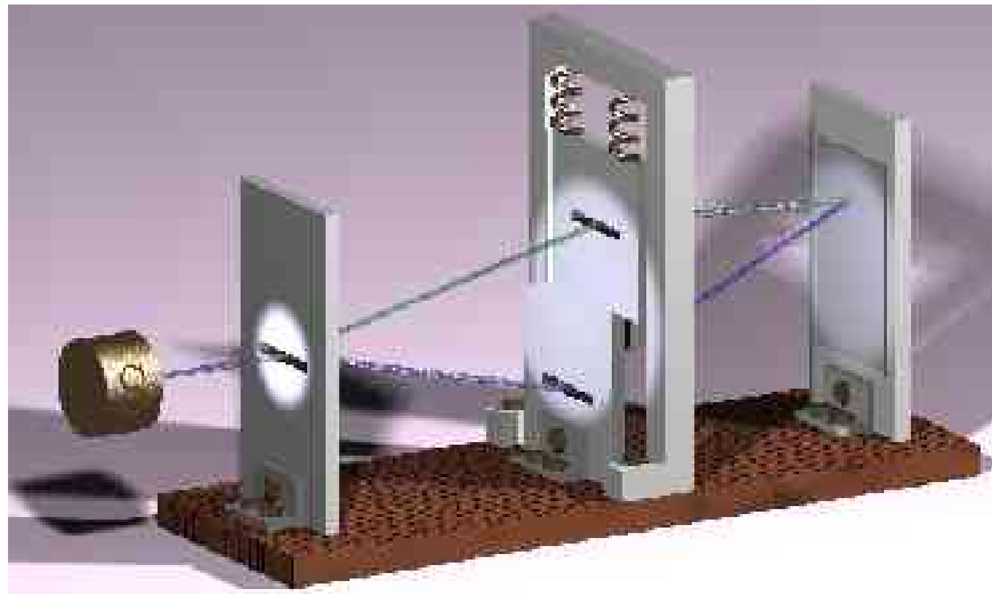
- 2 energy path separated by interaction with classical field



Destroying the fringes

- By gaining a “which-path” information on the system

One slit is mounted on springs

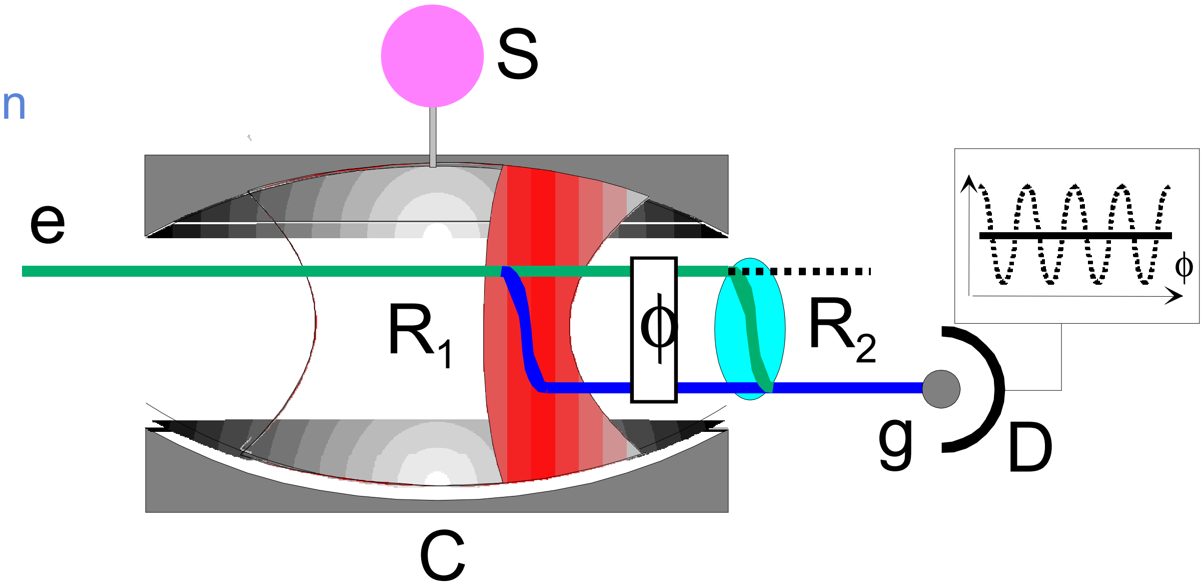


- Microscopic slit:
  - set in motion when deflecting particle.
  - Which path information and no fringes
- Macroscopic slit:
  - impervious to interfering particle.
  - No which path information and fringes
- Wave and particle are complementary aspects of the quantum object.
  - (From Einstein-Bohr at the 1927 Solvay congress)

Bohr's experiment with a Ramsey interferometer

- Illustrating complementarity: Store one Ramsey field in a cavity

Atom-cavity interaction time
Tuned for $\pi/2$ pulse
Possible even if C empty



- Initial cavity state $|\alpha\rangle$
- Intermediate atom-cavity state $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|e,\alpha_e\rangle + |g,\alpha_g\rangle\right)$

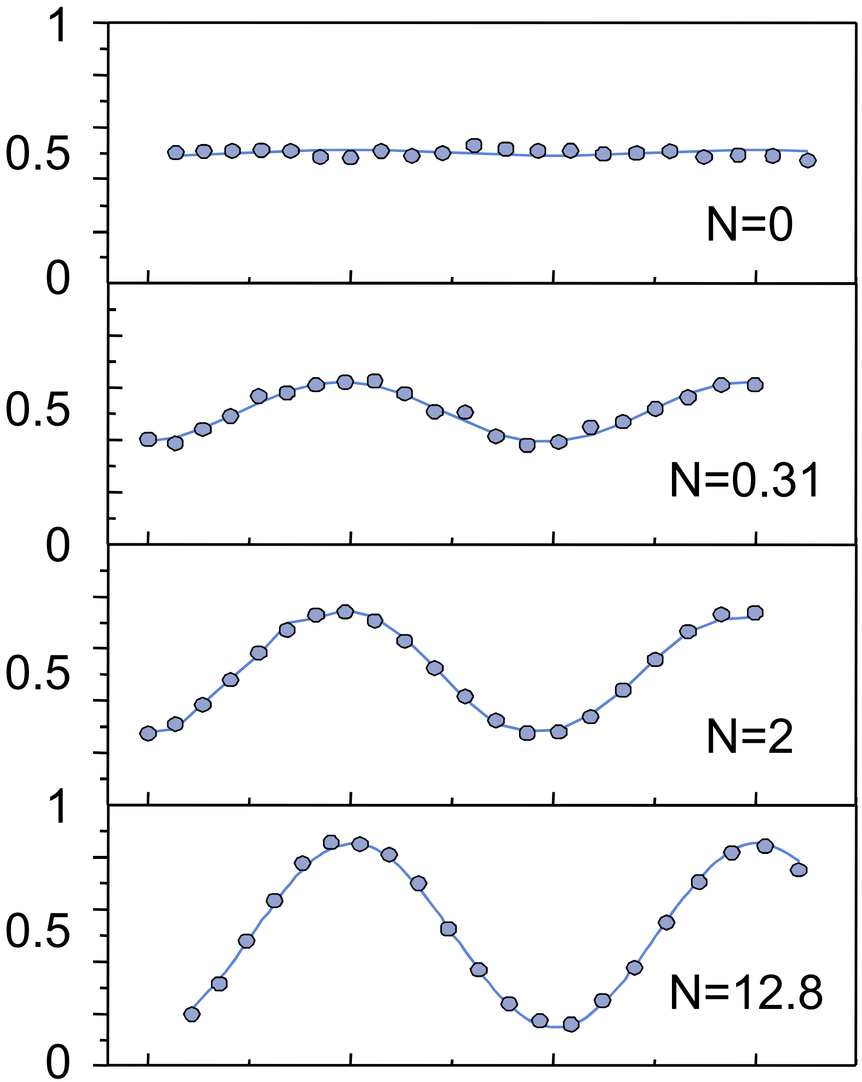
Ramsey fringes contrast $\left|\langle\alpha_e|\alpha_g\rangle\right|$

- Large field $|\alpha_e\rangle \approx |\alpha_g\rangle \approx |\alpha\rangle$ FRINGES

- Small field $|\alpha_e\rangle = |0\rangle, |\alpha_g\rangle = |1\rangle$

NO FRINGES

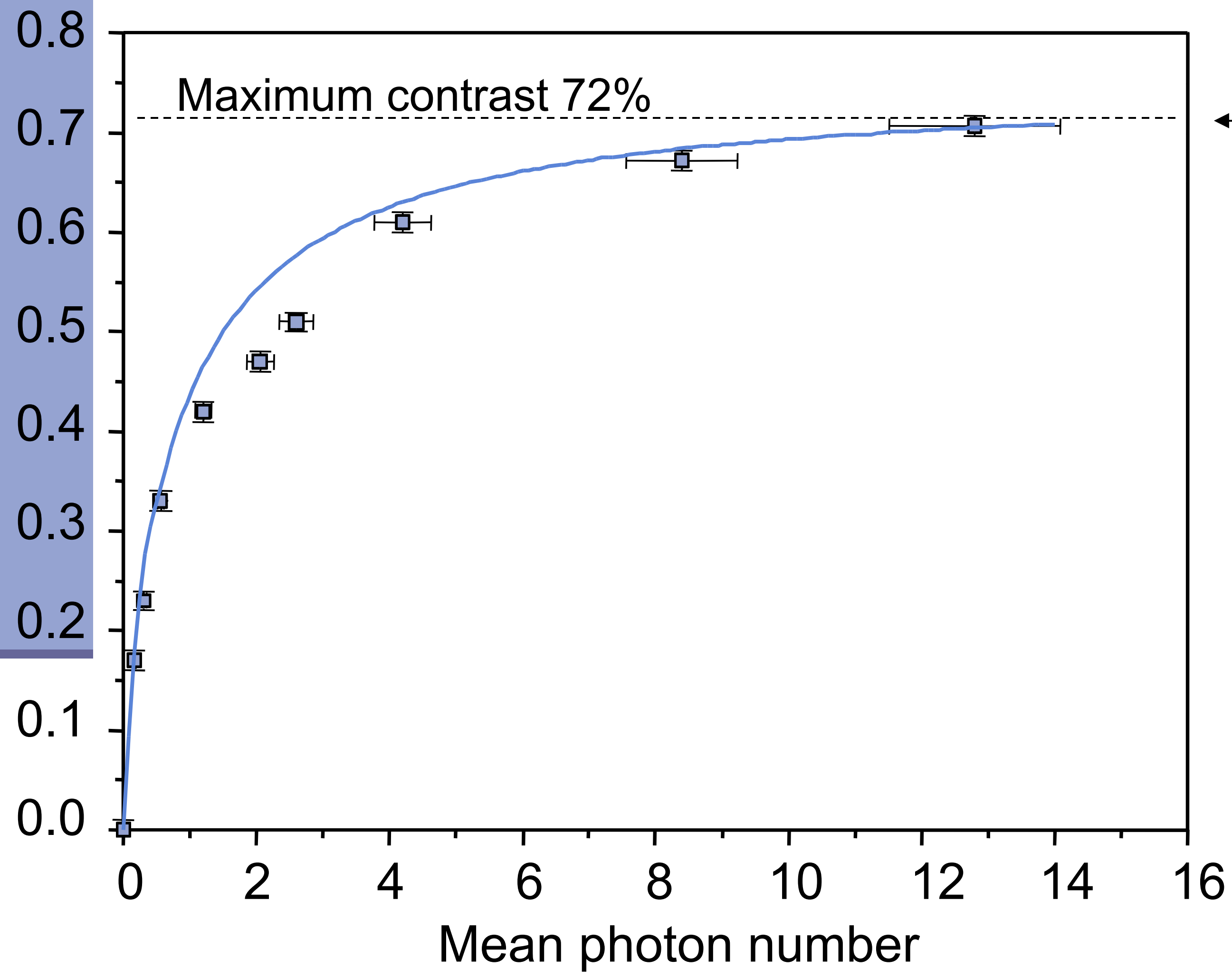
Interferences versus field size



- A microscopic field is strongly affected by the interaction
  - Stores an information about the atomic state
- For larger fields the « which path » information is lost
  - Fringes restored

Nature, **411**, 166 (2001)

Interference versus field size (2)



final contrast due to external imperfections

(velocity dispersion, 2 atom events…)

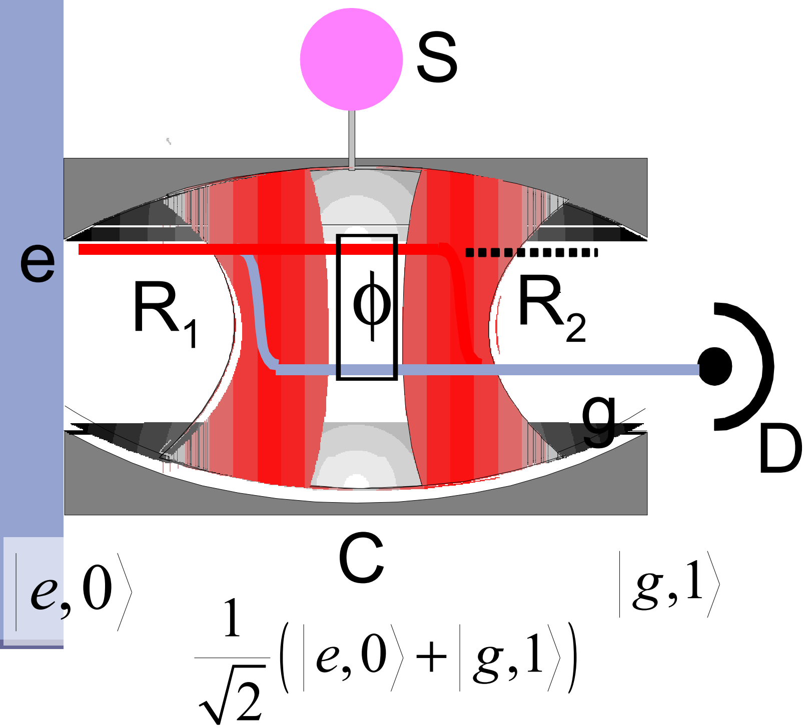
- Photon number acts as the mass of the slit in the Young's experiment
- Classical behaviour observed for about 10 photons

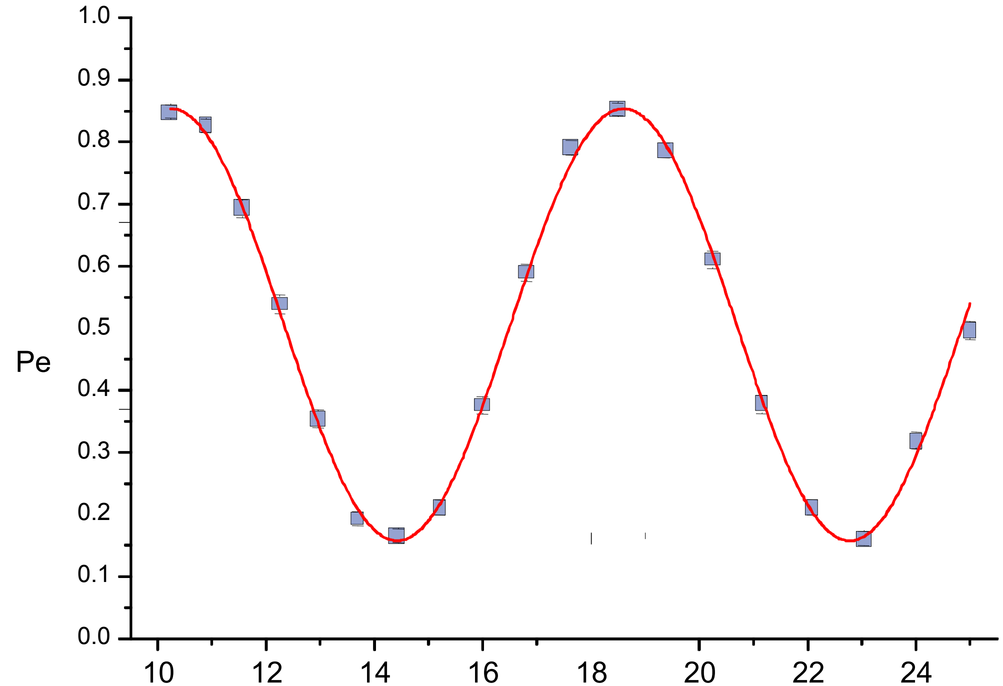
Important remarks

- No need to measure the field state
  - The mere fact that the information exists destroy the interferences
- Close link with entanglement
  - Atom and field no longer separable
  - Local operation on the atom cannot recover the whole information
- Fringes are recoverable
  - If one operates on the atom *and* the field…
  - … in order to erase the information

Ramsey “quantum eraser”

- A second interaction with the mode erases the atom-cavity entanglement





- Ramsey fringes without fields !
  - Quantum interference fringes without external field
  - A good tool for quantum manipulations

Intermediate conclusions

- Any coherent superposition can be created and probed in a interferometer experiment
- The fringe contrast is a direct measurement of how much « which-path » information one has about the system
- It is in principle possible to manipulate this amount of information to restore the coherence
- Cavity QED is an appropriate tool to manipulate those concepts in the case of an atom coupled to a few tens of photons

A general frame for decoherence

The environment as an unavoidable « which-path » meter

Effect of the size of the system

Link with the measurement theory

Why the box?

- Hide the cat
  - Physically: prevent diffusion of light on the system
- Isolate from its environment

- Coupling with the outside world
  - Metallic box?
    - Blackbody radiation
  - Surrounding gaz
    - Collision: brownian motion
    - Vibration: noise
- All those interactions provide a « which-path » information

Why the cat?

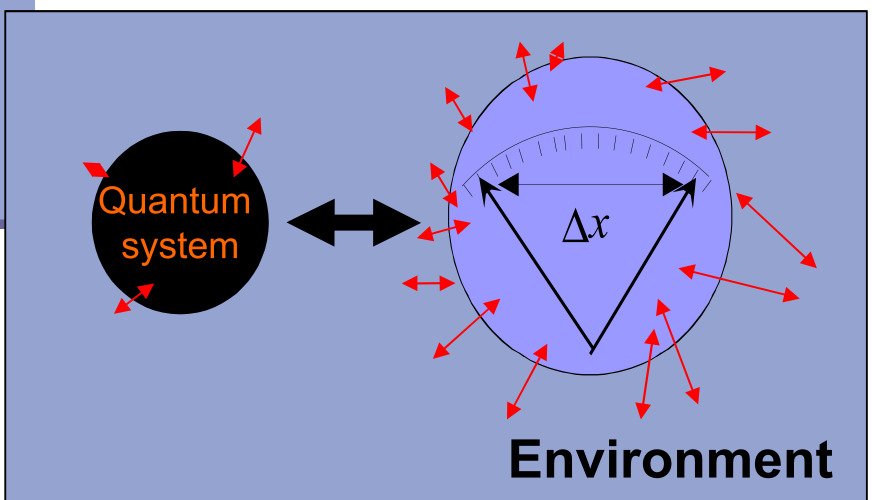
- What is the difference between a photon in an interferometer and a cat in a box
  - Strength of the coupling with the environment
  - Distance (in Hilbert space) between states

$$T_{dec}=T_{rel}/\text{Distance}$$

- How to evaluate the distance?
  - Depends on the nature of the coupling
    - Diffusion of photons: distance = (distance in space) / wavelength
- Not a trivial relaxation mechanism
  - But is explained by relaxation theory for simple models
- Final state
  - Trace over the environment: statistical mixture

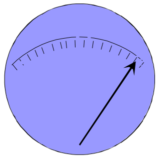
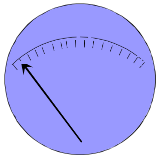
Link with measurement theory

- What is a good meter?
  - Coupled to the observed system
  - Gives an answer that you can see with your eye
    - Open, macroscopic system



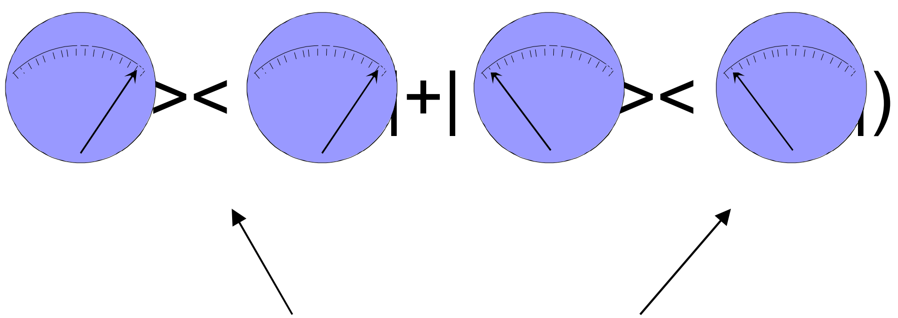
- A two step process
  - System-meter coupling
    - May be unitary
    - leads to an entangled state
  - « which – path » information gathered by environment
    - Coherence is lost
    - And unrecoverable if the environment is huge

Pointer states

- Only a small fraction of the Hilbert space is observed
  - Example:  or  , but no superpositions
- This « preferred basis » correspond to meter state which do not get entangled with the environment
  - Pointer states

What is the final state of the cat

- The coherence leaks into the environment
  - One has to account for the lack of knowledge about the environment
    - Density matrix description…
    - …with a trace over the environment
  - ρ_{fin}=1/2(|



Statistical mixture of two pointer state

The meter is either pointing one way *or* the other

A short summary: what is important in decoherence

- An open system whose interaction with the environment selects a preferred basis
- Pointer states which can be discriminated by a classical experience
  - Large distance in Hilbert space
  - The larger the distance, the faster the process

This is the case of every large system

  - Demonstration:
    - One never observes a cat dead and alive at the same time

Monitoring the decoherence

- Requirements to observe the phenomenon
  - A system as isolated as possible
    - Long relaxation time
  - A mesoscopic system
    - Whose « size » can be varied between macroscopic and microscopic dimensions
  - An easy way to prepare and analyse coherent superpositions of the system

A small coherent field in a cavity is a perfect example

Preparing and observing a Schrödinger kitten

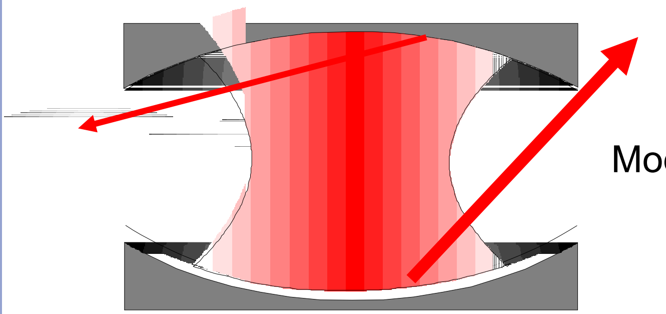
Decoherence for a superposition of coherent states of light

How to prepare such a state?

An interferometric experiment to test the coherence.

Cavity relaxation

- Due to diffraction on the surface
  - Not perfectly spherical over a few mm



  - Interaction Hamiltonian:

$$H=\sum g_i(ab_i^+ + a^+b_i)$$

- State at time t:
  - At T=0K:

$$|\alpha e^{-\gamma t/2}\rangle \prod_i |\beta_i(t)\rangle$$

  - A coherent state remains disentangled
    - Pointer state
  - Energy conservation

$$\sum |\beta_i(t)|^2 = |\alpha|^2(1-e^{-\gamma t})$$

$$= n(1-e^{-\gamma t})$$

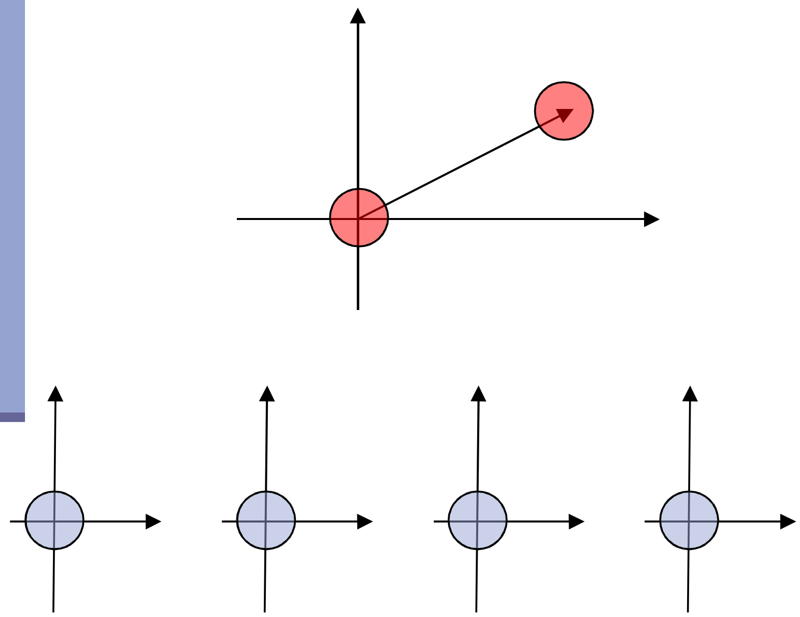
Decoherence of a coherent state superposition

- Initial state

$$1/N(|0\rangle + |\alpha\rangle)\prod |0\rangle_i$$

$$N = \sqrt{2 + \langle 0|\alpha\rangle} = \sqrt{2 + e^{-|\alpha|^2/2}}$$

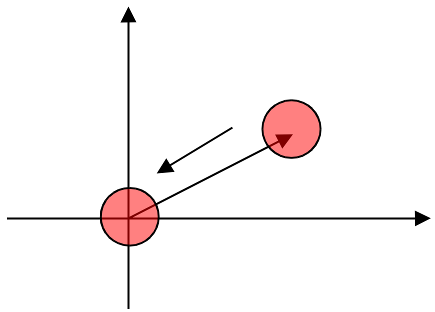
$$\approx \sqrt{2} \text{ if } |\alpha| >> 1$$

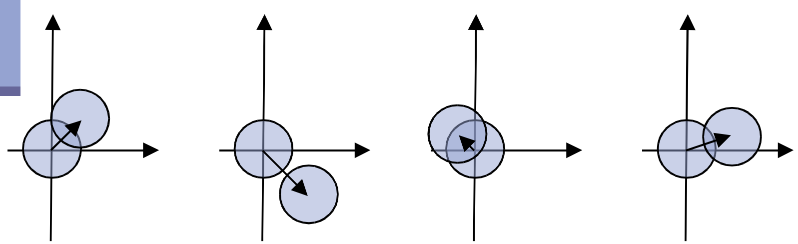


Decoherence of a coherent state superposition

- At time t:

$$|0\rangle \prod |0\rangle_i + |\alpha e^{-\gamma t/2}\rangle \prod |\beta_i(t)\rangle$$





- Entangled system
  - Which-path information has leaked to the environment
- Contrast:

$$C = \prod_i \langle 0|\beta_i(t)\rangle$$

$$= \prod e^{-|\beta_i(t)|^2/2} = e^{-\Sigma|\beta_i(t)|^2/2}$$

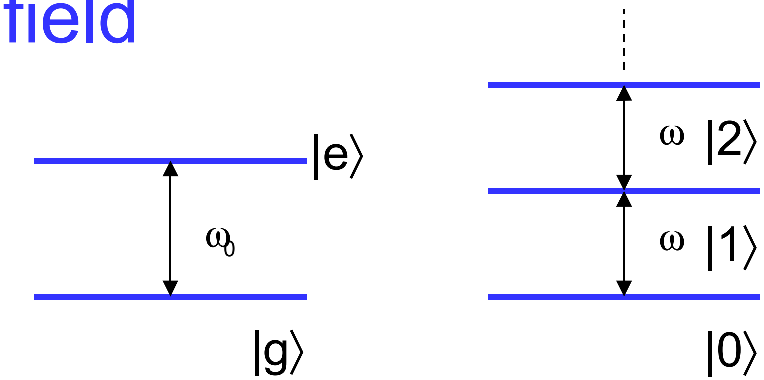
$$= e^{-|\alpha|^2 (1-e^{-\gamma t})/2} \approx e^{-|\alpha|^2 \gamma t/2}$$

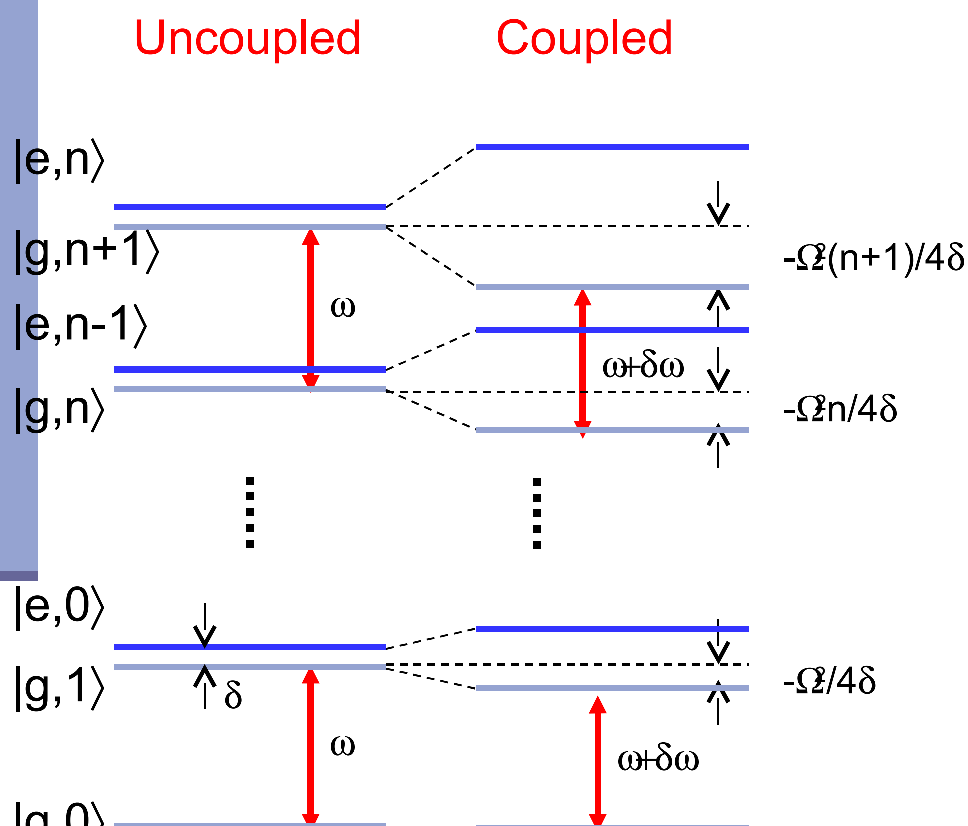
- Decoherence time

$$T_{dec} = 2\, T_r / |\alpha|^2$$

A single atom in a coherent field

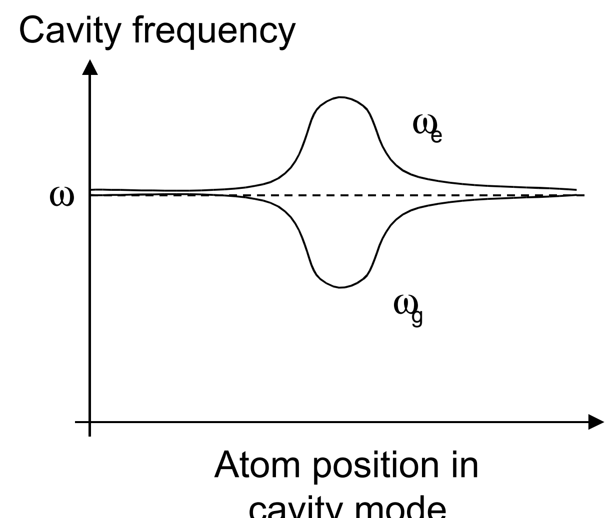
- Non resonant interaction
  - No exchange of energy





- If $\delta \gg \Omega$

$|+,n\rangle \approx |g,n\rangle$ and $\Delta E_{+,n} = -\Omega^2 n/4\delta$

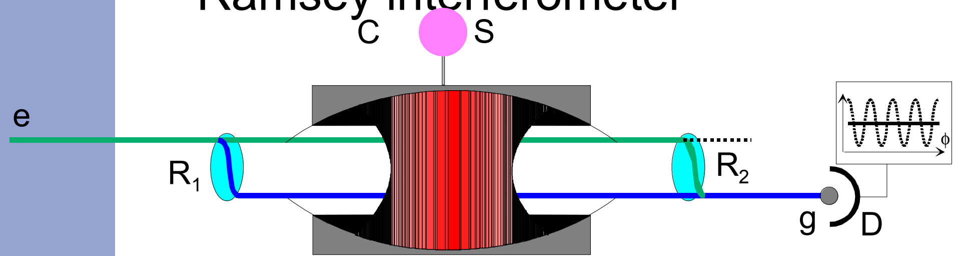


Atom acts as a dielectric that phase shifts the field in the cavity…

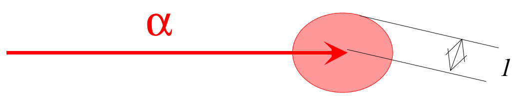
… depending on its state

Another experiment on complementarity

Cavity as an external detector in the Ramsey interferometer



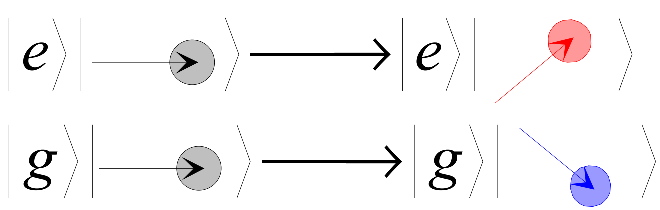
Cavity contains initially a coherent field



Non-resonant atom-field interaction:

Atom modifies the cavity field phase

(index of refraction effect)



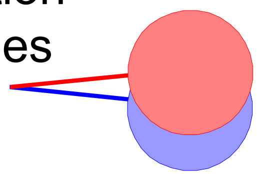
Phase shift $\propto 1/\delta$ (δ:atom-cavity detuning)

"Which path" information:

- Small phase shift (large δ)

(smaller than quantum phase noise)

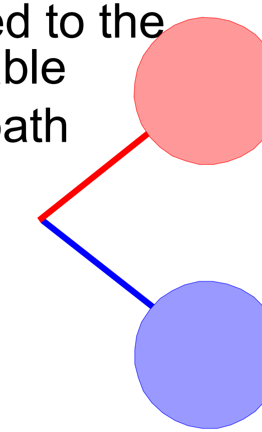
  - field phase almost unchanged
  - No which path information
  - Standard Ramsey fringes



- Large phase shift (small δ)

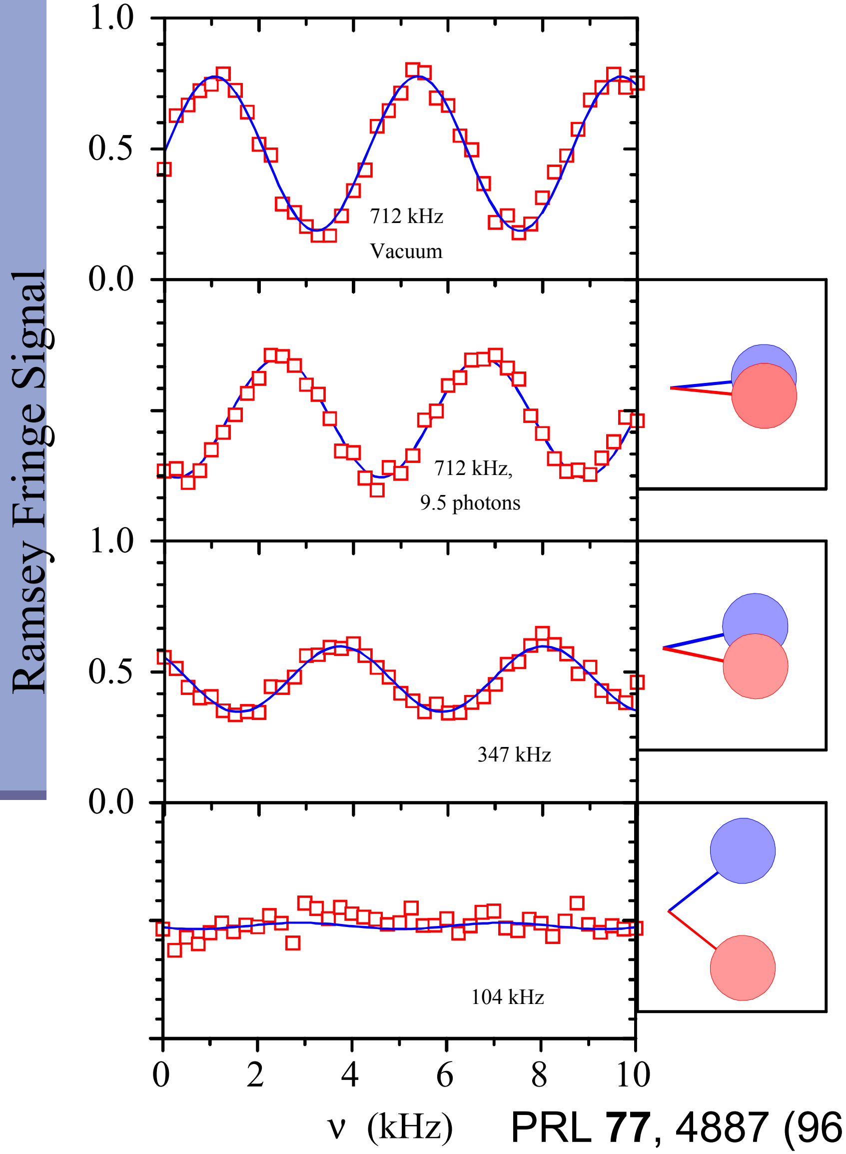
(larger than quantum phase noise)

  - Cavity fields associated to the two paths distinguishable
  - Unambiguous which path information
  - No Ramsey fringes



Fringes and field state

Complementarity



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State transformations

R1 $|e\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|e\rangle + |g\rangle\right)$

R2 $|e\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|e\rangle + e^{i\varphi}|g\rangle\right)$

$|g\rangle \rightarrow \frac{1}{\sqrt{2}}\left(-e^{-i\varphi}|e\rangle + |g\rangle\right)$

C $|e,\alpha\rangle \rightarrow e^{i\Phi}|e,\alpha e^{i\Phi}\rangle \qquad |g,\alpha\rangle \rightarrow |g,\alpha e^{-i\Phi}\rangle$

Before R1 $|e,\alpha\rangle$

Before C $\frac{1}{\sqrt{2}}\left(|e\rangle + |g\rangle\right)|\alpha\rangle$

After C $\frac{1}{\sqrt{2}}\left(e^{i\Phi}|e,\alpha e^{i\Phi}\rangle + |g,\alpha e^{-i\Phi}\rangle\right)$

After R2 $\frac{1}{2}|e\rangle e^{i\Phi}\left\{|\alpha e^{i\Phi}\rangle - e^{-i(\varphi+\Phi)}|\alpha e^{-i\Phi}\rangle\right\}$

$+\frac{1}{2}|g\rangle e^{i(\varphi+\Phi)}\left\{|\alpha e^{i\Phi}\rangle + e^{-i(\varphi+\Phi)}|\alpha e^{-i\Phi}\rangle\right\}$

Detection probabilities

$$P_{g,e} = \frac{1}{2}\left[1 \pm \mathrm{Re}\, e^{-i(\varphi+\Phi)}\langle \alpha e^{i\Phi}|\alpha e^{-i\Phi}\rangle\right]$$

Ramsey fringes signal multiplied by

$$\langle \alpha e^{i\Phi}|\alpha e^{-i\Phi}\rangle$$

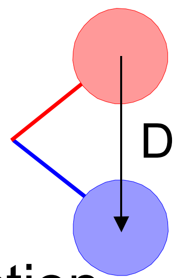
Signal analysis

Fringe signal multiplied by

$$\langle \alpha e^{i\Phi} | \alpha e^{-i\Phi} \rangle$$

- Modulus

$$e^{-2\bar{n}\sin^2\Phi} = e^{-D^2/2}$$



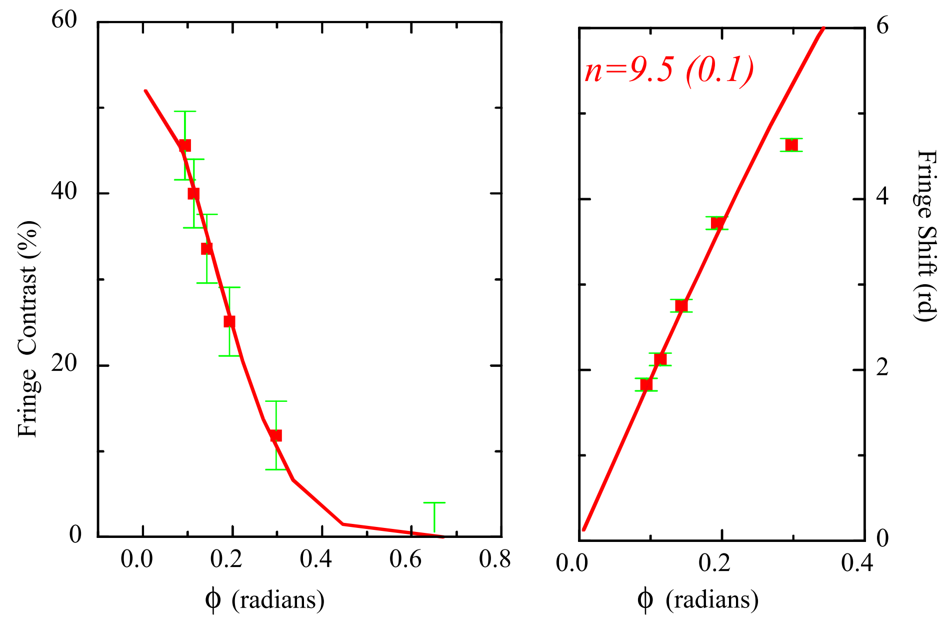
  - Contrast reduction

- Phase $2\bar{n}\sin\Phi$

  - Phase shift corresponding to cavity light shifts

Phase leads to a precise (and QND) measurement of the average photon number

Fringes contrast and phase

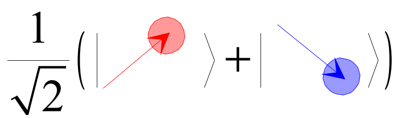


- Excellent agreement with theoretical predictions.
- Not a trivial fringes washing out effect

Calibration of the cavity field 9.5 (0.1) photons

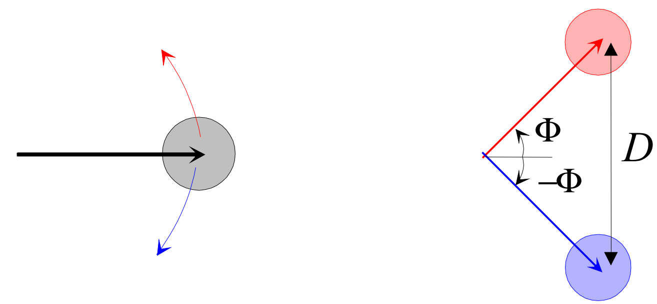
Probing the coherence

- Field state after atomic detection

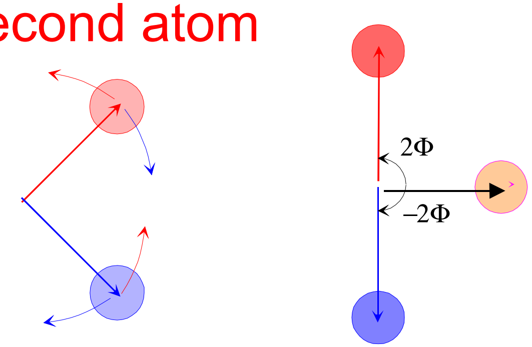


  - A coherent superposition of two 'classical' states.
- Decoherence will transform this superposition into a statistical mixture
  - Is it possible to study the dynamics of this phenomenon
  - Requires to perform an interferometry experiment with the cavity state
- A second atom to probe the field

- First atom



- Second atom



- Two indistinguishable quantum paths to the same final state
  - e_1g_2 and e_2g_1
  - Quantum interferences

Atomic correlations

- A correlation signal

$$\eta = \Pi_{e,e} - \Pi_{g,e}$$

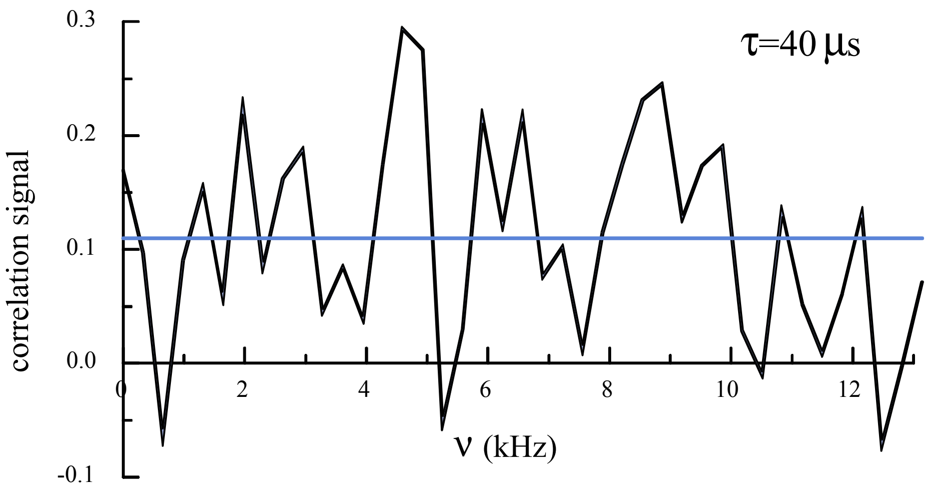
$$= \frac{P_{e,e}}{P_{e,e}+P_{e,g}} - \frac{P_{g,e}}{P_{g,e}+P_{g,g}}$$

  - Independent of Ramsey interferometer frequency
    - when Φ is neither 0 nor $\pi/2$
  - 0.5 for a quantum superposition

$$\eta = \frac{1}{2}\mathrm{Re}\langle\alpha|\alpha\rangle$$

  - 0 for a statistical mixture
  - 0 for an empty cavity

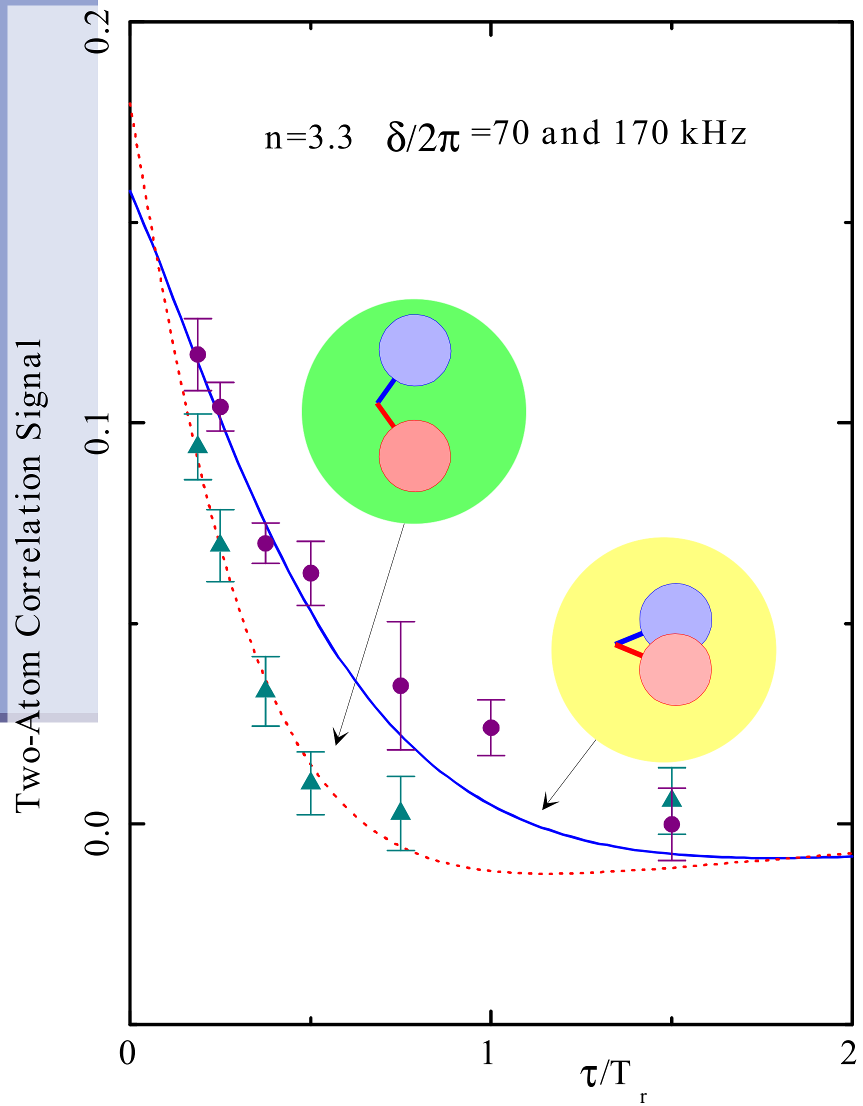
- Principle of the experiment
  - Send a first atom to prepare the cat
  - Wait for a delay τ
  - Send a second probe atom
  - Measure η versus τ
- Raw correlation signals



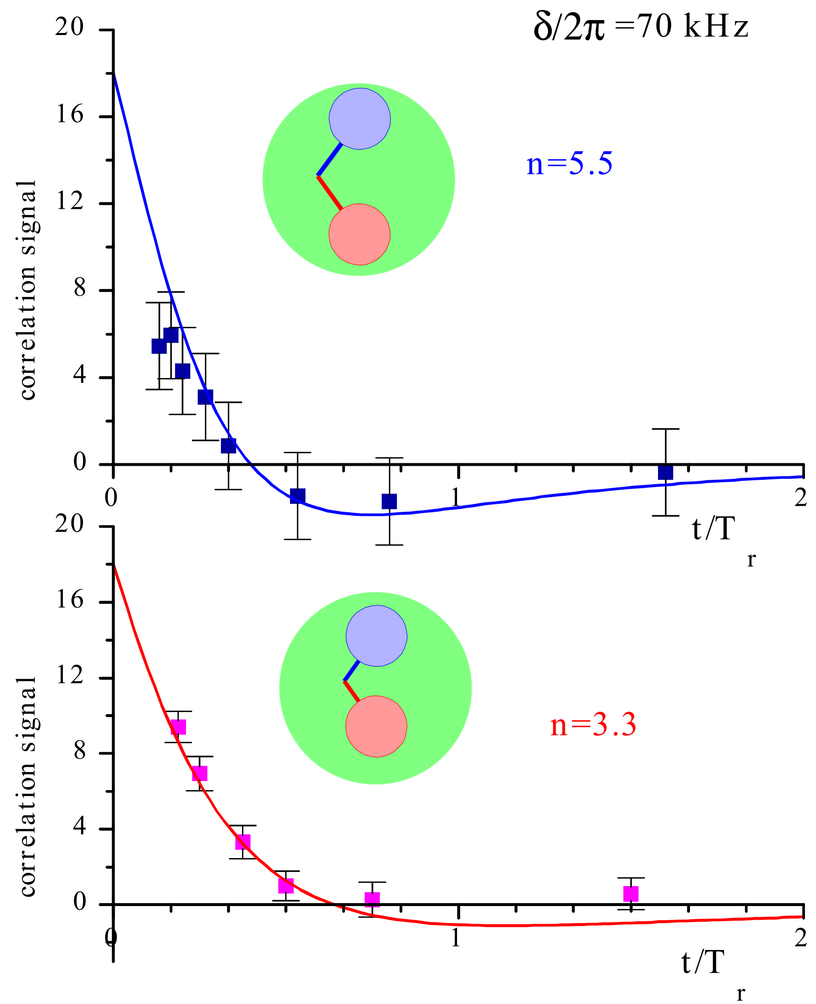
15000 coincidences

Seeing the decoherence over time

Effect of atomic phase shift



Effect of field amplitude



PRL **77**, 4887 (1996)

What can we say from those results

- It is possible to prepare mesocopic superpositions of states
  - Schrödinger kitten
- However the coherence is lost very very very very fast
  - The larger the cat, the faster the phenomenon
  - Experimental data in good agreement with theory $T_{dec}=T_{rel}/D$
  - In our case D goes up to 9 photons

How to obtain larger cat states?

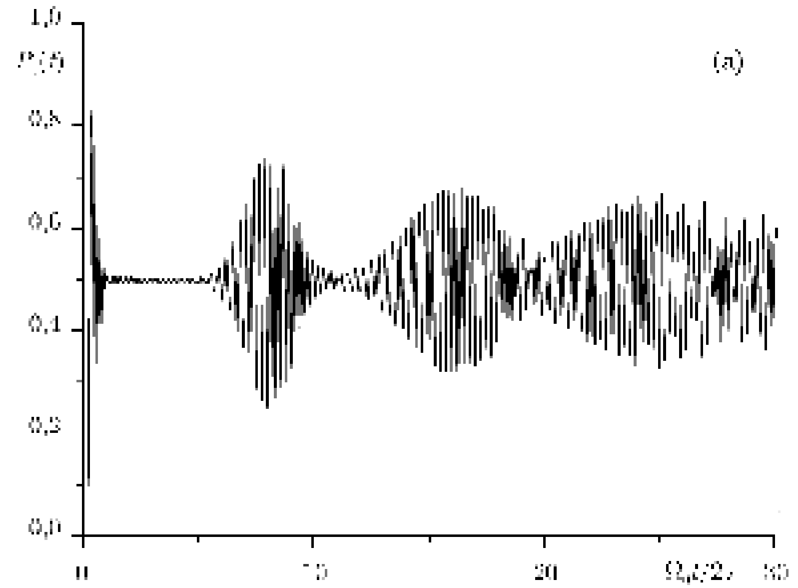
- Increase cavity lifetime (i.e. T_{rel})
- Increase coupling
  - Dispersive interaction $\propto \Omega^2/\delta$ with $\delta >> \Omega$
  - Resonant interaction $\propto \Omega$
- Increase interaction time (lower atomic velocity)
- What is the effect of a resonant atom on a mesoscopic coherent field?

Rabi oscillation in a mesoscopic field

- Intermediate regime of a few tens of photons. A first insight $|\alpha\rangle = \sum_n c_n|n\rangle$;
- A simple theoretical problem

$$|\Psi(t)\rangle = \sum_n c_n \cos\frac{\Omega_0\sqrt{n+1}t}{2}|e,n\rangle + c_n \sin\frac{\Omega_0\sqrt{n+1}t}{2}|g,n+1\rangle \; ;$$

$$P_e(t) = \sum_n p_c(n)\frac{1+\cos\Omega_0\sqrt{n+1}t}{2}$$



Collapse and revival

- Collapse: dispersion of field amplitudes due to dispersion of photon number

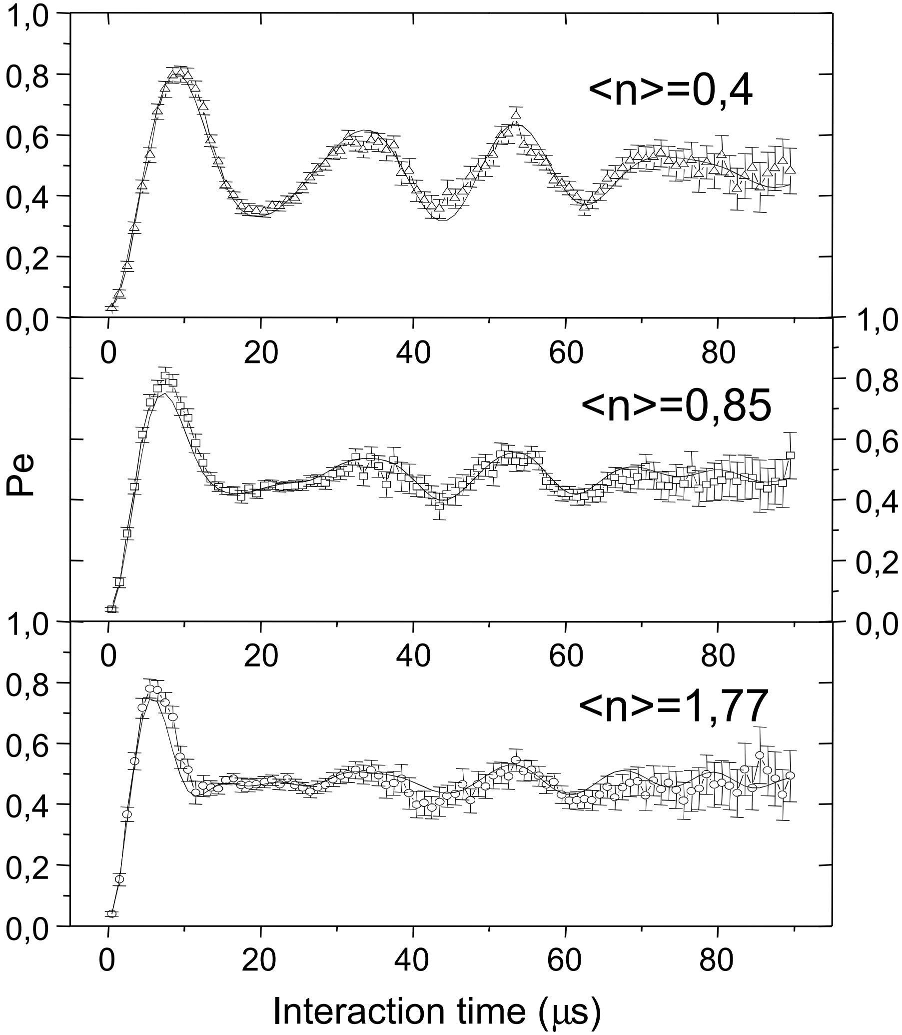
$$t_c \approx \pi / \Omega_0 \; .$$

- Revival: rephasing of amplitudes at a finite time such that oscillations corresponding to n and n+1 come back in phase

$$t_r \approx \frac{4\pi}{\Omega_0} \sqrt{\overline{n}} \; .$$

- Revival is a genuinely quantum effect

Oscillations in a small coherent field

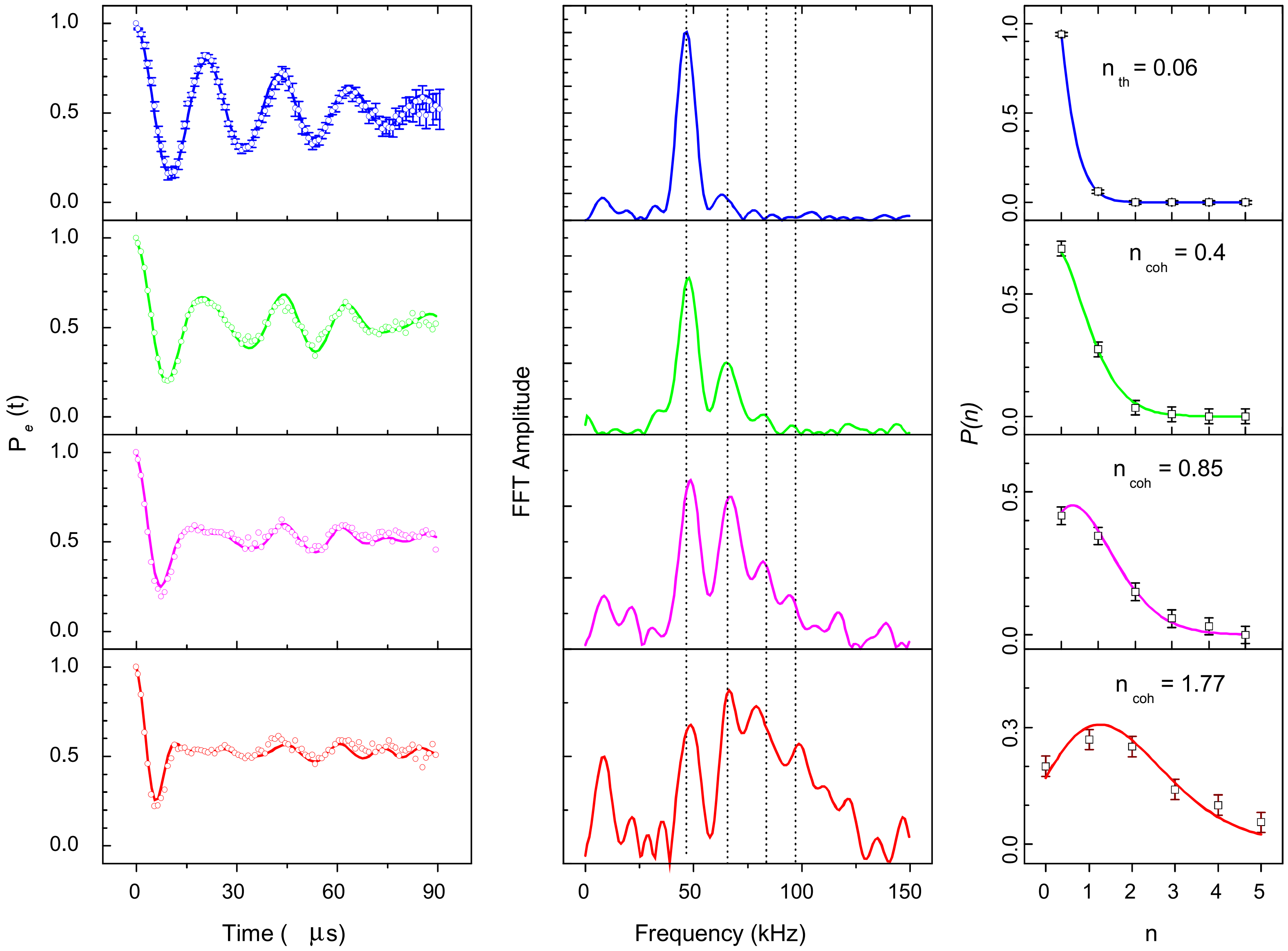


Brune *et al.*, PRL **76**, 1800 (1996)

- Initial state $|e,\alpha\rangle$
  - $|\alpha|^2=\langle n\rangle$
- Observation of the collapse and revival
  - A fourier transform reveals the different frequencies $\Omega(n)=\Omega_0\sqrt{n}$
- Open questions
  - How is the field affected?
  - What is happening if <n> increases (classical limit)

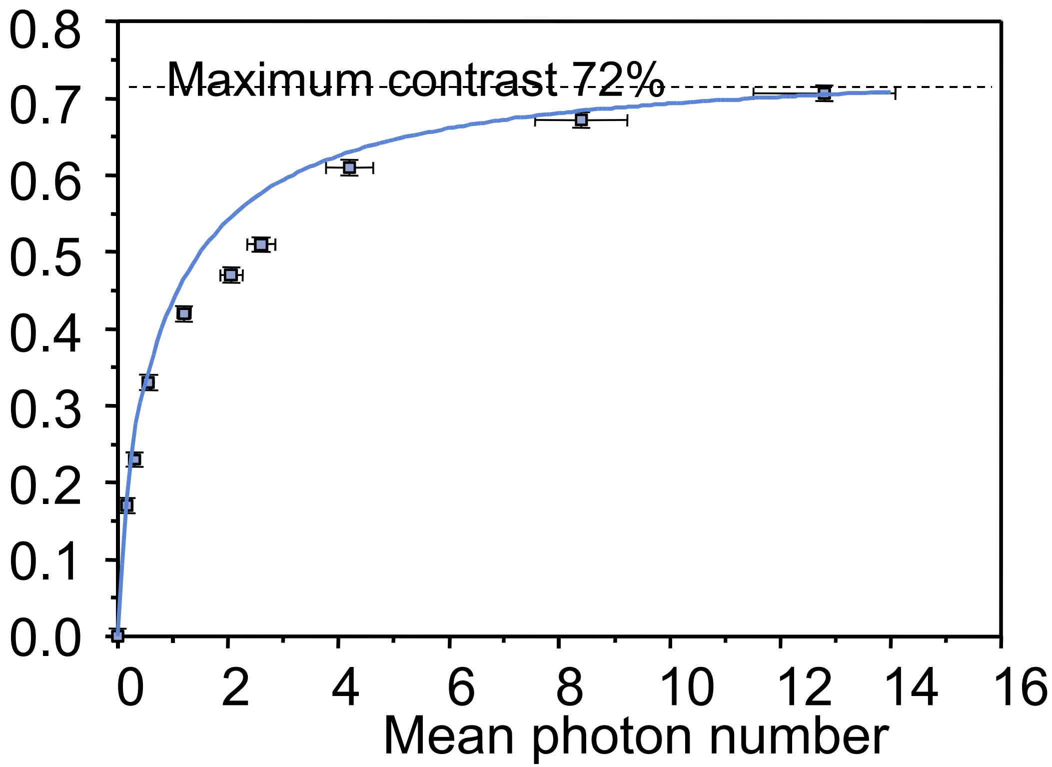
All results

N=0
0.4
0.85
1.77



Remark at short time

- time necessary for doing a $\pi/2$ pulse
  - Complementarity experiment shows that the field is not affected



A more detailed analysis of Rabi oscillation

- Valid in the case of mesoscopic fields $\Delta n \ll \bar{n}$
- Expansion of $|\Psi\rangle$ around n=<n> (Gea Banacloche PRL **65**, 3385, Buzek et al PRA **45,** 8190)
  - First non-trivial order: $\left|\Psi(t)\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\Psi_a^+(t)\right\rangle\left|\Psi_c^+(t)\right\rangle + \left|\Psi_a^-(t)\right\rangle\left|\Psi_c^-(t)\right\rangle\right]$

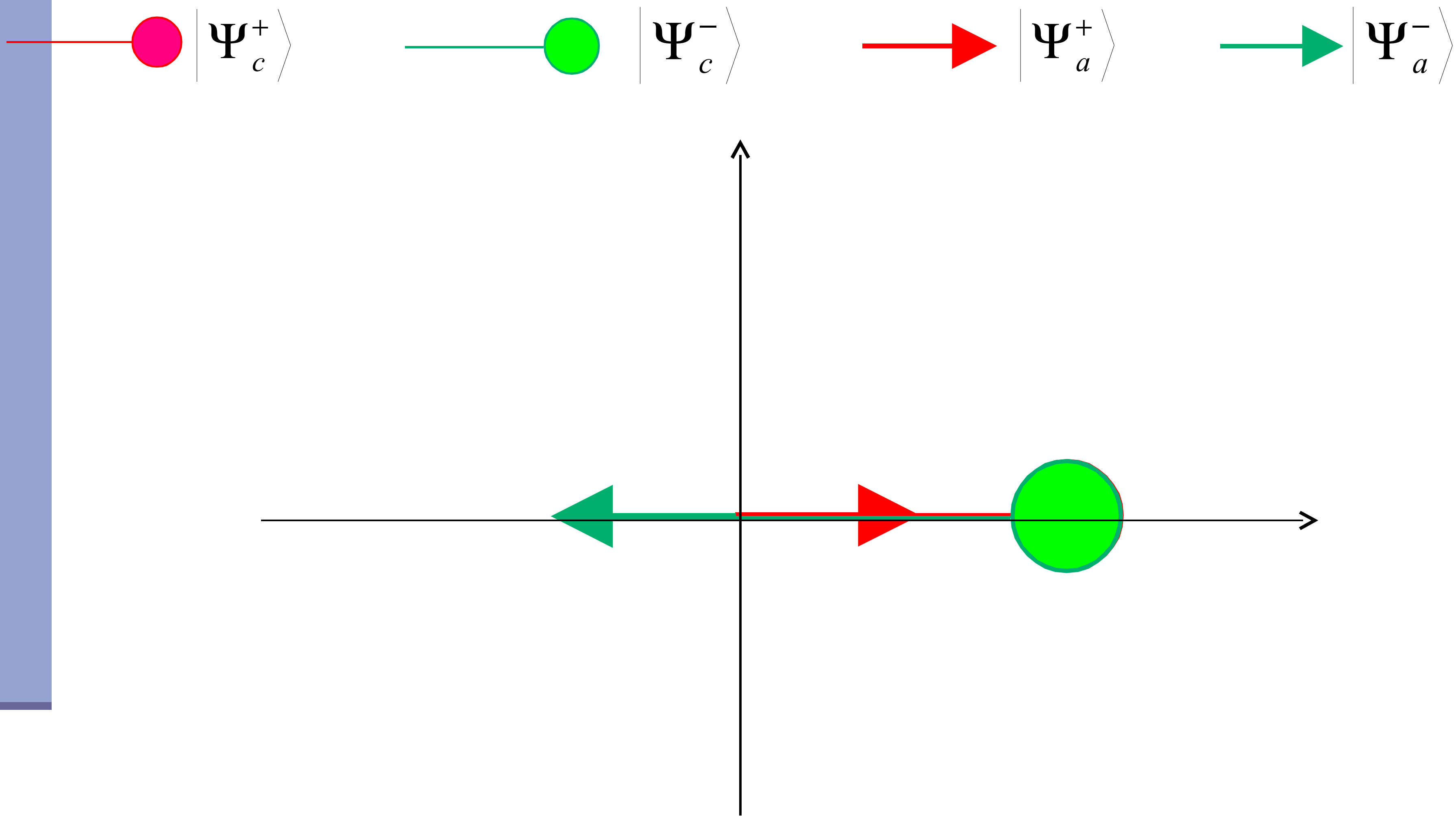
$$\left|\Psi_a^{\pm}\right\rangle = \frac{1}{\sqrt{2}} e^{\pm i\Omega_0\sqrt{\bar{n}}t/2}\left[e^{\pm i\Phi}\left|e\right\rangle \mp i\left|g\right\rangle\right] \qquad \Phi = \frac{\Omega_0 t}{4\sqrt{\bar{n}}}$$

  - Atomic states slowly rotating in the equatorial plane of the Bloch sphere (<n> times slower than Rabi oscillation)

$$\left|\Psi_c^{\pm}\right\rangle = e^{\mp i\Omega_0\sqrt{\bar{n}}t/4}\left|\alpha e^{\pm i\Phi}\right\rangle$$

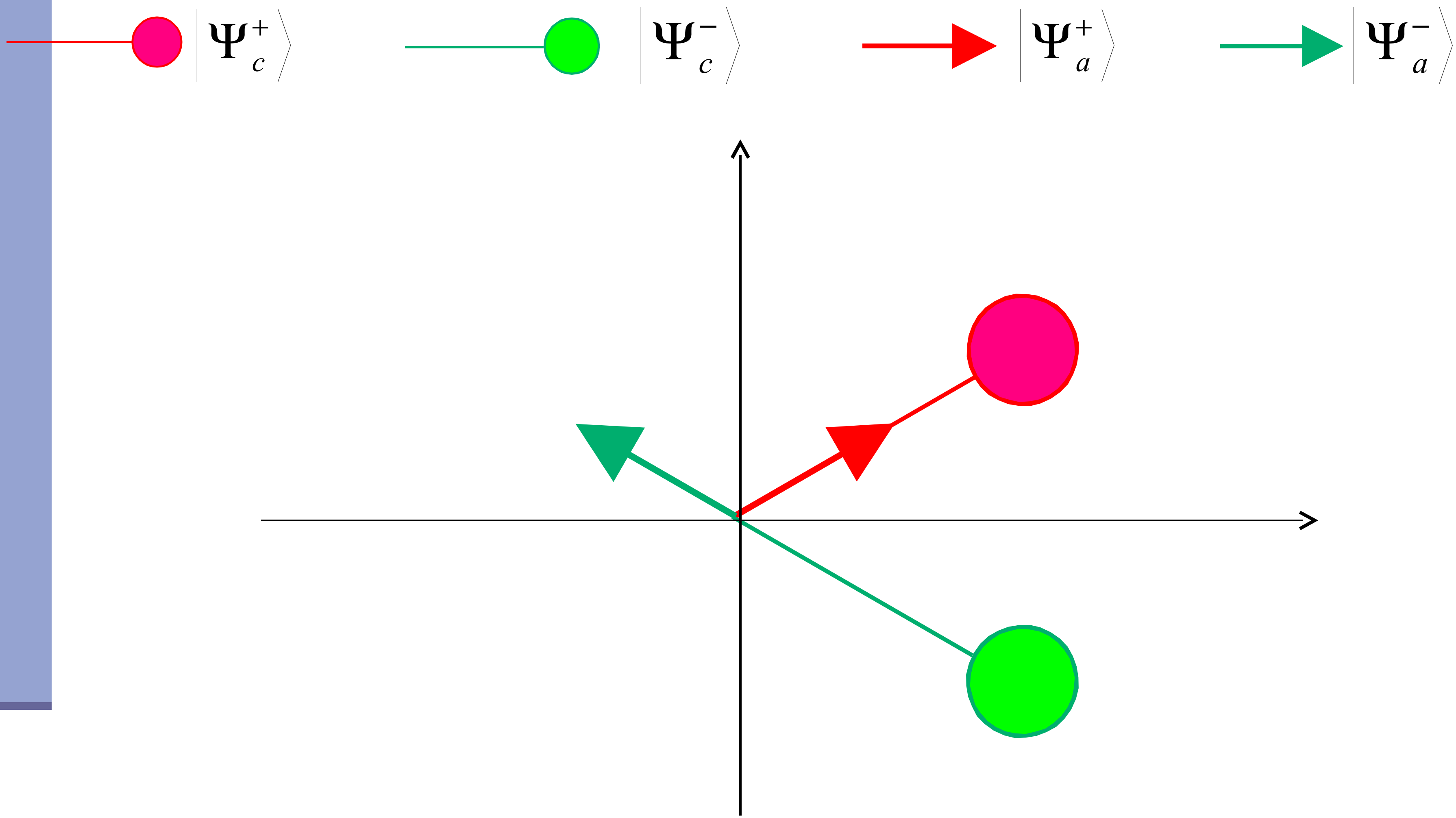
  - A slowly rotating field state in the Fresnel plane
- Graphical representation of the joint atom-field evolution in a plane
  - t=0:
    - both field states coincide with original coherent state
    - Atomic states are the classical eigenstates

Atom-field states evolution



Initial state: a coherent superposition of two states

Atom-field states evolution

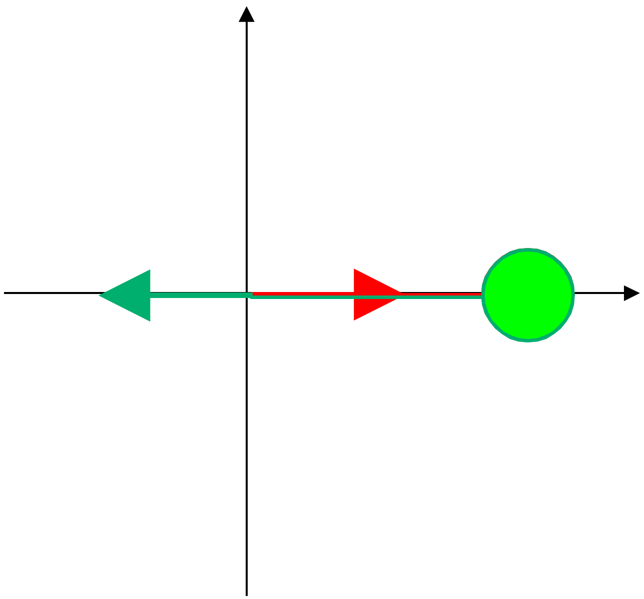


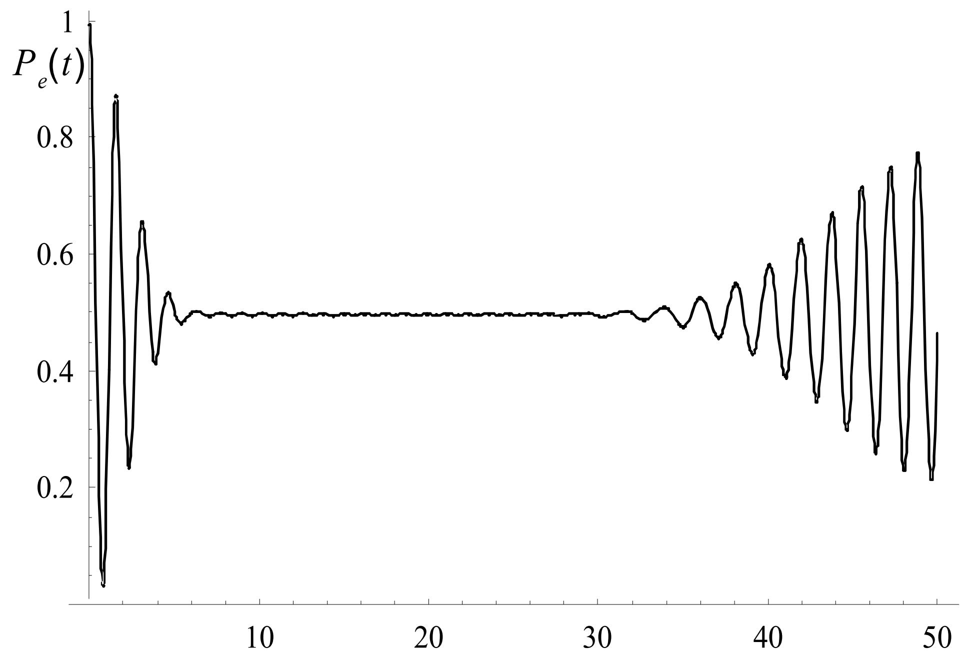
- At most times: $\langle \Psi_c^- | \Psi_c^+ \rangle = 0$ an atom-field entangled state
- In spite of large photon number: considerable reaction of the atom on the field

Link with collapses and revival

Rabi oscillation: interference between $|\Psi_a^+\rangle$ and $|\Psi_a^-\rangle$

- Contrast vanishes when $\langle \Psi_c^- | \Psi_c^+ \rangle$
  - A direct link between Rabi collapse and complementarity





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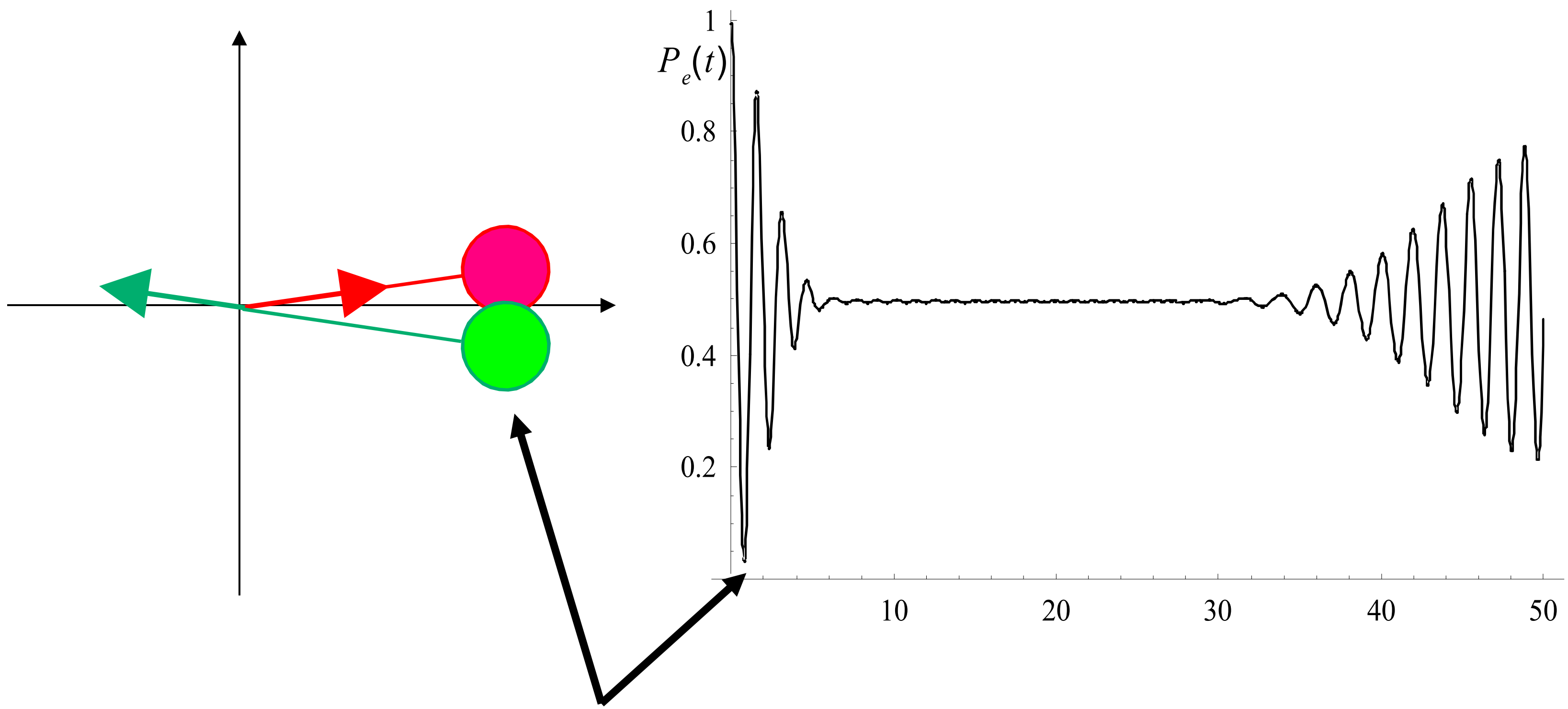

Loss of contrast as the field gets which-path information

Link with collapses and revival

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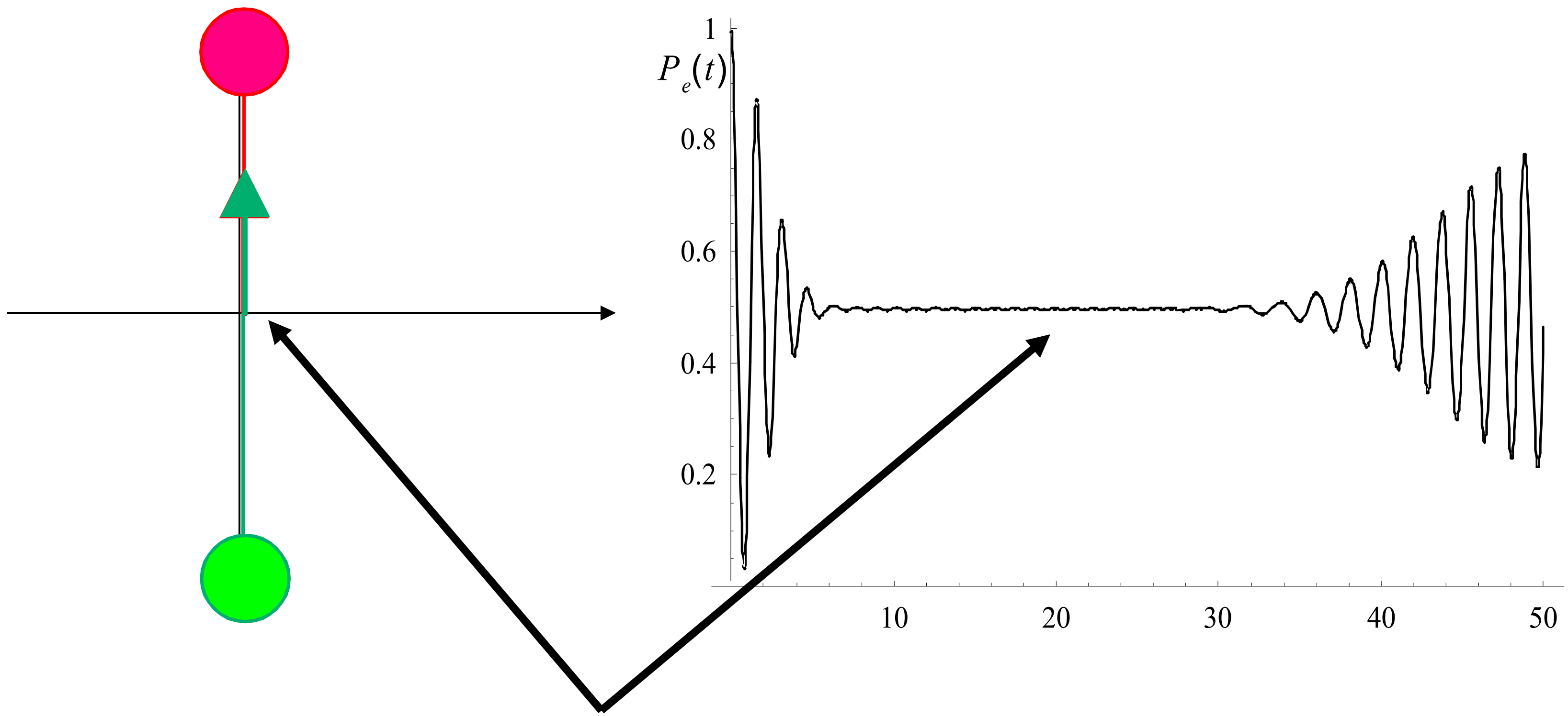

$t_{rev}/2$: atom disentangled from a field in a Schrödinger cat state

Link with collapses and revival

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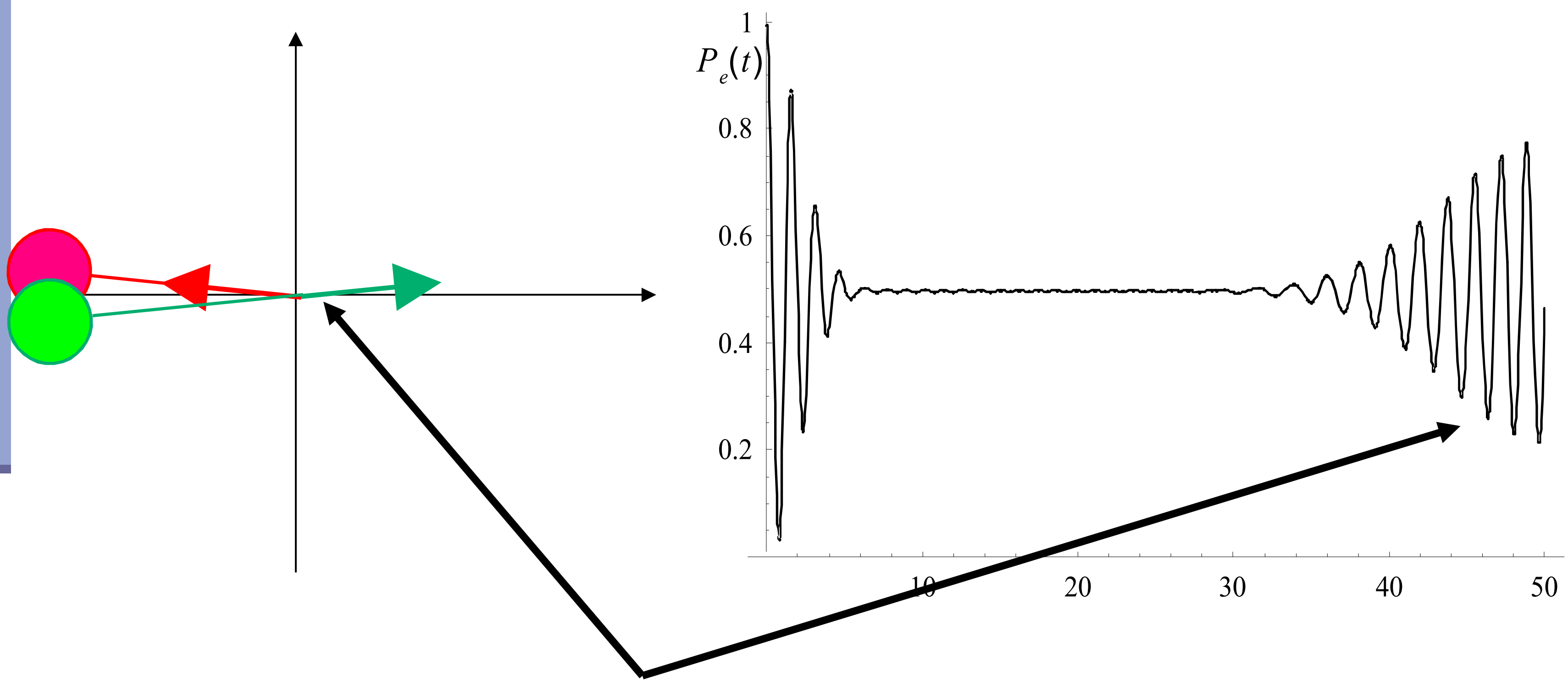
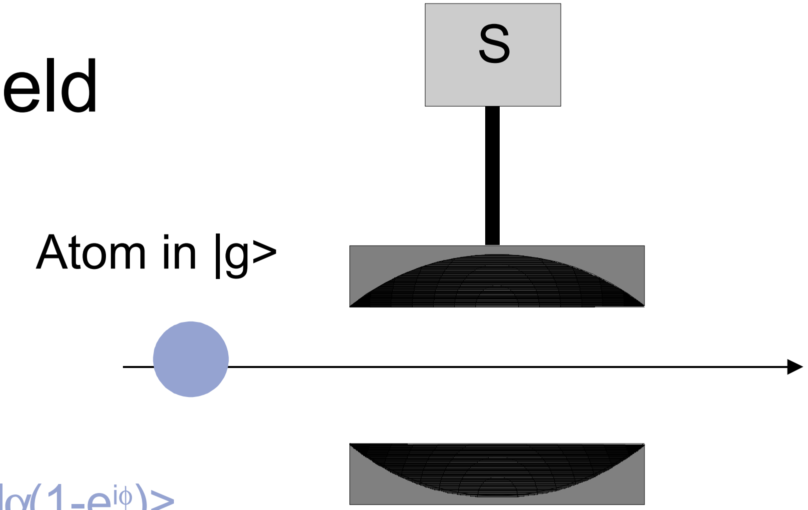


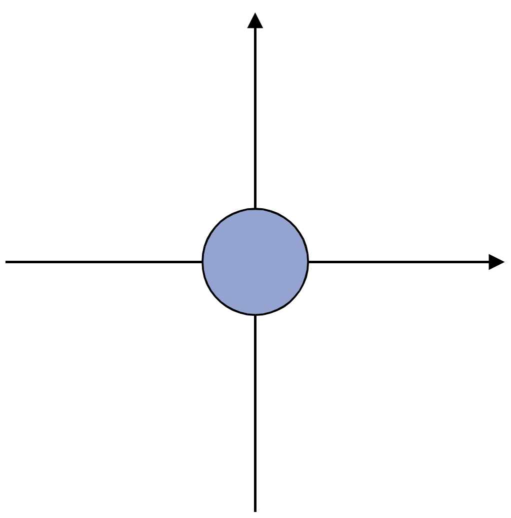
revival: field phase information is erased

Phase distribution measurement

Heterodyning a coherent field



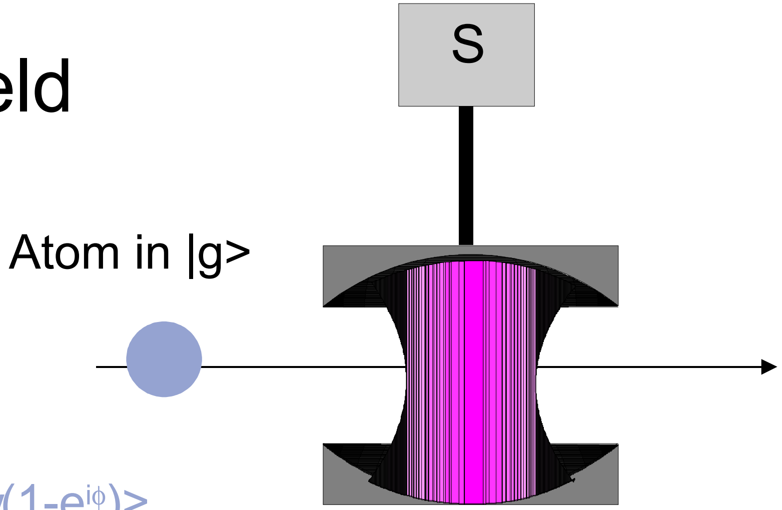
- Inject a coherent field $|\alpha\rangle$
- Add a coherent amplitude $-\alpha e^{i\phi}$
    - Resulting field (within global phase) $|\alpha(1-e^{i\phi})\rangle$
    - Zero final amplitude for $\phi=0$



- Probe final field amplitude with atom in *g*
    - $P_g=1$ for a zero amplitude
    - $P_g=1/2$ for a large amplitude
- More generally: $P_g(\phi)$ reveals field phase distribution
    - In technical terms, $P_g(\phi)$=Q distribution

Phase distribution measurement

Heterodyning a coherent field



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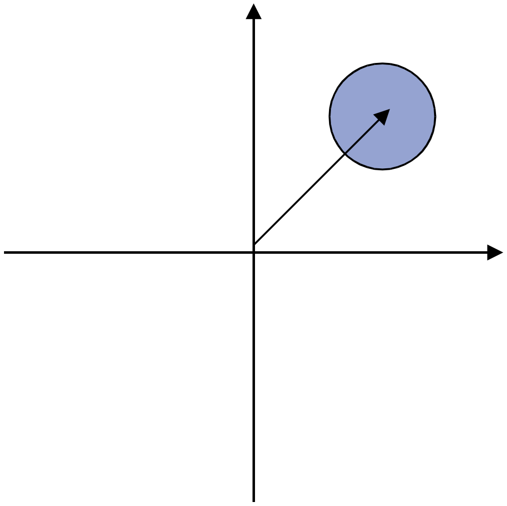

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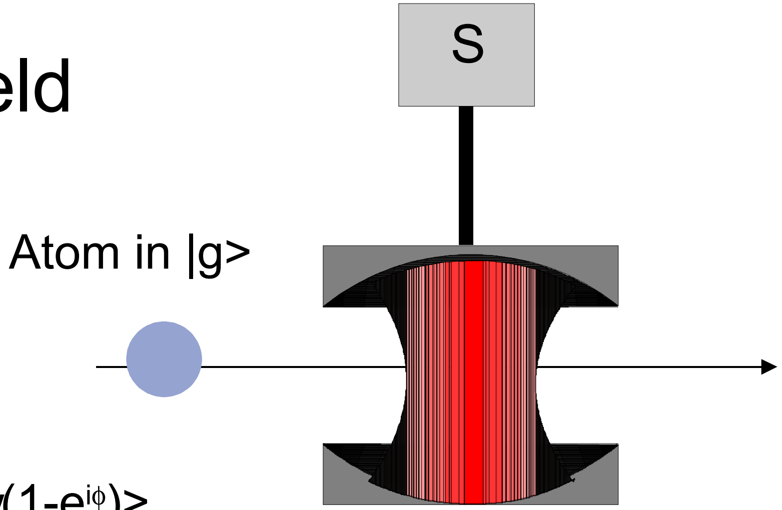

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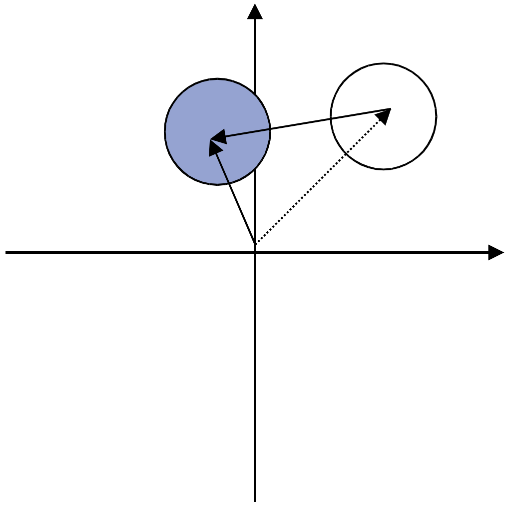

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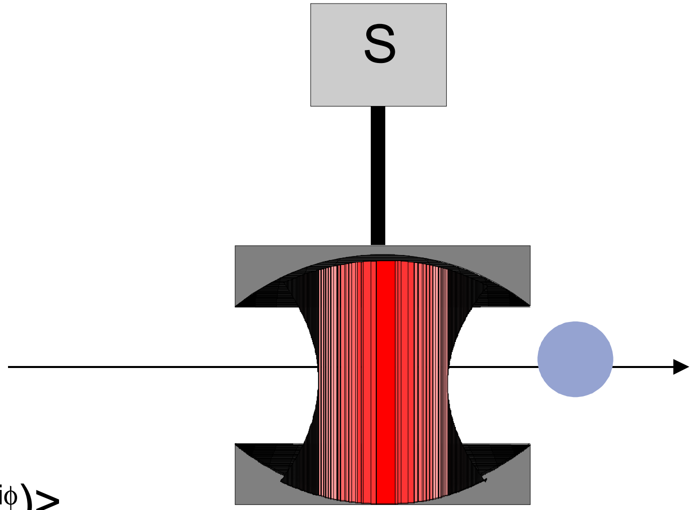

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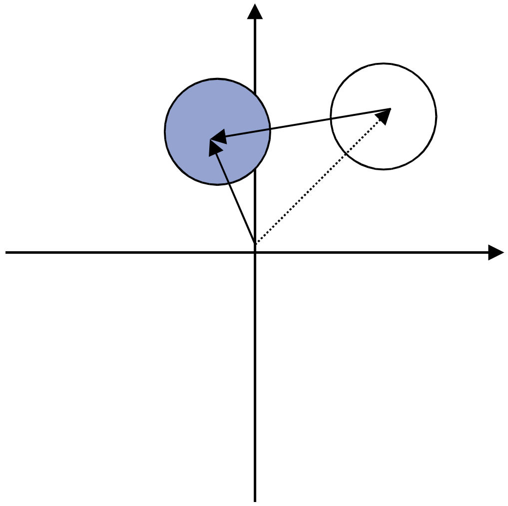
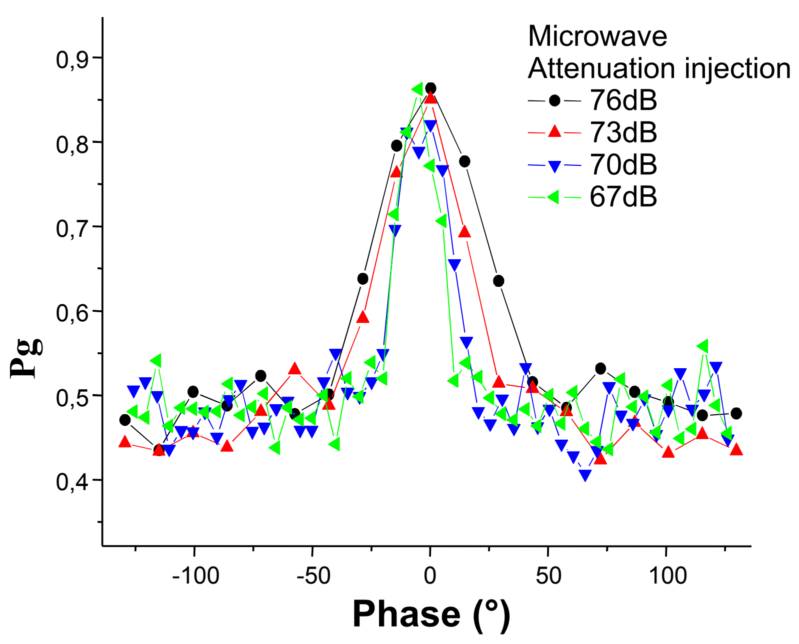


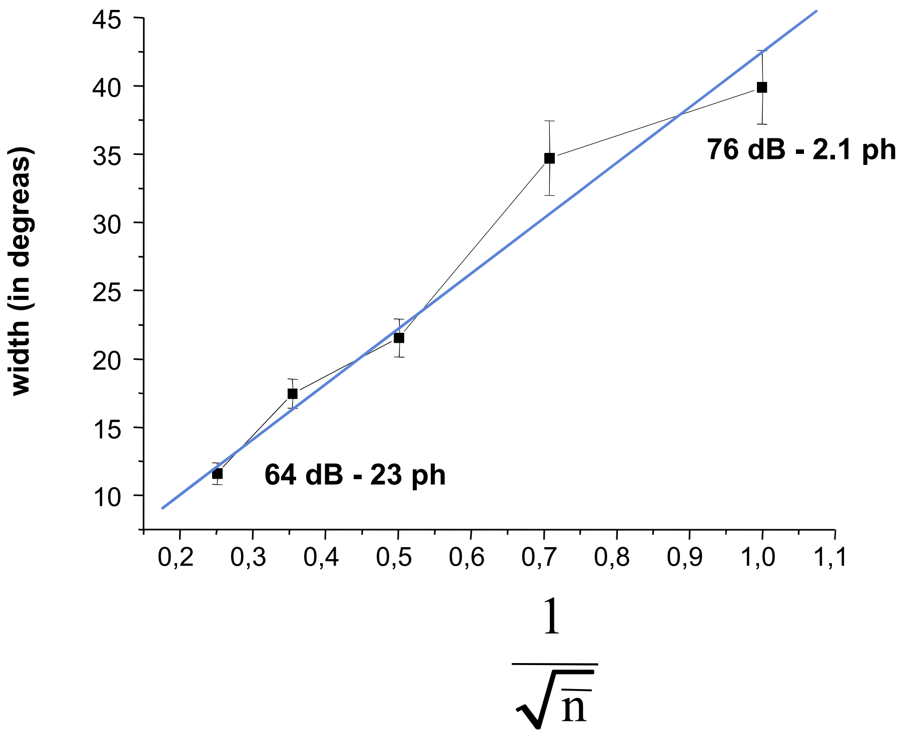
- More generally: $P_g(\phi)$ reveals field phase distribution
  - In technical terms, $2xP_g(\phi)-1=Q(\alpha)$ distribution

Coherent field phase distribution

Homodyning signal



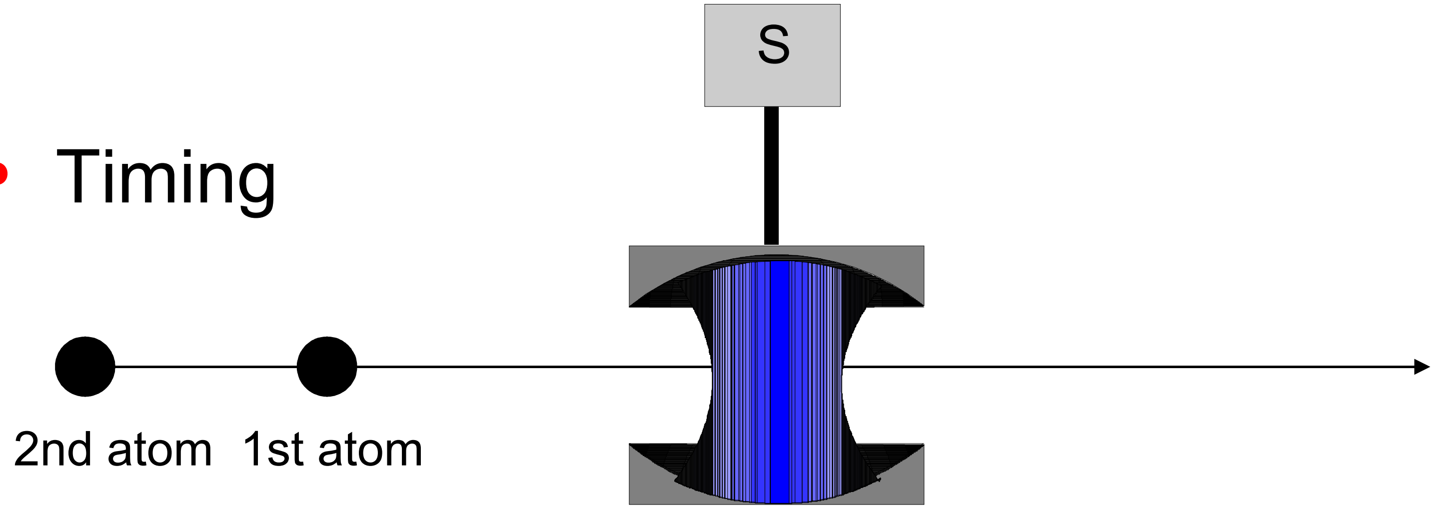
Width versus <n>



- One can apply the same phase measurement after interaction with an atom
  - Detection of states $|\Psi_c^+\rangle$ and $|\Psi_c^-\rangle$

Phase splitting in quantum Rabi oscillation

- Timing



•Inject a coherent field

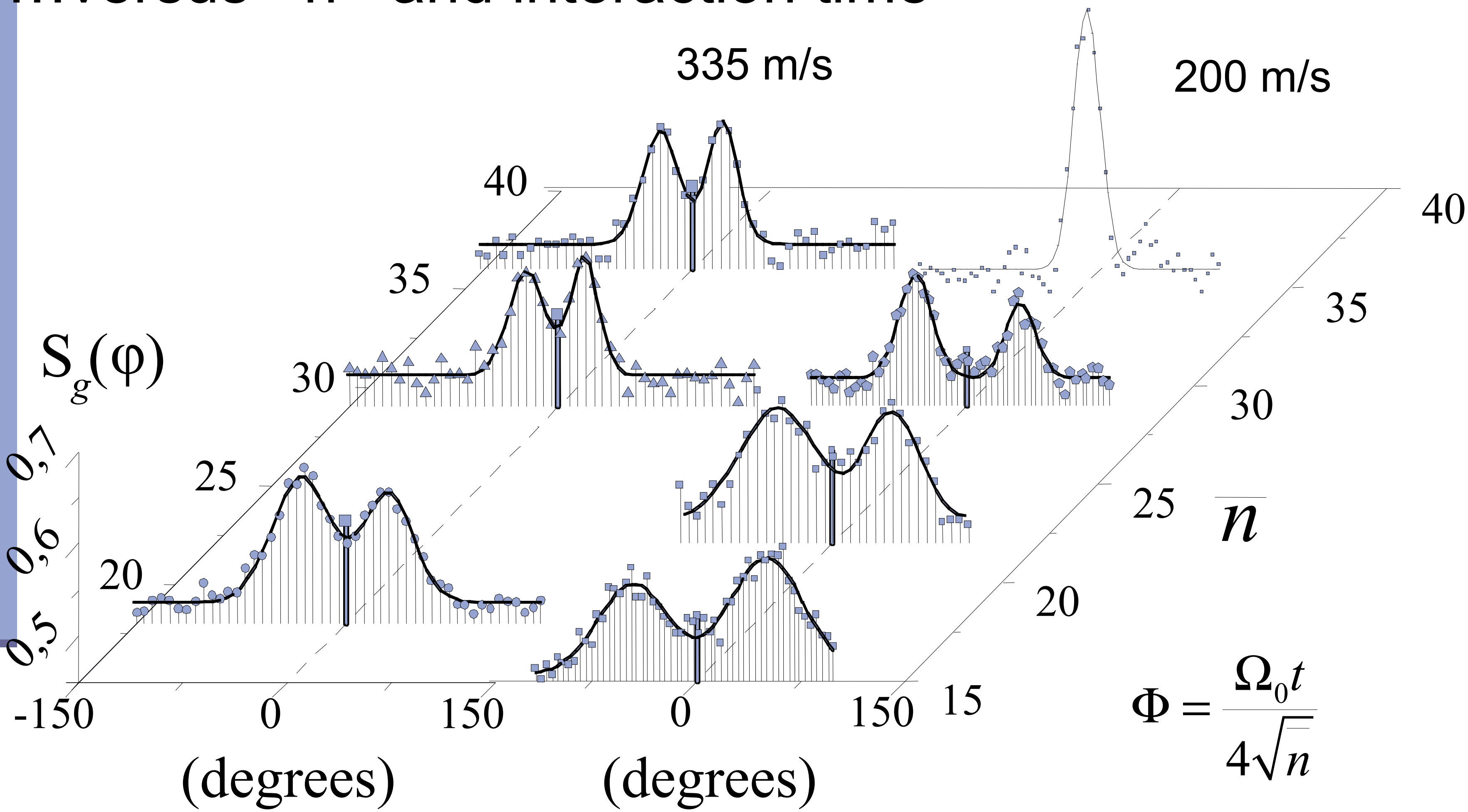
•Send a first atom: Rabi oscillation and phase shift

•Inject a phase tunable coherent amplitude

•Send a second atom in *g*: final amplitude read out

Phase splitting of a mesoscopic field

…versus <n> and interaction time

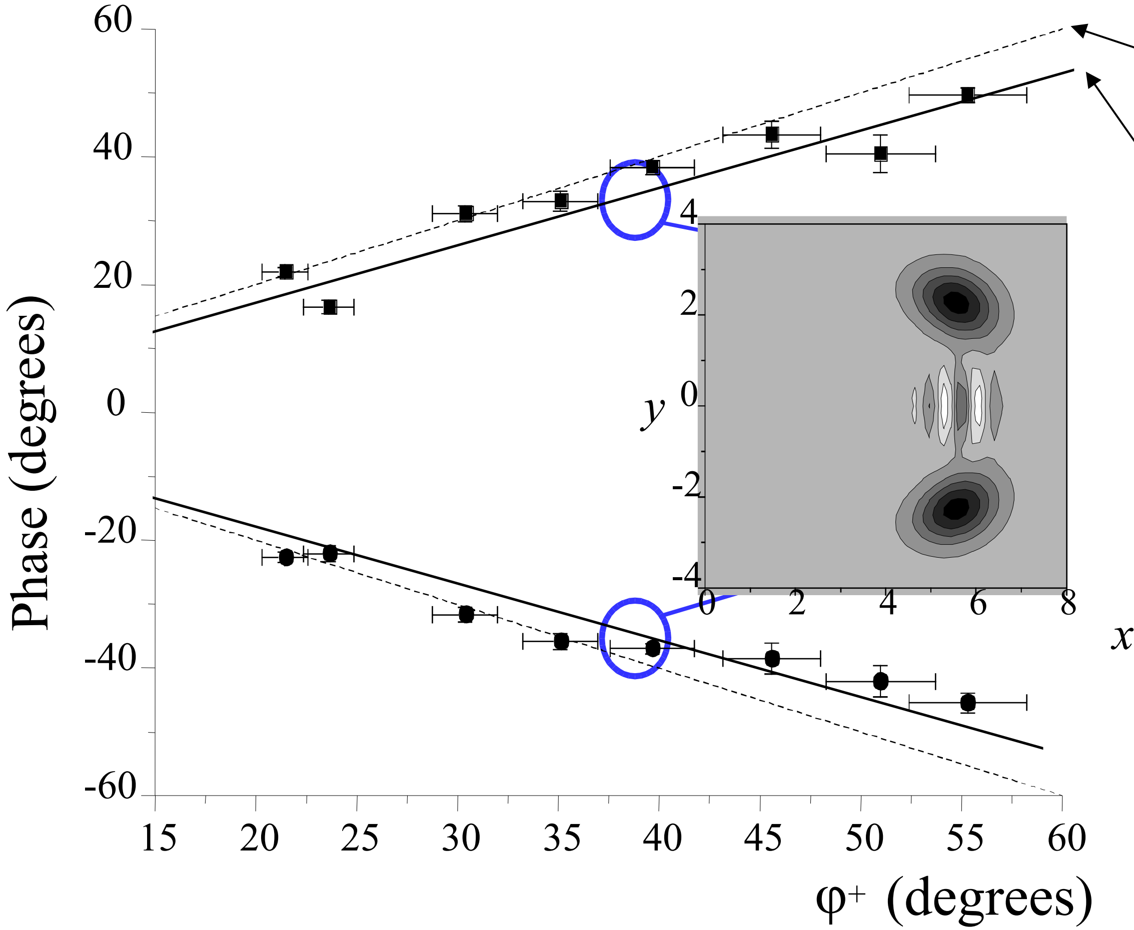


$$\Phi = \frac{\Omega_0 t}{4\sqrt{n}}$$

A. Auffèves *et al.*, Phys. Rev. Lett. **91**, 230405 (2003)

Phase splitting in quantum Rabi oscillation

Observed phase versus theoretical phase



$$\Phi = \frac{\Omega_0 t}{4\sqrt{n}}$$

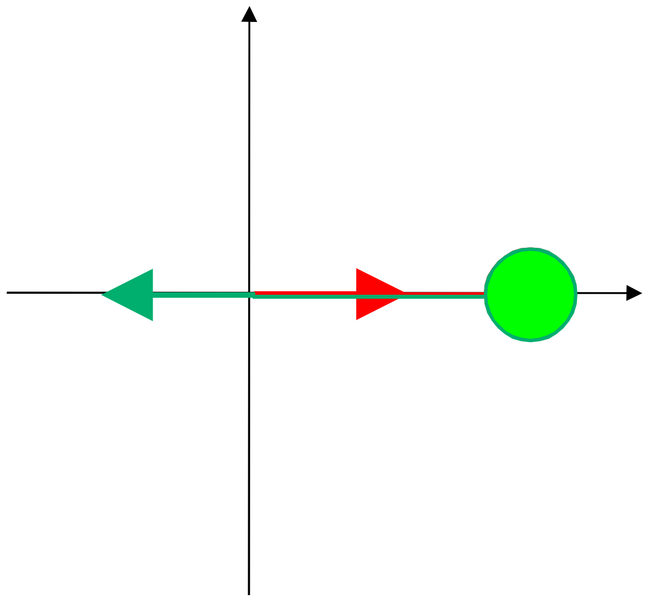
Numerical simulation of field master equation

Large Shrödinger cat states (up to 40 photons separation)

How to test that one has a coherent superposition for the field state

- Interference between the two states
- Waiting for the revival will do the trick
  - If atomic populations oscillates again then it proves the coherence
- But interaction time is limited
  - Maximum interaction time 100μs for atoms at 100 m/s
  - Revival time for n=25: 300μs

Test of coherence: induced quantum revivals



Initial Rabi rotation,

Collapse

And slow phase rotation

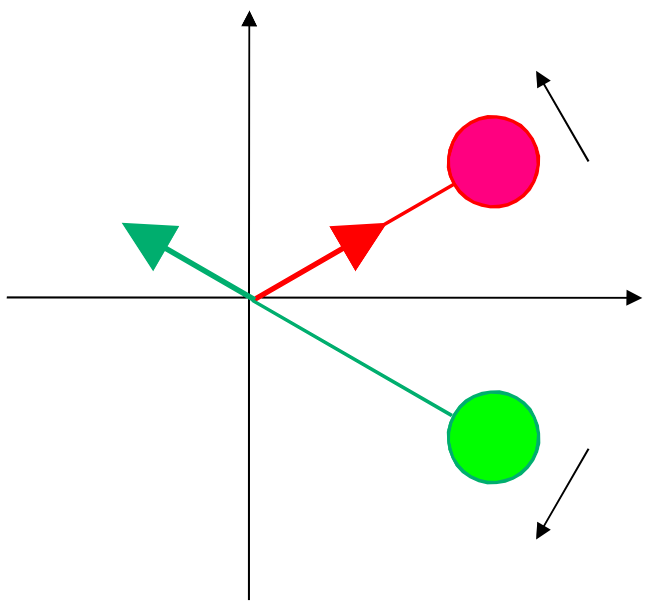
Stark pulse (duration short compared to phase rotation).

Equivalent to a Z rotation by π

Reverse phase rotation

Recombine field components and resume Rabi oscillation

Test of coherence: induced quantum revivals



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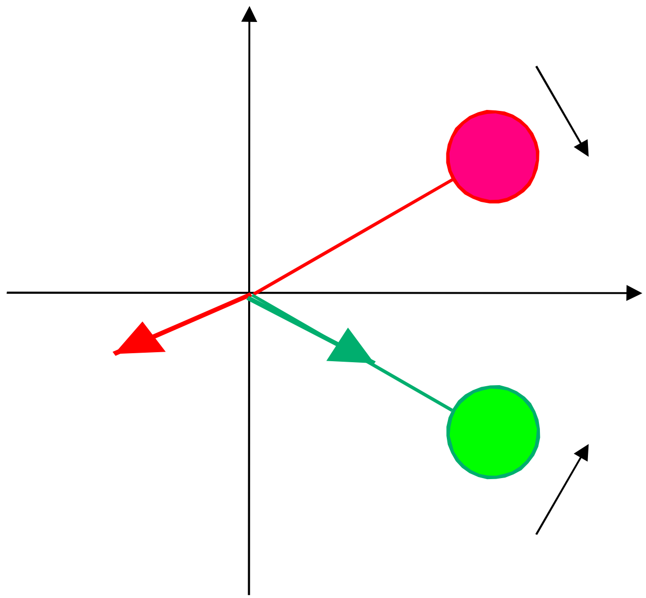

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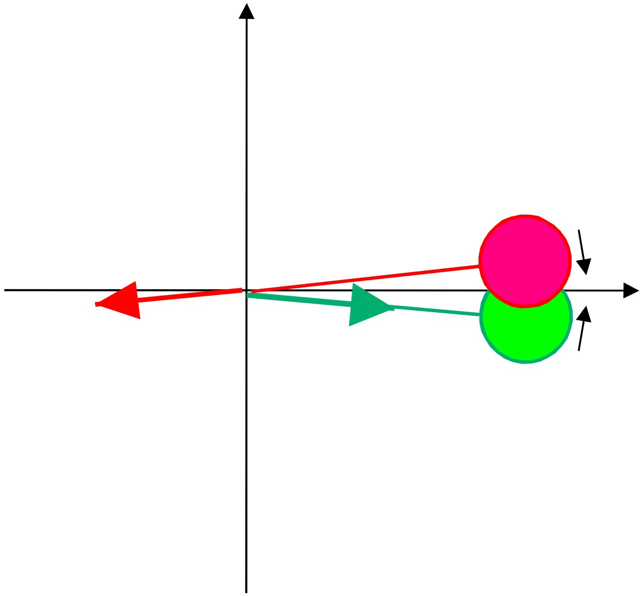

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Collapse

And slow phase rotation

Stark pulse (duration short

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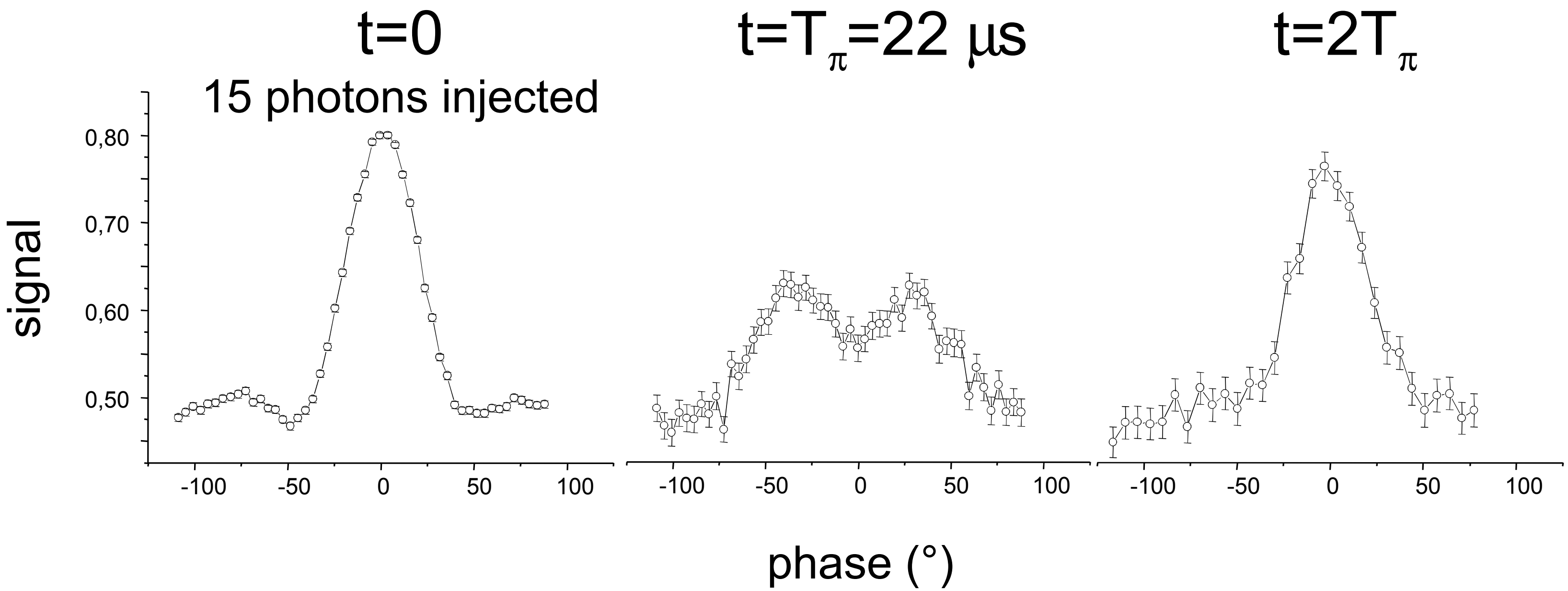
Equivalent to a Z rotation by π

Reverse phase rotation

Recombine field components and
resume Rabi oscillation

Induced quantum revivals (I)

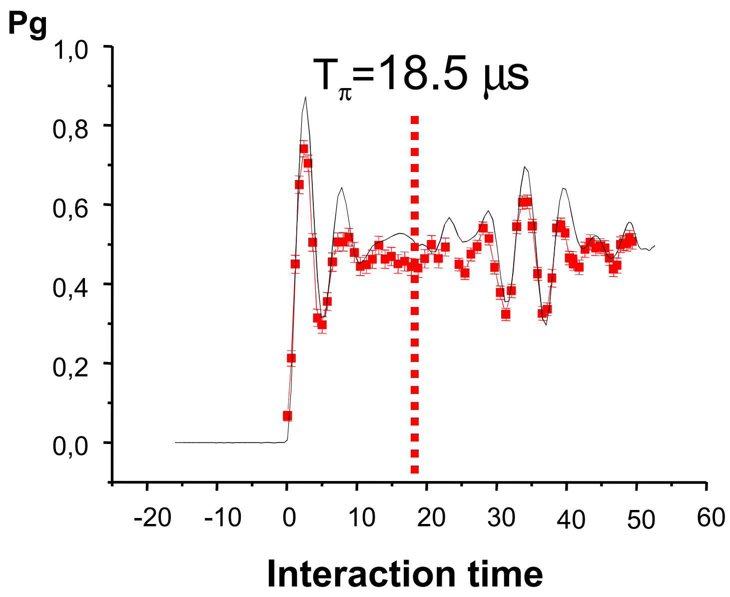
- Effect on the field observed by homodyning

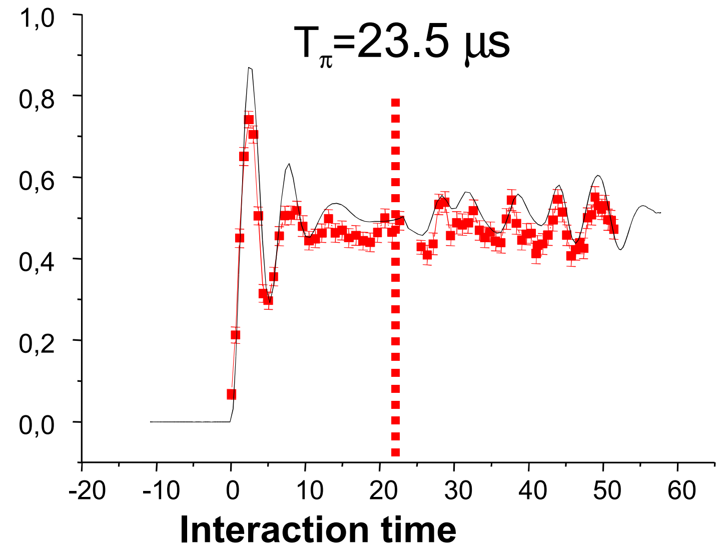


- Two components clearly resolved
  - Separated in phase space by a distance of 9 photons
  - Associated decoherence time 100 μs

Induced quantum revivals (II)

- Atomic population as a function of π pulse delay





- Contrast decreases when T_π increases
  - simulation of the experimental data
  - Main limitation due to inhomogeneous effect (velocity dispersion)
  - Additional decoherence term

Conclusion

- Study of decoherence in the frame of cavity QED
  - A Schödinger cat for a « trapped field »
- In good agreement with theory
  - Effect of the size of the cat
- Close relation with the theory of measurement in quantum physics
  - Possible application and testground for basic Quantum information processing experiments

Lecture 4: future directions

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Collaboration

Luiz Davidovich
Nicim Zagury
Wojtek Gawlik
Daniel Estève

Permanent

Gilles Nogues *
Michel Brune
Jean-Michel Raimond
Serge Haroche

*: atom chip team

Outline

- More about the field
  - QND detection of more than one atom
  - Measurement of the Wigner distribution
- More about the atom
  - QND detection of the atomic state
  - A Schrödinger cat state for a mesoscopic ensemble of atoms
- New experimental tools
  - A two-cavity setup

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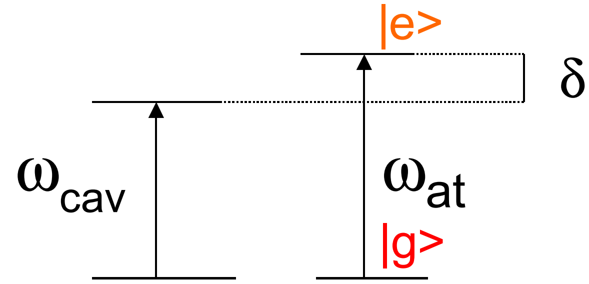
QND detection experiment

- Only works for |0> and |1>
- If one has 2 photons in the cavity
  - Rabi frequency $\Omega_0\sqrt{2} \approx 1.5 x \Omega_0$
  - A 3π pulse is performed $|g,2\rangle \rightarrow |e,1\rangle$
  - One photon is absorbed and atom is neither in *g* or *i*
- 2 main problems
  - A measured qubit can only provide one bit of information
  - Dispersive interaction is required to prevent energy exchange for all photon number

Dispersive interaction

Dispersive regime : $\delta = \omega_{at} - \omega_{cav} > \Omega/2$

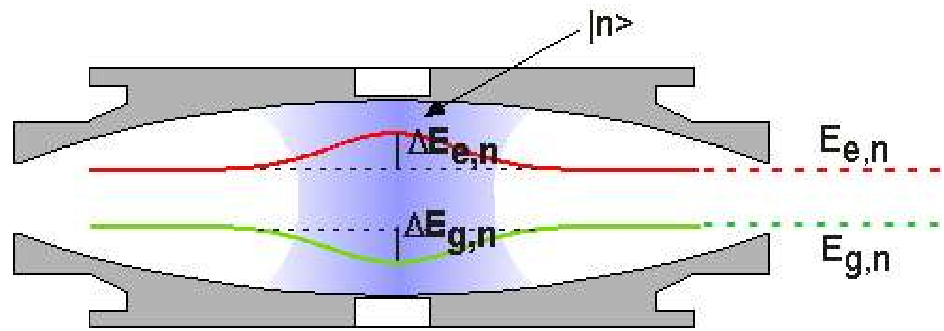


➡ No energy exchange

But : **light shift**

$$\Delta E_{e,n} = \hbar \frac{\Omega^2}{4\delta}(n+1)$$

$$\Delta E_{g,n} = -\hbar \frac{\Omega^2}{4\delta} n$$

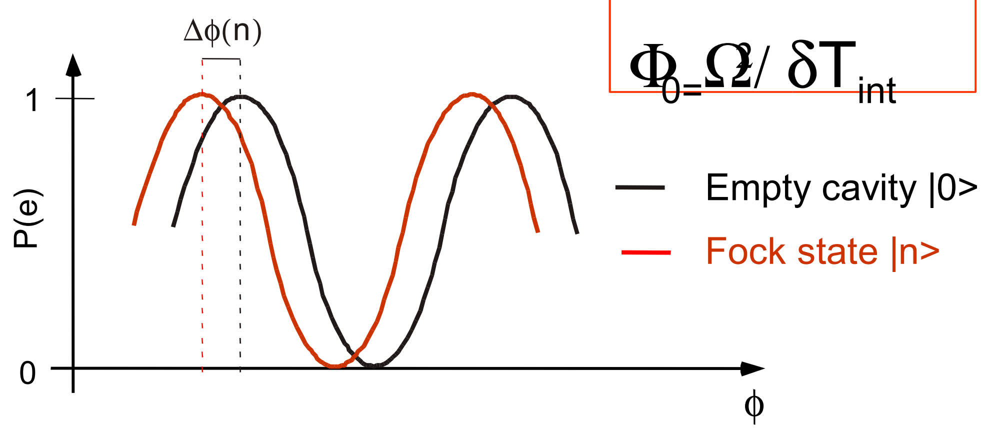


➡ $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \longrightarrow \frac{1}{\sqrt{2}}(|e\rangle + e^{i\Delta\Phi(n)}|g\rangle)$

$\Delta\Phi(n) = \Phi_0 n$

$\Phi_0 = \Omega^2/\delta T_{int}$

➡ **Phase** shift
of Ramsey fringes
on the e-g transition



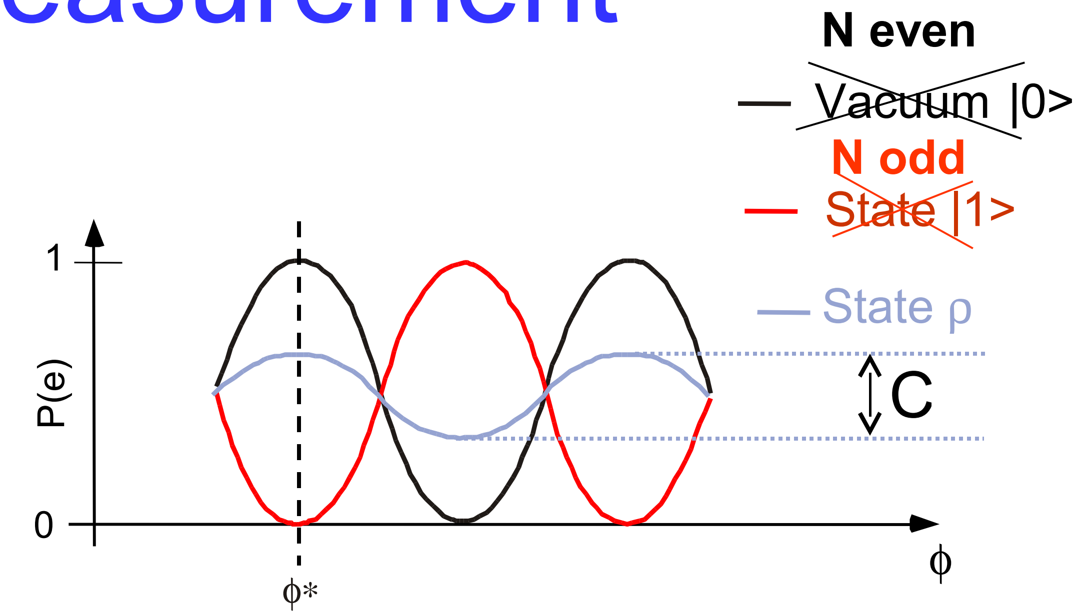
Parity Measurement

$\Phi_0=\pi$

For $\phi=\phi^*$:

If N even, detection in e

If N odd, detection in g



The measurement of the final atomic state gives the parity operator value

$P=e^{ia^+a}$

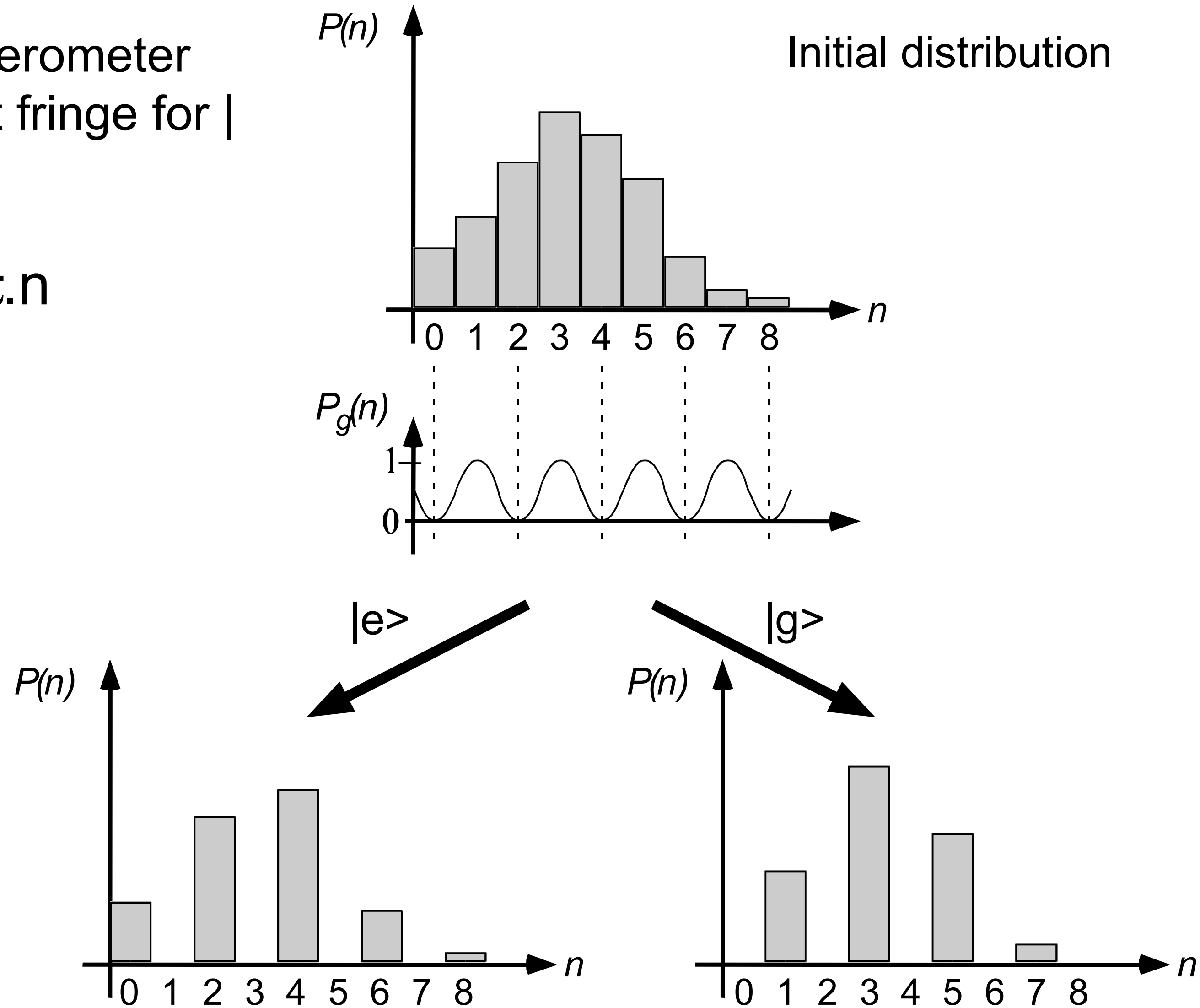
- P=+1 for state |2n>
- P=-1 for state |2n+1>

$$\langle \hat{P} \rangle = \sum_n (-1)^n P(n) = P_{\phi^*}(e) - P_{\phi^*}(g) = C$$

Photon number decimation

Ramsey interferometer set on a bright fringe for |0>

$\Delta\Phi(n)=\Phi_0 n=\pi.n$



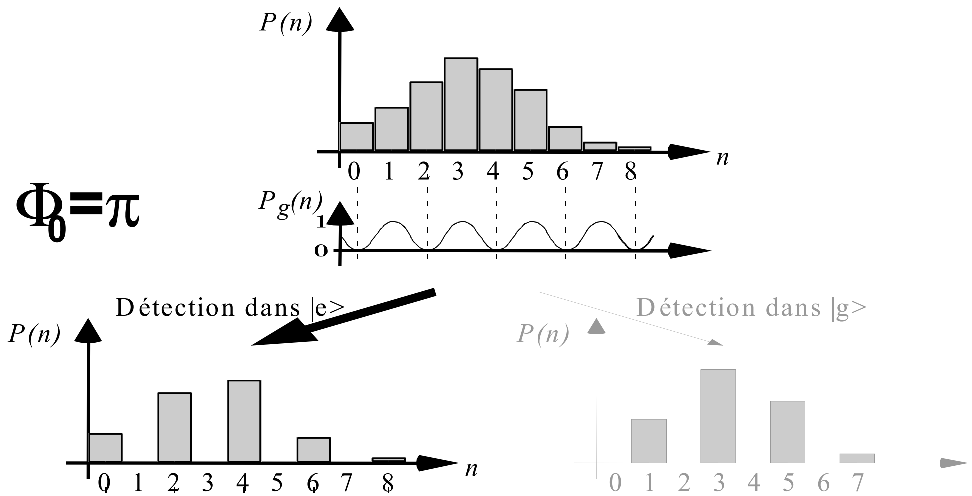
Final conditional probabilities

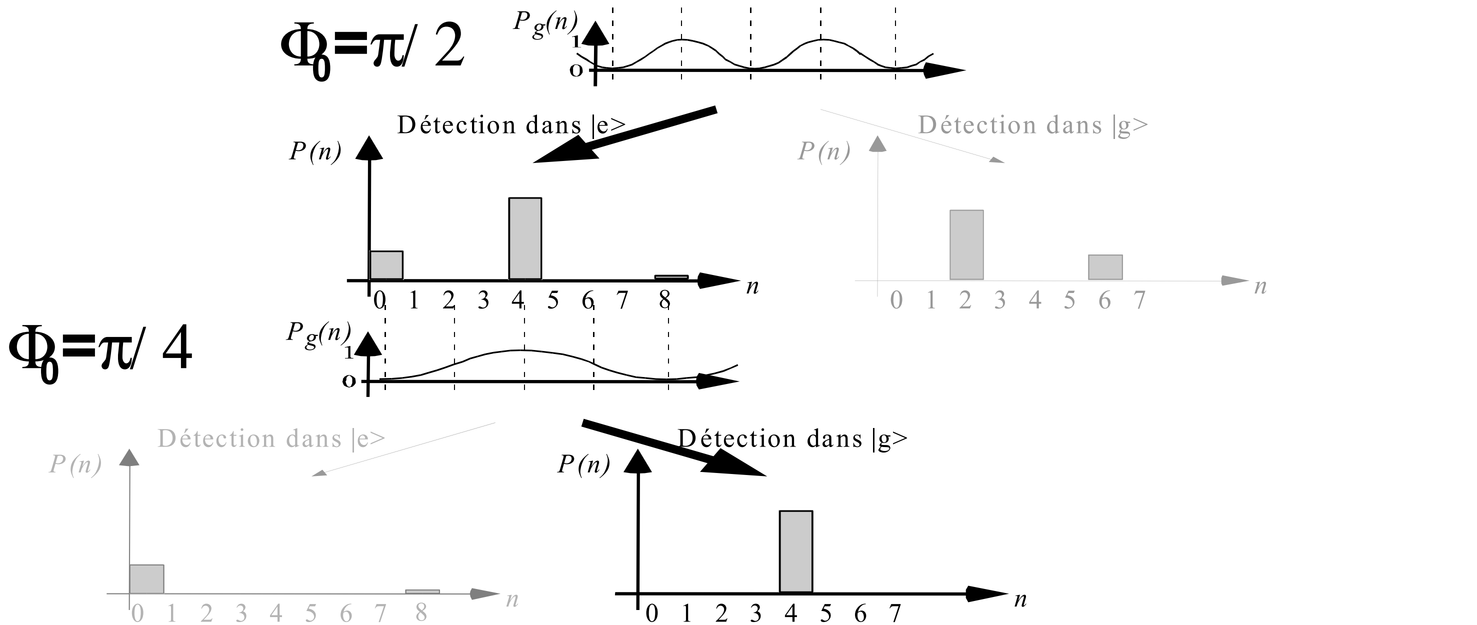
A complete QND measurement

Adapt the phase shift per photon and the interferometer phase at each step

Number of steps:

$N_s = \mathrm{Log}_2(<n>)$





The first step of the QND measurement

Measurement of the Wigner distribution

Wigner function: an insight into a quantum state

JUNE 1, 1932 PHYSICAL REVIEW VOLUME 40

On the Quantum Correction For Thermodynamic Equilibrium

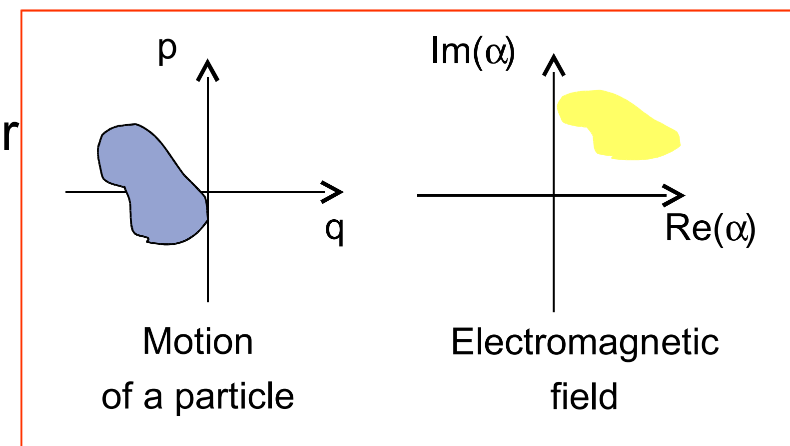
By E. WIGNER
Department of Physics, Princeton University
(Received March 14, 1932)

The probability of a configuration is given in classical theory by the Boltzmann formula exp $[-V/hT]$ where V is the potential energy of this configuration. For high temperatures this of course also holds in quantum theory. For lower temperatures, however, a correction term has to be introduced, which can be developed into a power series of h. The formula is developed for this correction by means of a probability function and the result discussed.

A quasi-probability distribution in phase space.

- Characterizes completely the quantum state
- Negative for non-classical states.

Describes the motion of a particle or a quantum single mode field



Properties

Definition $$W(x,p) = \frac{1}{\pi}\int dx' e^{-2ix'p}\langle x + \frac{x'}{2}|\rho|x - \frac{x'}{2}\rangle \ .$$

By inverse Fourier transform $$\langle x + \frac{x'}{2}|\rho|x - \frac{x'}{2}\rangle = \int dp e^{2ix'p} W(x,p)$$

In particular
$$\langle x|\rho|x\rangle = \int dp W(x,p)$$

The probability distribution of x is obtained by integrating W over p

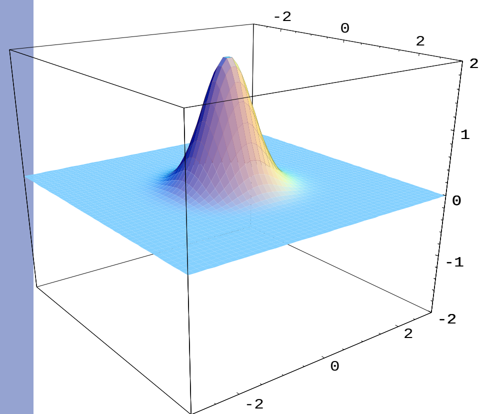
This property should obviously be invariant by rotation in phase space

$$\int W\left(q_\theta \cos\theta - p\ \sin\theta, q\ \sin\theta + p\ \cos\theta\right) dp$$
$$= P\left(q_\theta\right) = \left\langle q \left| \hat{U}^\dagger(\theta)\,\hat{\rho}\,\hat{U}(\theta) \right| q \right\rangle$$

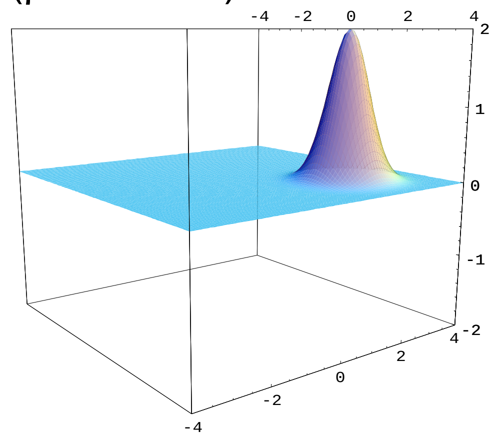
All elements of density matrix derived from W: contains all possible information on quantum state.

Examples of Wigner functions

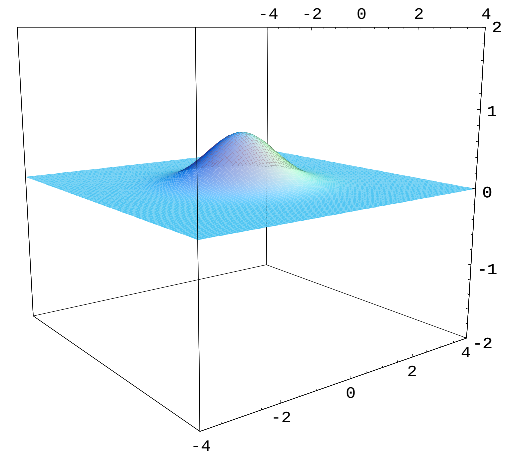
Vacuum |0>



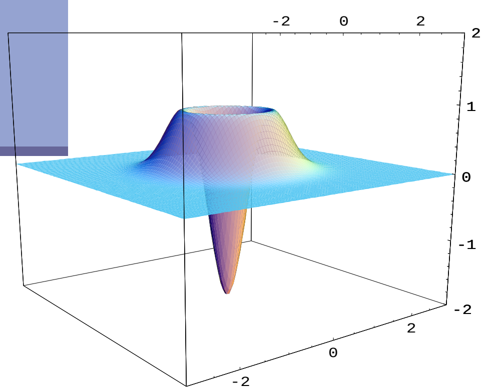
Coherent state |β>
(β=1.5+1.5i)



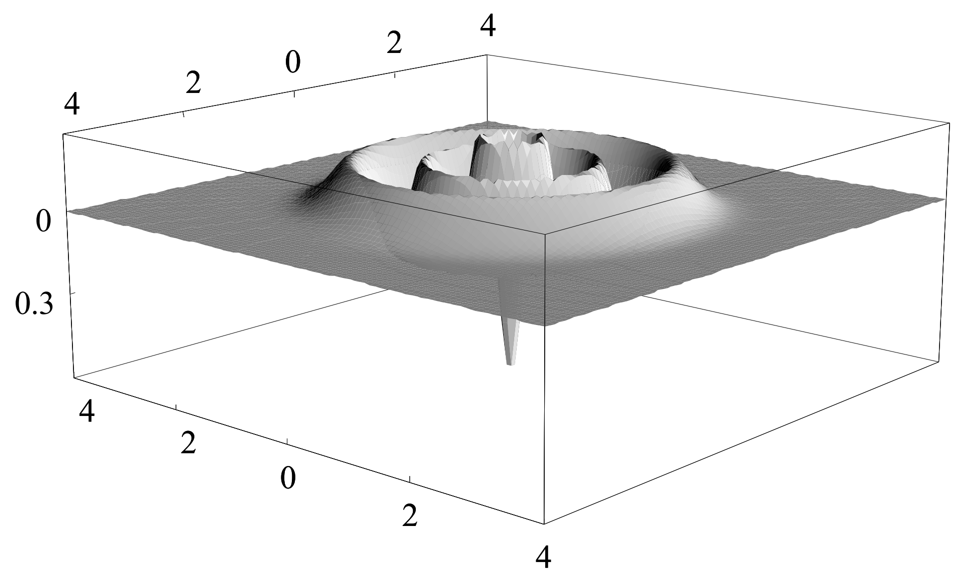
Thermal field n_{th}=1



Fock state |1>



5 photons Fock state



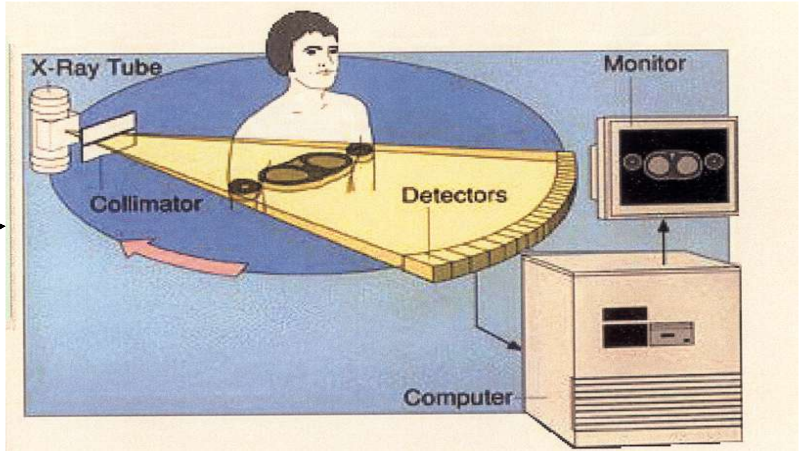
How to measure W for the electromagnetic field ?

Propagating fields : « Tomographic » methods

Principle : - Homodyning measures marginal distributions $P(q_\theta)$ for different θ

- inverse Radon transform allows reconstruction of W(q,p)

(⟺ medical tomography)



Refs : - Coherent and squeezed states : - Smithey et al., PRL **70**, 1244 (1993)

- Breitenbach et al., Nature **387**, 471 (1997)

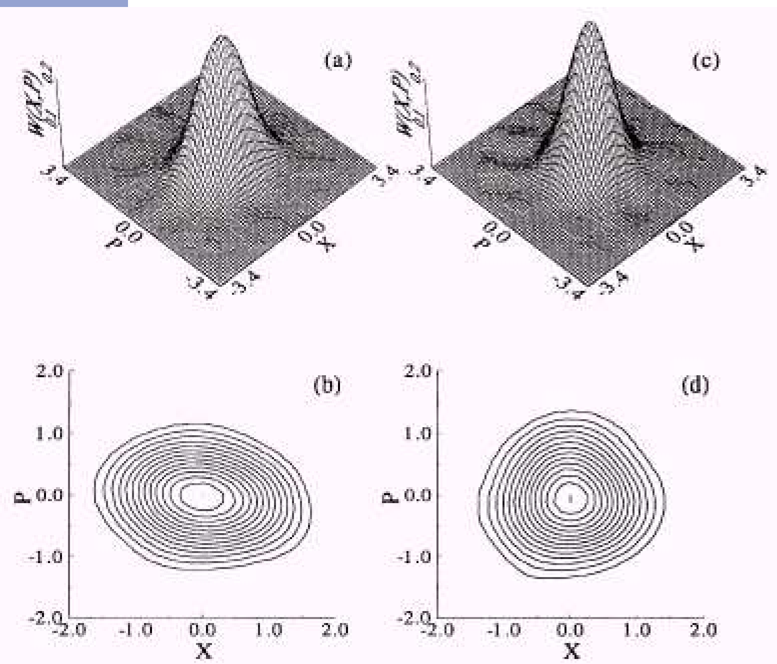
- One-photon Fock state : Lvovsky et al., PRL **87**, 050402 (2001)

- $\alpha|0\rangle+\beta|1\rangle$: Lvovsky et al., PRL **88**, 250401-1 (2002)

RESULTATS EXPERIMENTAUX

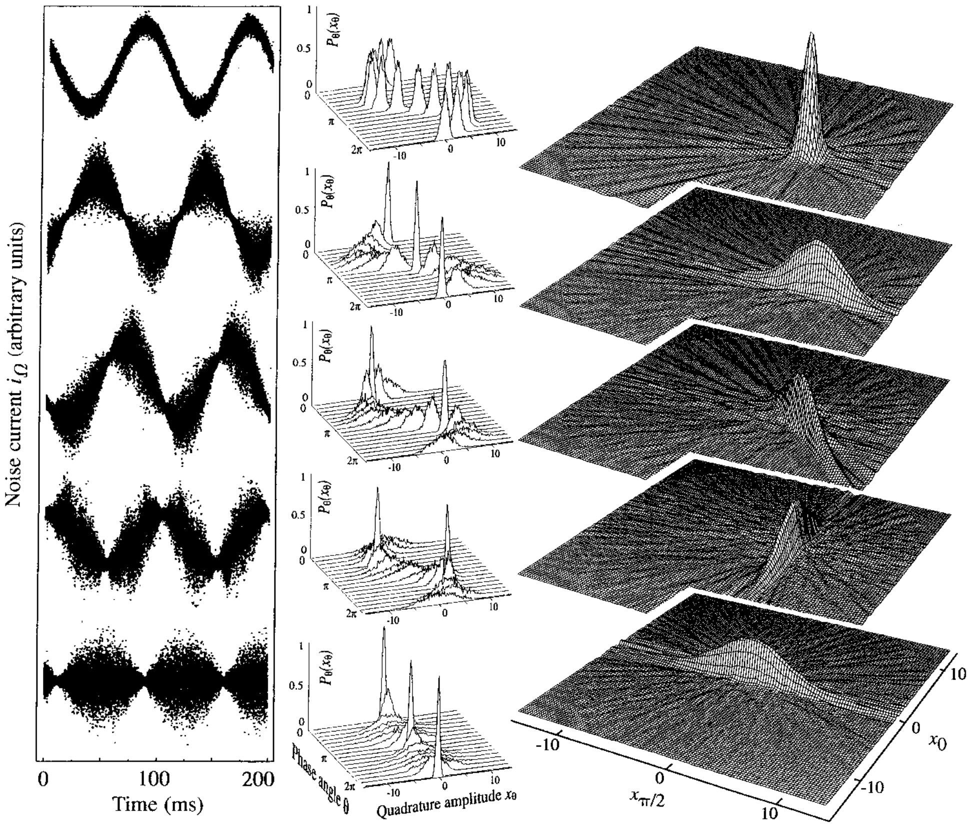
Smithey *et al.*, PRL **70**, 1244 (1993)

Breitenbach *et al*, Nature **387**, 471 (1997)



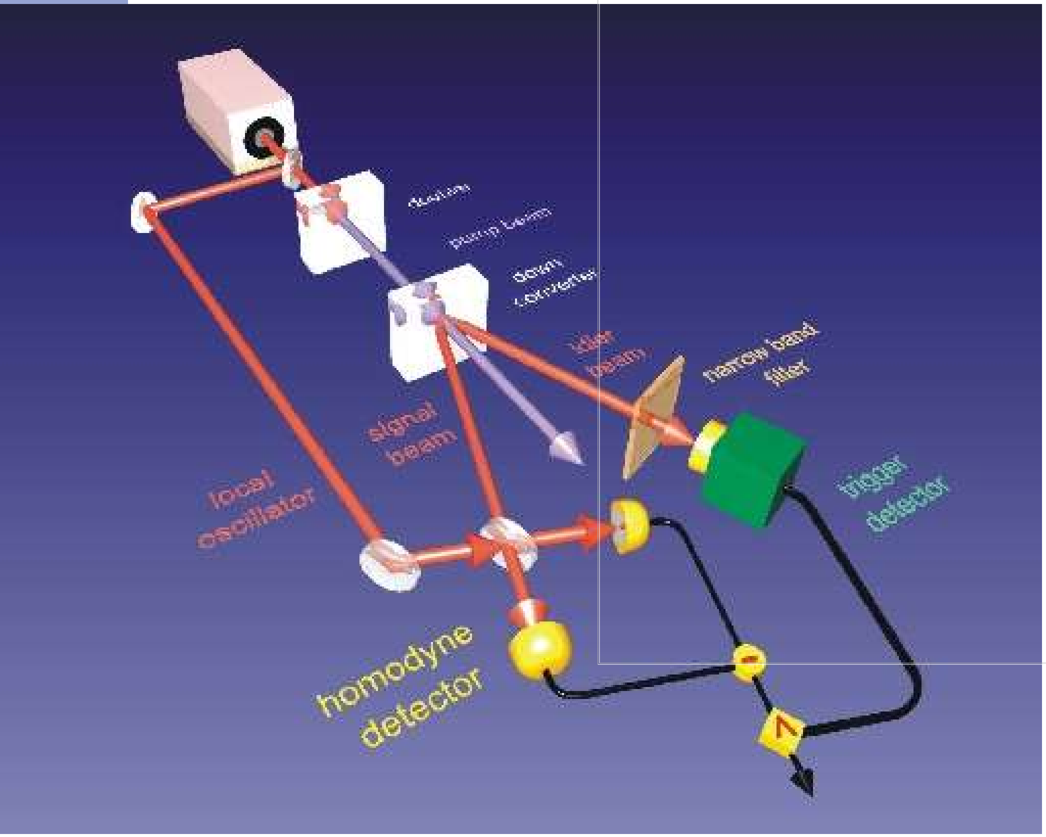
Comprimé

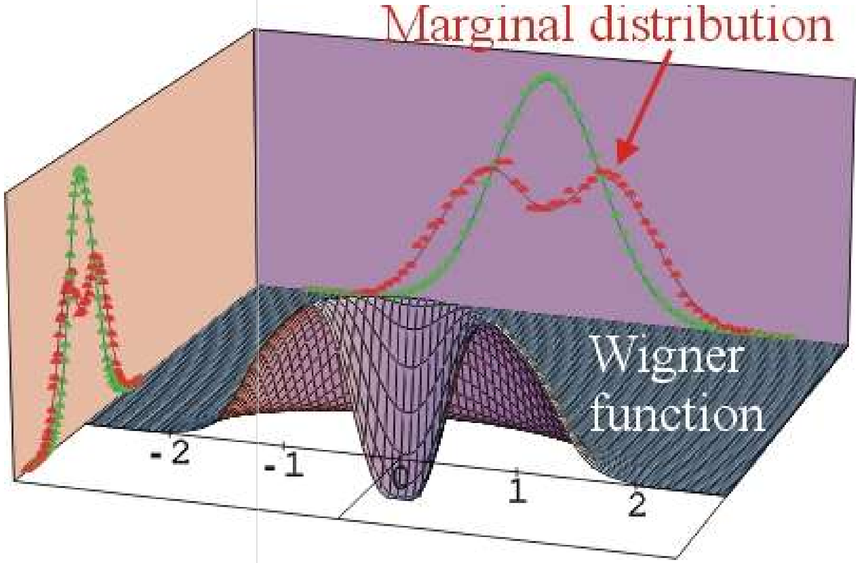
Vide



MESURE COMPLETE DE LA DISTRIBUTION DE WIGNER POUR UN PHOTON

Lvovsky *et al*, PRL **87**, 050402 (2001)





Other methods

Use the link between W and parity operator

$$W(\alpha) = 2\mathrm{Tr}(\hat{D}(-\alpha)\rho\hat{D}(\alpha)(-1)^{\hat{N}})$$

Displace the field and measure parity by determination of photon number probability

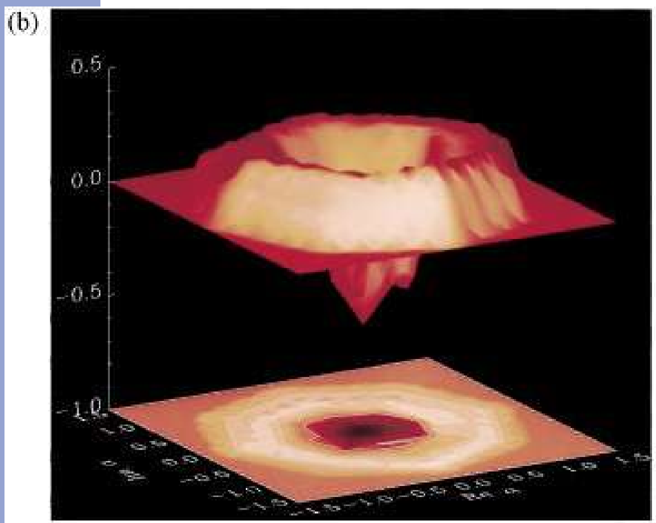
- Direct counting (Banaszek et al for coherent states)
- Quantum Rabi oscillations for an ion in a trap (Wineland)

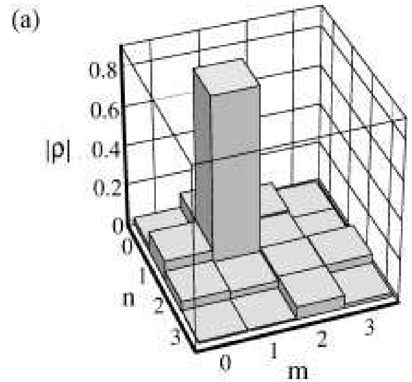
A demanding method. Much more information than the mere average parity needed

Wigner distribution for a trapped ion

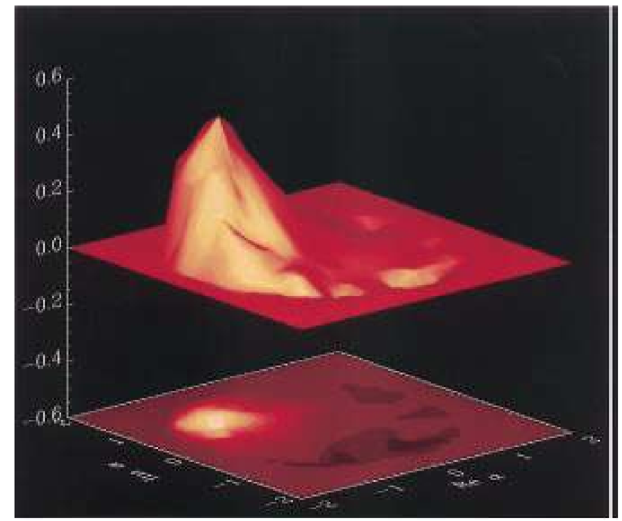
Etat nombre $|n=1\rangle$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$





Matrice densité



D. Liebfried et al, PRL 77, 4281 (1996), NIST, Boulder

- Same outcome for trapped neutral atom:
  - *G.Drobny and V. Buzek, PRA **65** 053410 (2002)*

  From the data of C. Salomon et I. Bouchoule

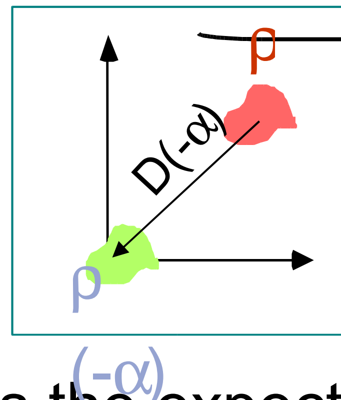
Our approach

- Proposed by Lutterbach and Davidovich (Lutterbach et al. PRL 78 (1997) 2547)

- Based on : $W(\alpha) = 2\mathrm{Tr}(\hat{D}(-\alpha)\rho\hat{D}(\alpha)(-1)^{\hat{N}})$

« parity » operator

$\rho(-\alpha)$



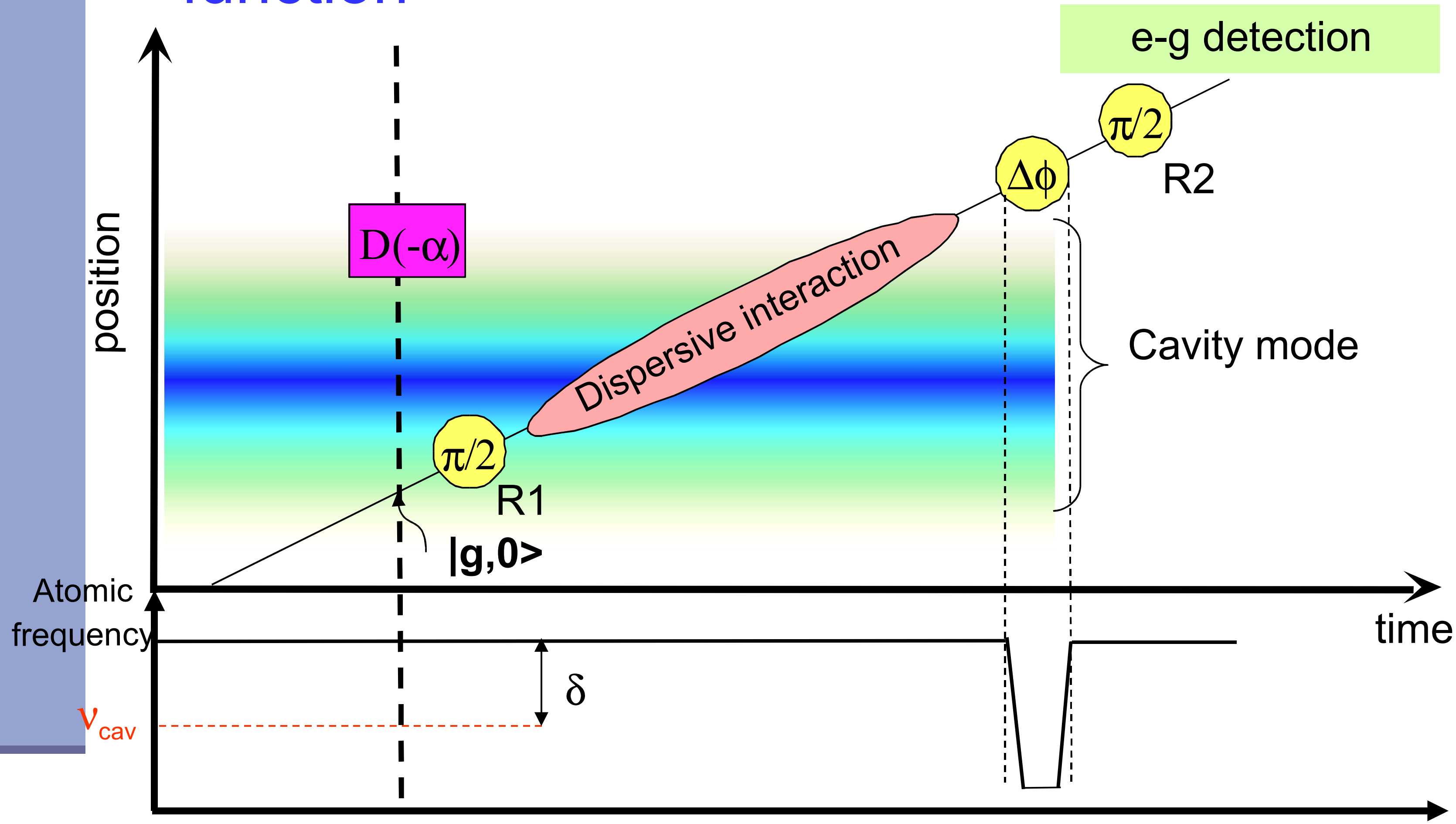
$$(-1)^{\hat{N}}|n\rangle = \begin{cases} +|n\rangle & \text{if } n=2k \\ -|n\rangle & \text{if } n=2k+1 \end{cases}$$

⇒ W is the expectation value of the Parity operator $(-1)^{\hat{N}}$ in the displaced state $\rho(-\alpha)$

A) Apply D(-α) ⟺ Inject –α in cavity mode OK

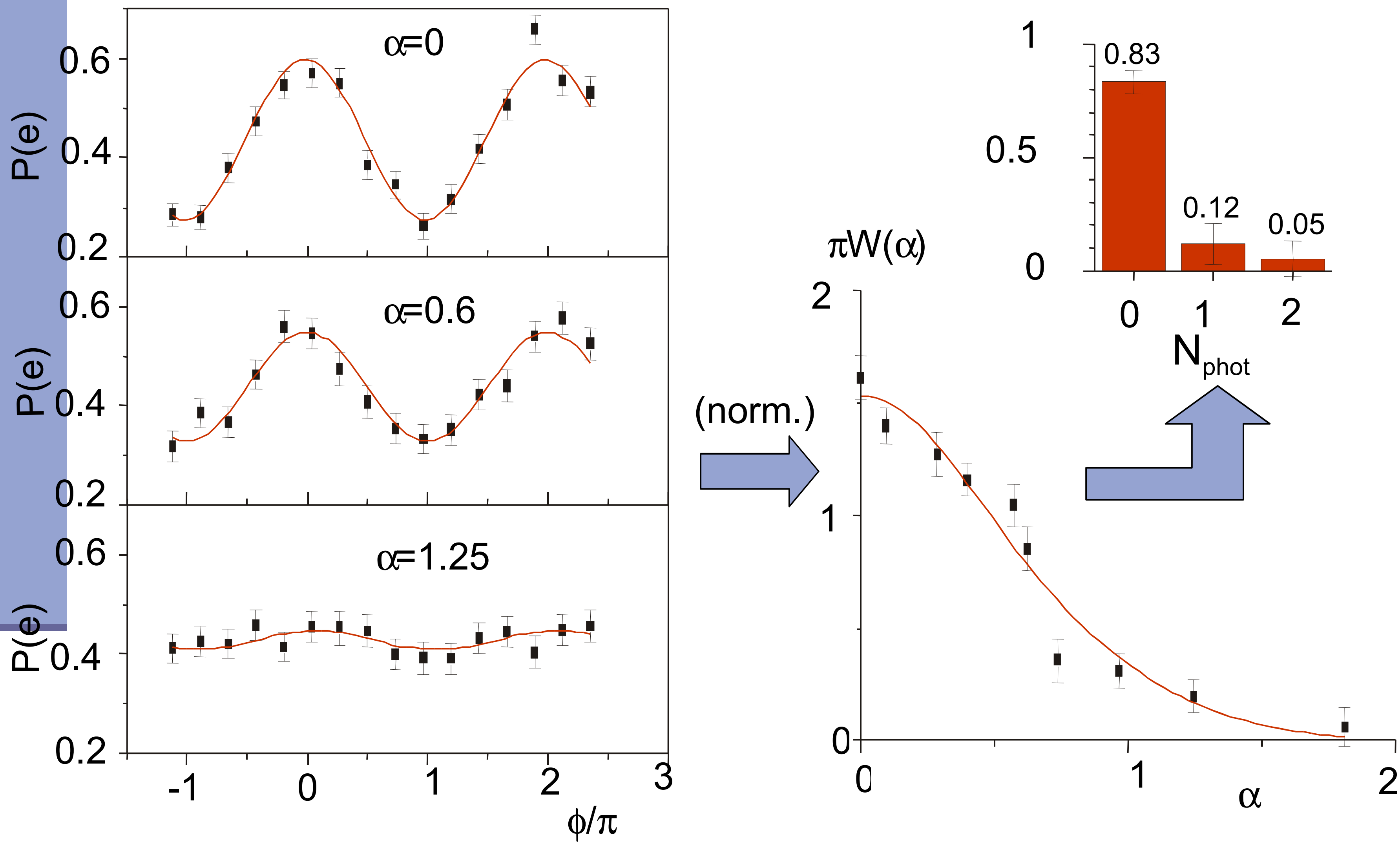
B) Parity measurement directly gives $\langle(-1)^{\hat{N}}\rangle$

Testing the method: vacuum state Wigner function

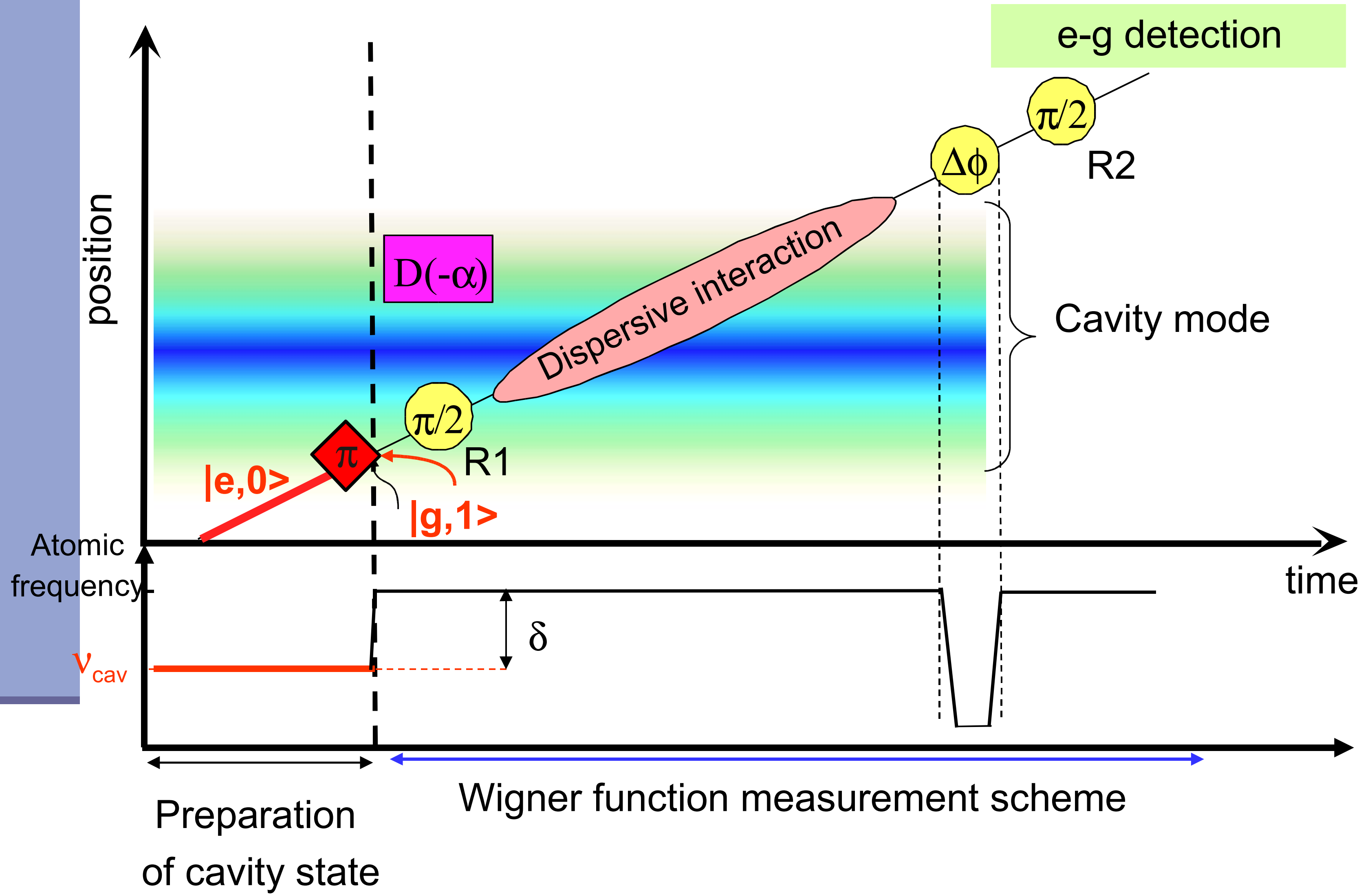


- Use Stark effect to tune interferometer phase
- No phase information in cavity field: injected field phase irrelevant
- Finite intrinsic contrast of the Ramsey interferometer

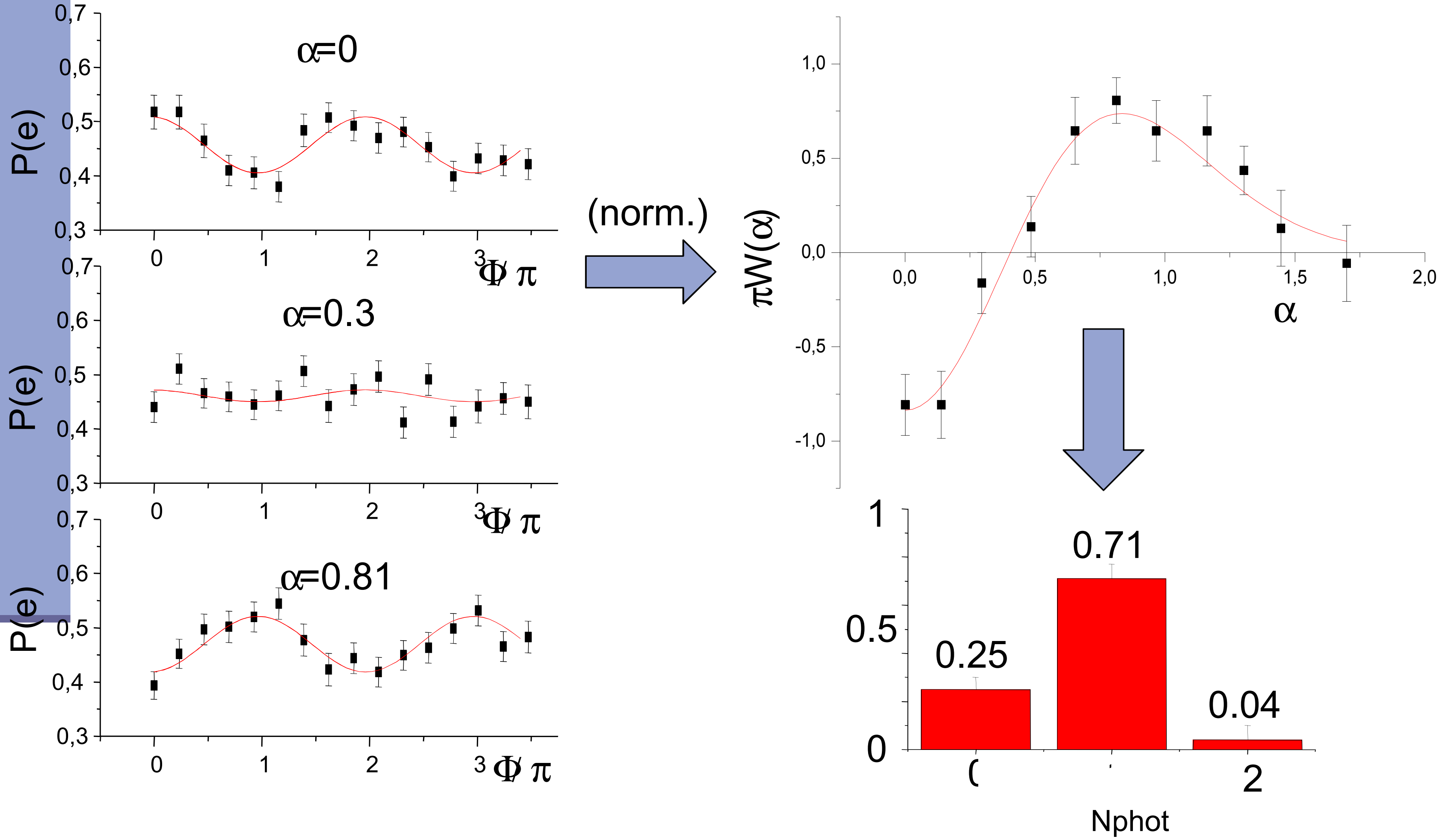
Wigner function of the "vacuum"



Single photon Wigner function measurement

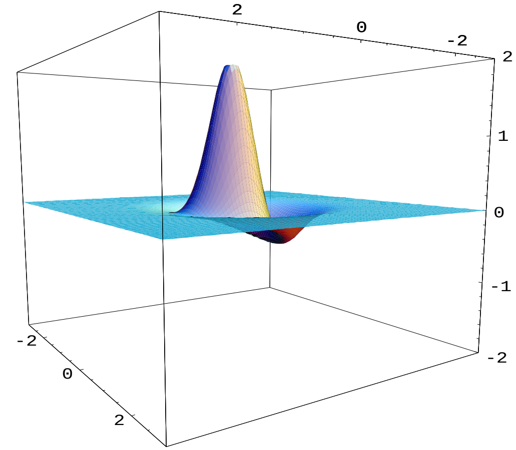


Wigner function of a "one-photon" Fock state



Towards other states

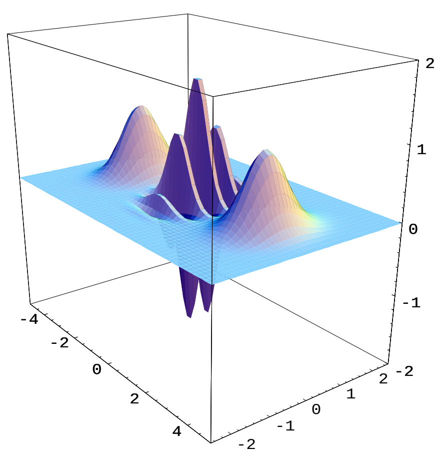
- Cavity QED setup : direct measurement of the field



- Next improvements : - better isolation
  - better detectors

→ More complex states : ex $(|0\rangle+|1\rangle)/\sqrt{2}$

- In the future : « movie » of the decoherence of a Schrödinger cat



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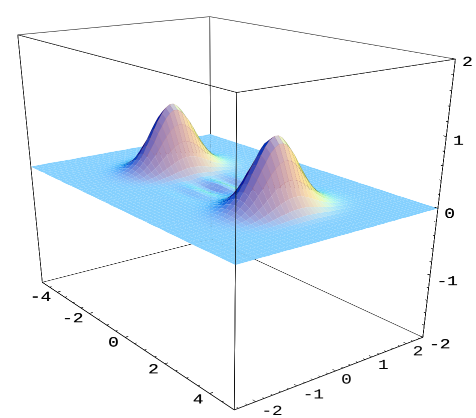

Outline

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  - QND detection of more than one atom
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Phase shift with dispersive atom-field interaction

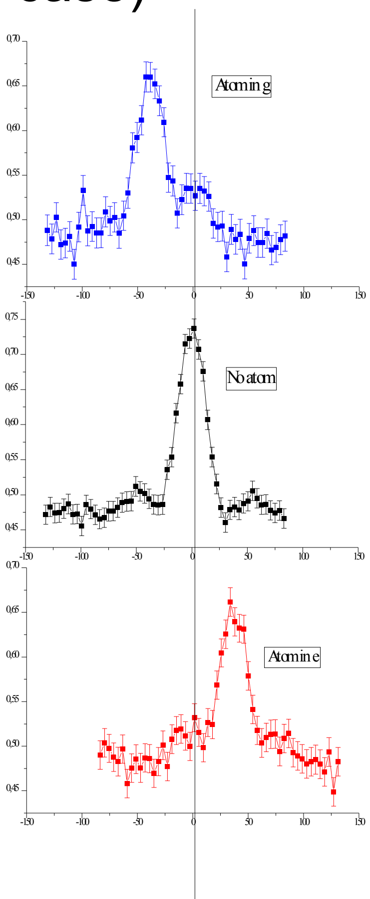
Non resonant atom: no energy exchange but cavity mode frequency shift (atomic index of refraction effect).

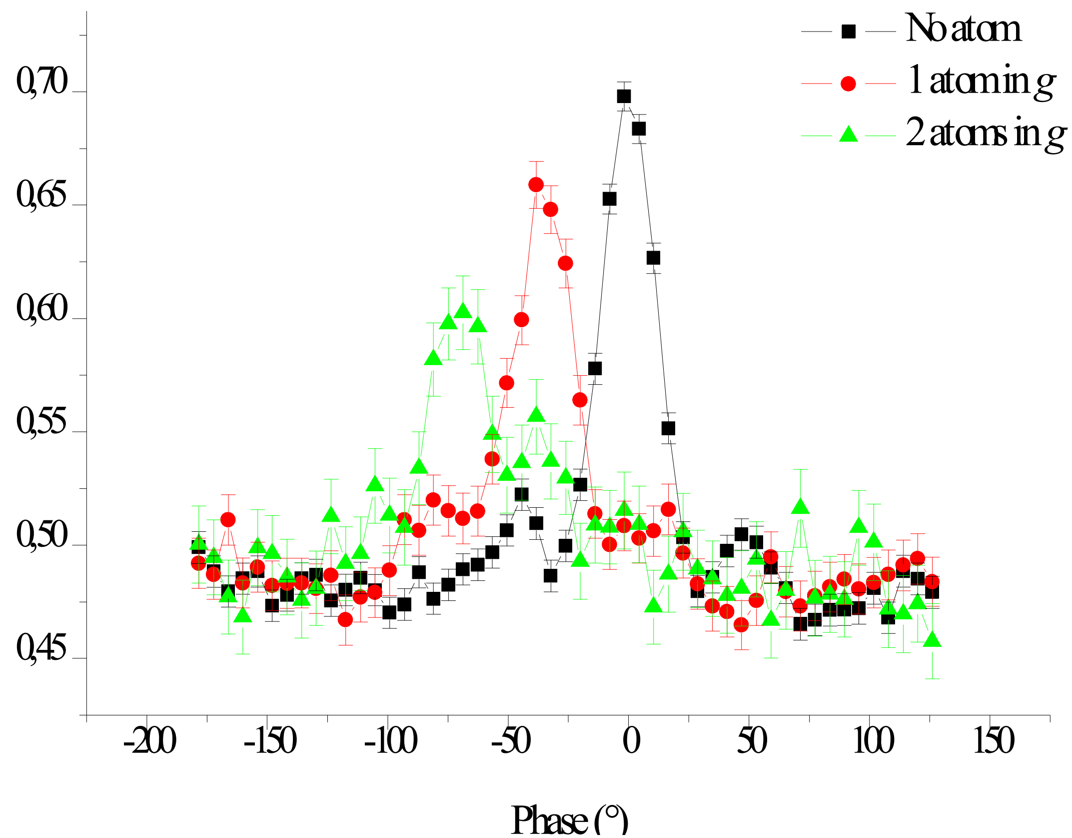
Phase shift of the cavity field (slower than in the resonant case)

Atom in *g*

No atom

Atom in *e*





Opposite values for *e* and *g*

Proportional to atom number

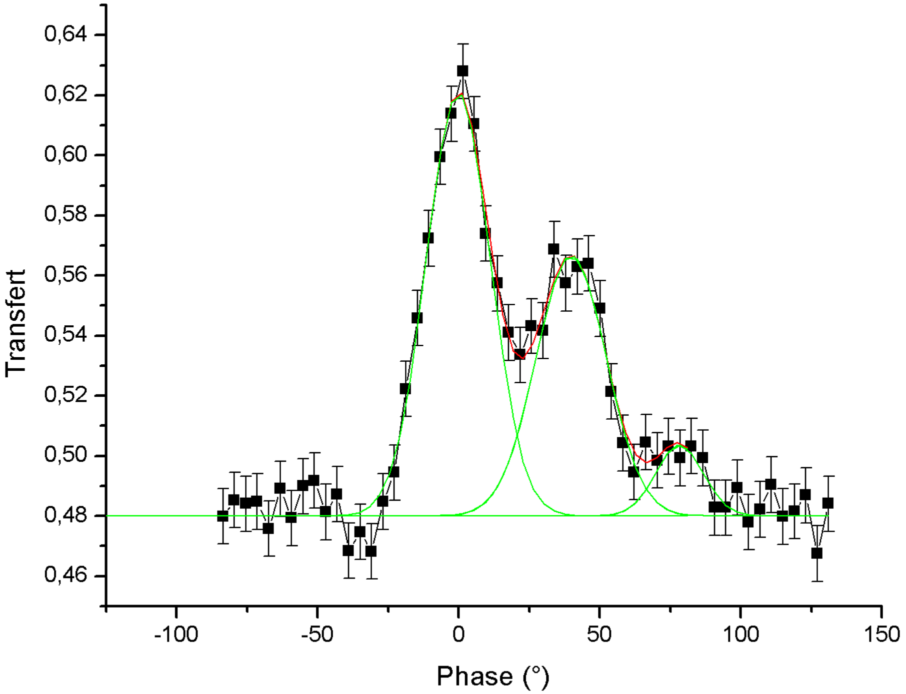
Absolute measurement of atomic detection efficiency

Histogram of field phase reveals exact atom count

Comparison with detected atom counts provides field ionization detectors efficiency in a precise and absolute way

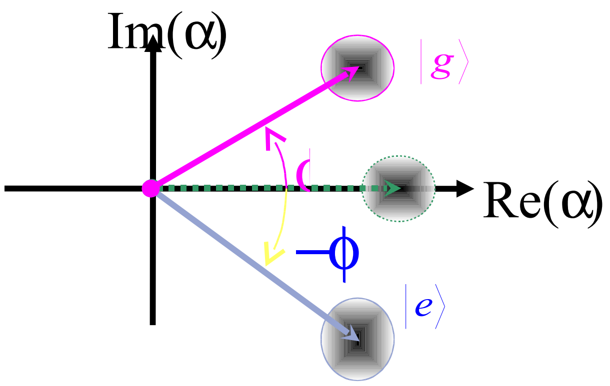
0.4 atoms samples:

70-90 % detection efficiency



Towards a 100% efficiency atomic detection

Inject a very large coherent field in the cavity



Send an atomic sample

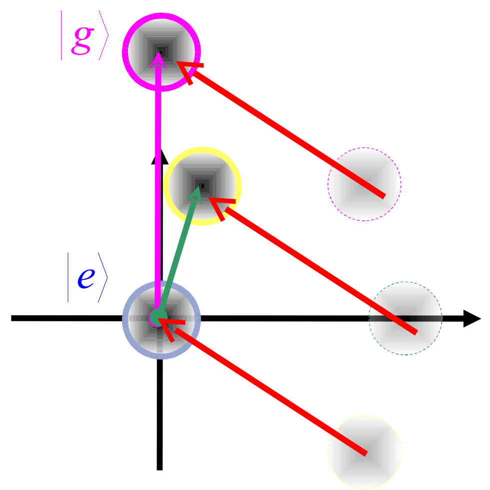
Different phase shifts for *e*, *g* or no atom

Inject homodyning amplitude

Zero amplitude for *e*.

Larger for no atom.

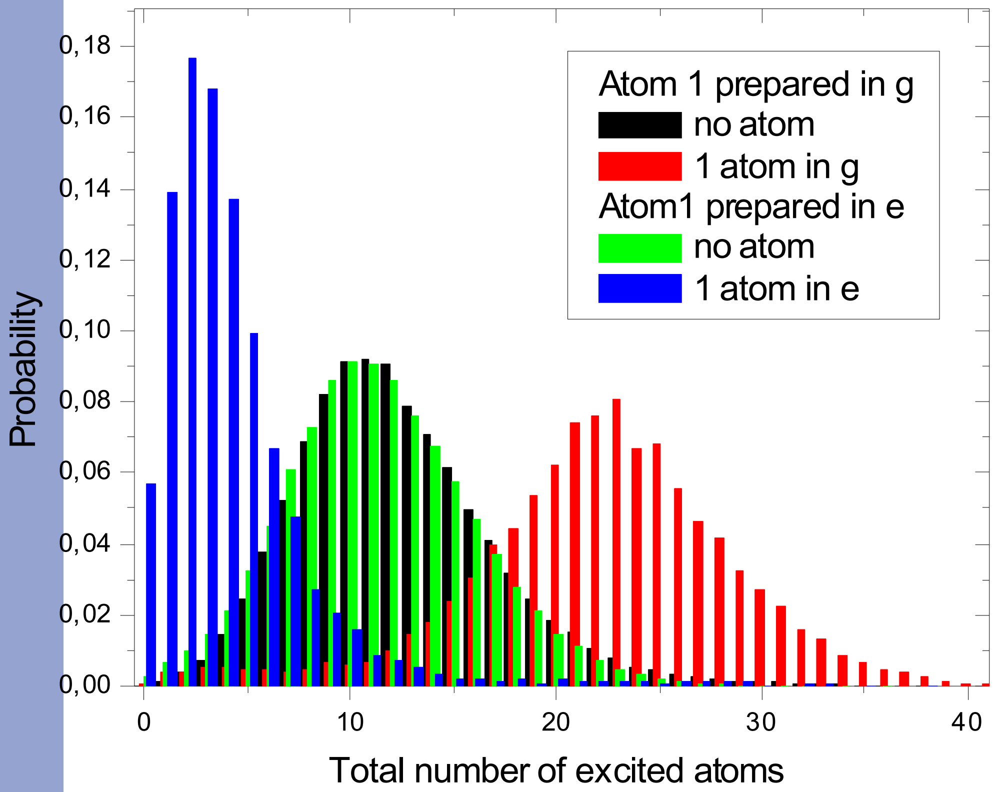
Still larger for *g*



Read final field amplitude by sending a large number of atoms in *g*

Final number of atoms in *e* proportional to photon number

Preliminary experimental results



Experimental conditions:

- 75 photons initially
- v=200 m/s
- δ=50 kHz
- 70 absorber atoms

detection efficiency: 87%

error probability: 0 atom detected as 1: 10% (main present limitation)

e in g: 1.6%

g in e: 3%

100% detection efficiency within reach with slower atoms: v=150 m/s
….experiment in progress.

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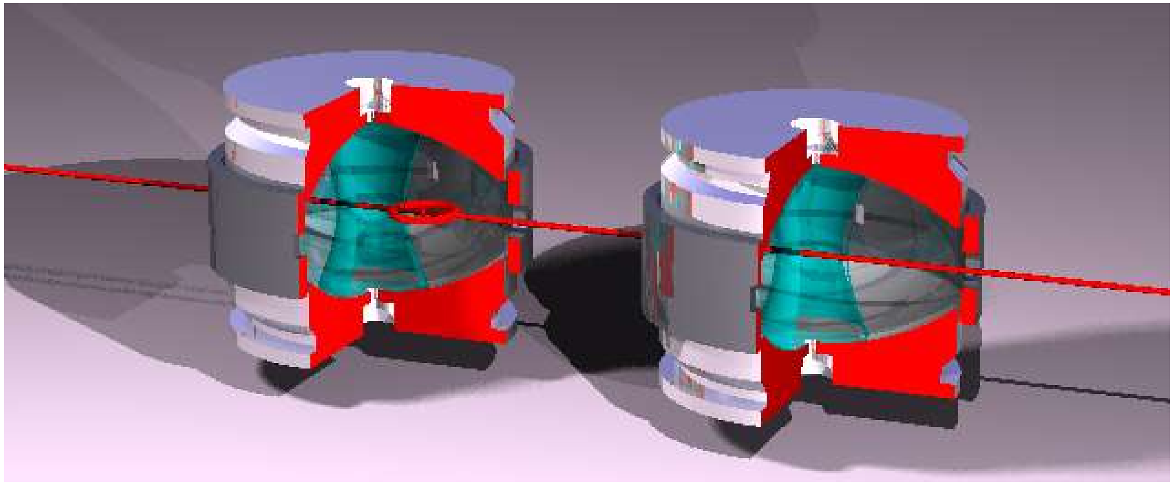
A two-cavity experiment

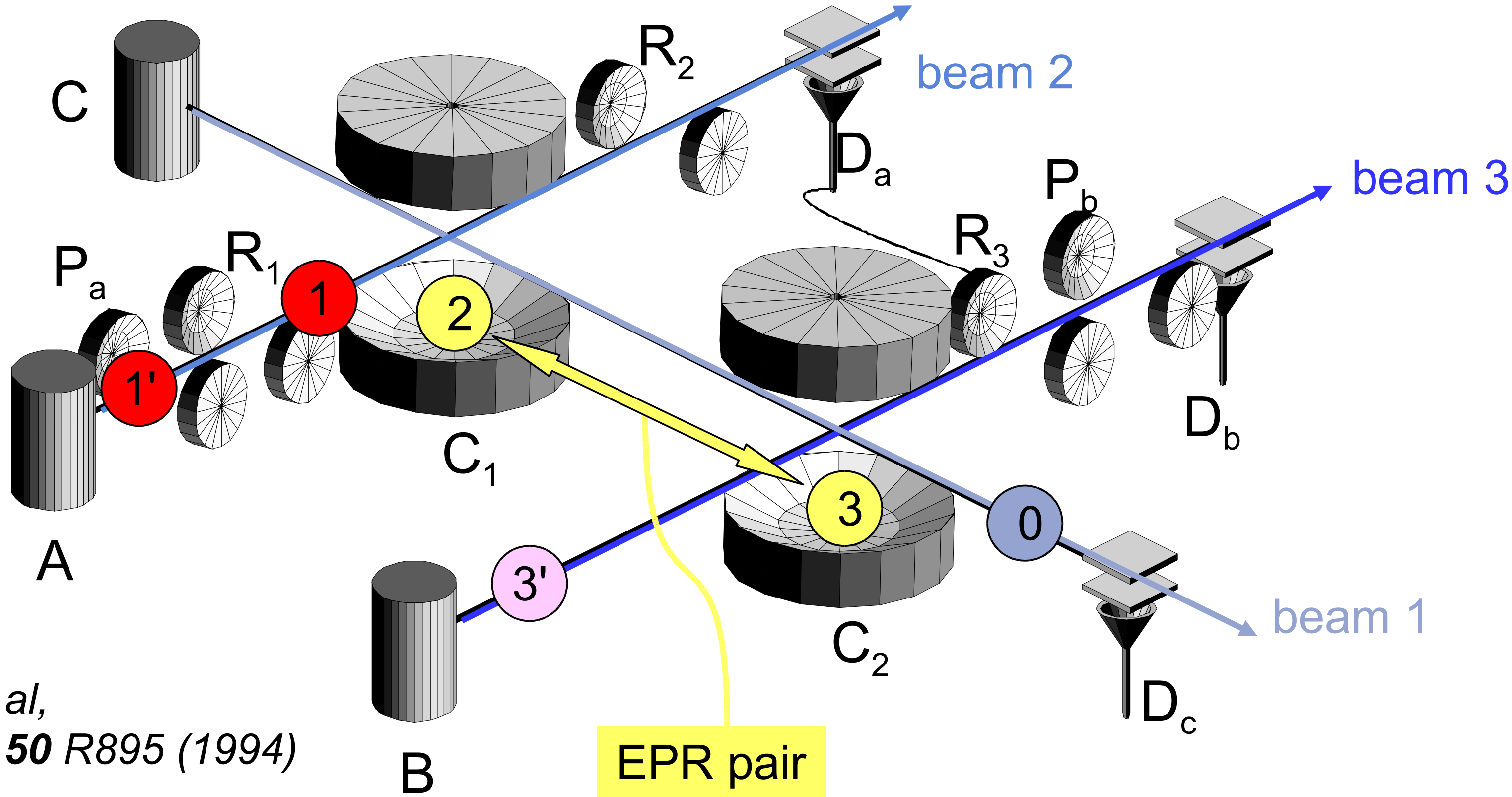
Rydberg atoms and superconducting cavities:

Towards a two-cavity experiment



- Creation of non-local mesoscopic Schrödinger cat states
  - Non-locality and decoherence (real time monitoring of *W* function)
- Complex quantum information manipulations
  - Quantum feedback
  - Simple algorithms
  - Three-qubit quantum error correction code

Teleportation of an atomic state



Davidovich et al,
*PHYS REV A **50** R895 (1994)*

- This scheme works for massive particles
- Detection of the 4 Bell states and application of the "correction" to the target is possible using a C-Not gate (beam 2 and 3)
- The scheme can be compacted to 1 cavity and 1 atomic beam

Entangling two modes of the radiation field

Principle:

First atom

Initial state $|e,0,0\rangle$

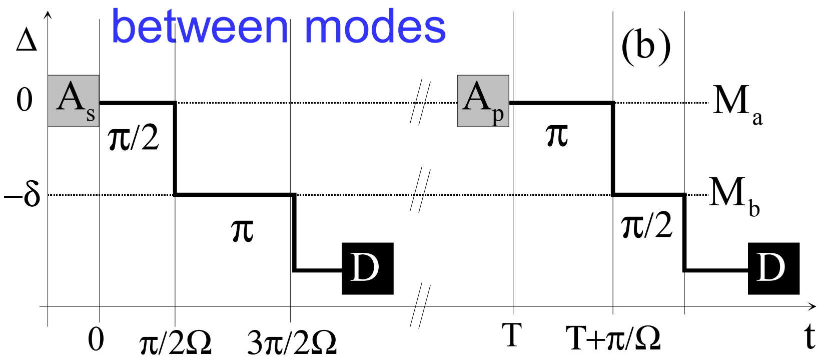
$\pi/2$ pulse in M_a $\quad \frac{1}{\sqrt{2}}\left(|e,0,0\rangle + |g,1,0\rangle\right)$

π pulse in M_b $\quad \frac{1}{\sqrt{2}}|g\rangle\left(|0,1\rangle + |1,0\rangle\right)$

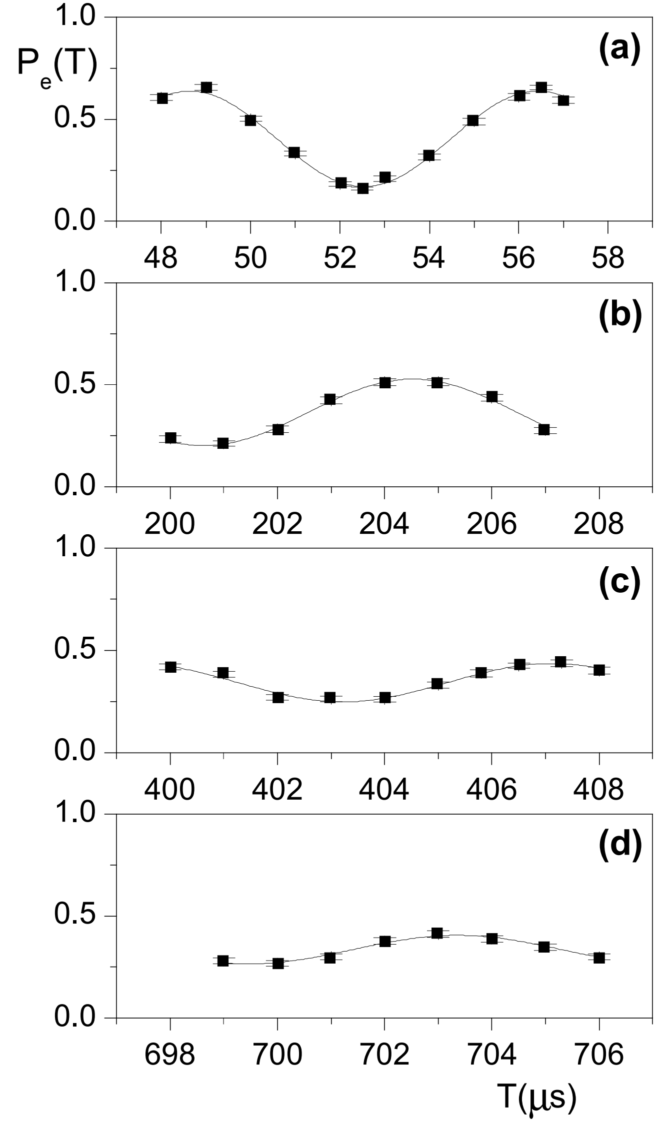
Second atom:

probes field states

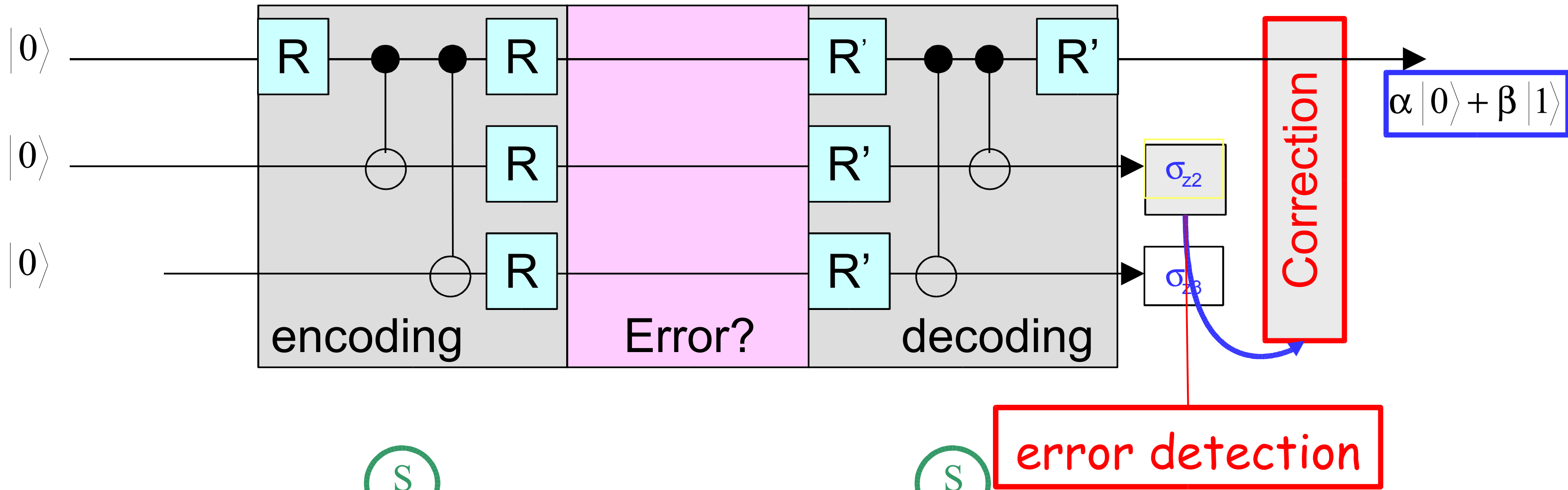
Final transfer rate modulated versus the delay at the beat note between modes

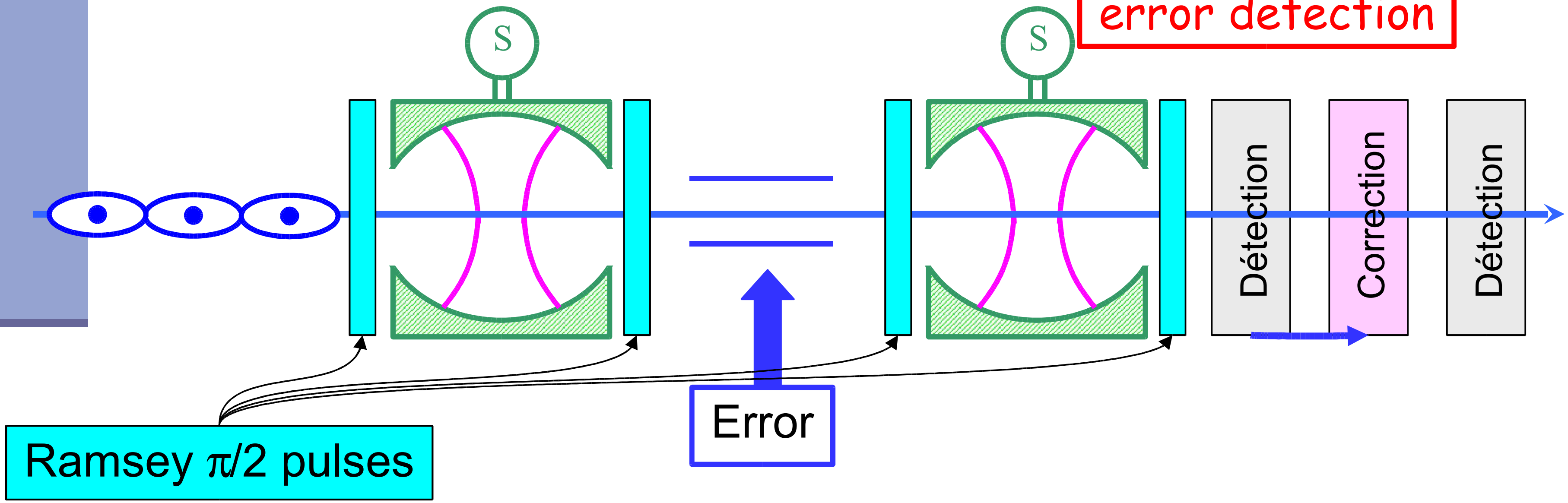


Single photon beats



Implementation of 3 qubit error correction





encoding and decoding: preparation of a GHz triplet

all the tools exist!

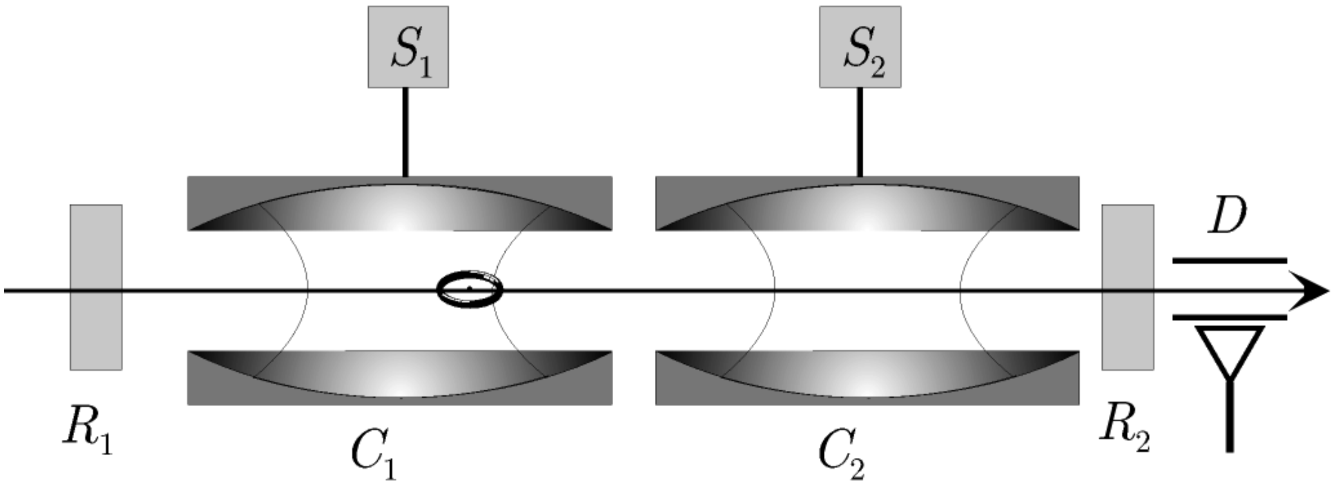
New non-locality explorations

Use a single atom to entangle two mesoscopic fields in the cavity

$$|\Psi^{\pm}\rangle = \frac{1}{N_{\Psi^{\pm}}}\left(|\pm\gamma,\gamma\rangle + |\mp\gamma,-\gamma\rangle\right)$$

A non-local Schrödinger cat or a mesoscopic EPR pair

Easily prepared via dispersive atom-cavity interaction



Mesoscopic Bell inequalities

A Bell inequality form adapted to this situation

$$\Pi(\alpha,\beta) = (\pi^2/4)W(\alpha,\beta)$$

$$\mathcal{B} = |\Pi(\alpha',\beta') + \Pi(\alpha,\beta') + \Pi(\alpha',\beta) - \Pi(\beta,\alpha)| \leq 2,$$

$$|\Psi^{\pm}\rangle = \frac{1}{N_{\Psi^{\pm}}}\left(|C_+,C_+\rangle \pm |C_-,C_-\rangle\right)$$

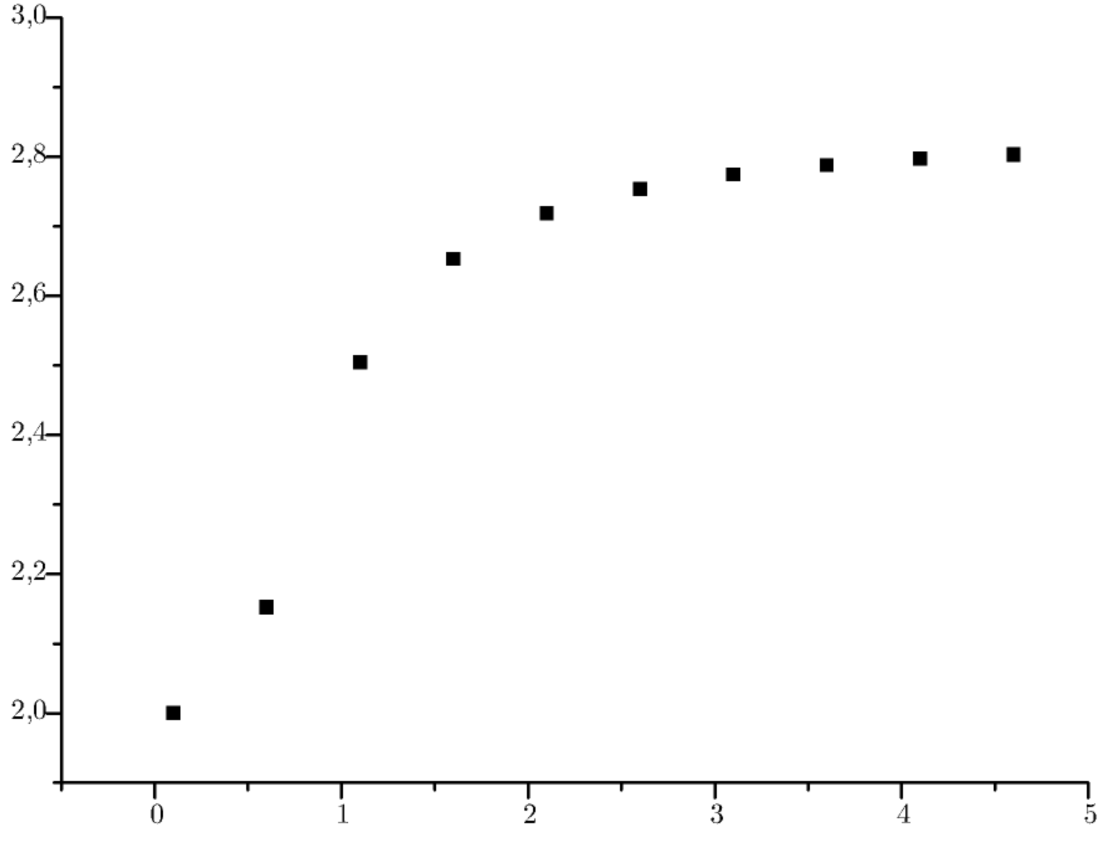
Here, Π is the parity operator average.

Dichotomic variable for which the Bell inequalities argument can be used (transforms the continuous variable problem in a spin-like problem) $|C_{\pm}\rangle = 1/N_{\pm}\left(|\gamma\rangle \pm |-\gamma\rangle\right)$

Maximum violation for parity entangled states:

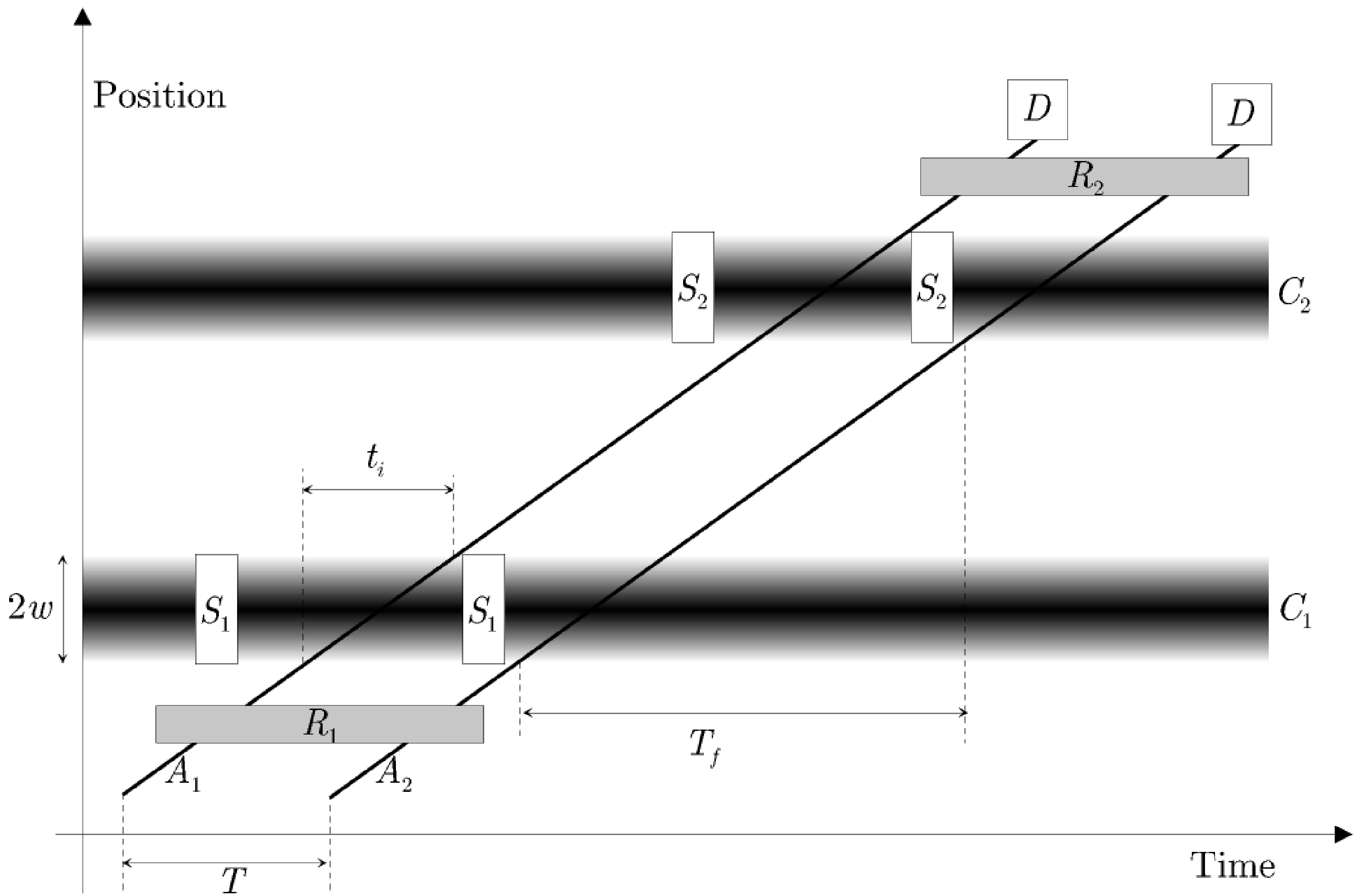
Bell inequalities violation

Optimum Bell signal versus γ



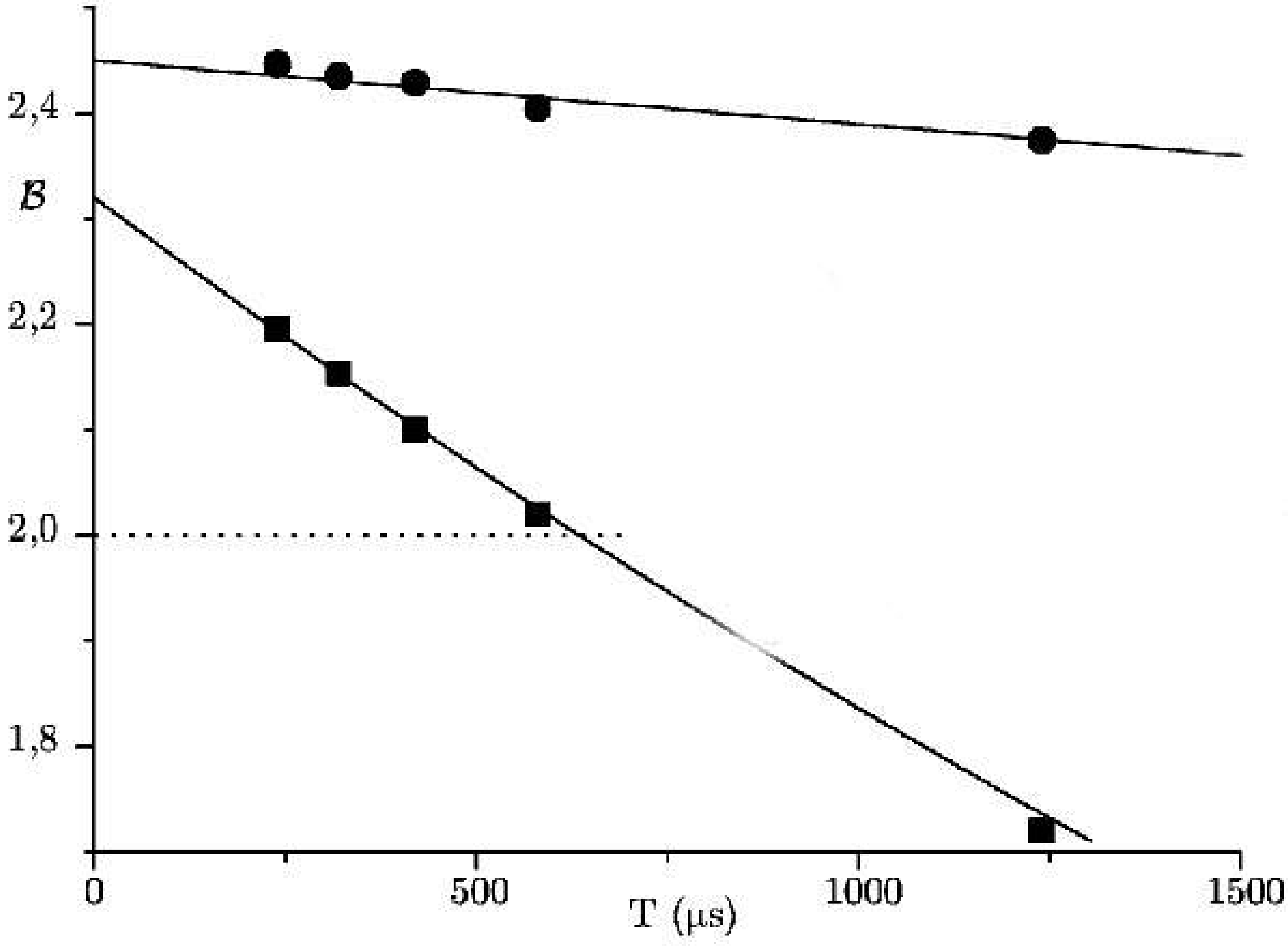
Probing the Wigner function

A second atom to read out both cavities (same scheme as for single mode Wigner function)



A difficult but feasible experiment

Bell signal versus time T_c=30 and 300 ms



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