On non-signalling quantum boxes

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Motivation

- Understand better the relation between *locality* and *causality*
- Investigate the *non-local* features of quantum mechanics
- Analyze the *communication* needed to perform a bipartite quantum operation and allowed by it

Non-locality & causality

- Testing and evaluating non-locality

$$A_0, A_1, B_0, B_1 \qquad \text{Dichotomic } \textit{experiments}\text{: } \pm 1$$

$$\langle Bell \rangle = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

$$\langle Bell \rangle \quad \leqslant 2 \qquad \text{LHV models} \quad p_\lambda(a, b | A, B) = p_\lambda(a|A)\, p_\lambda(b|B)$$

$$\leqslant 2\sqrt{2} \qquad \text{quantum mechanics} \qquad \langle AB \rangle = Tr(\rho\, AB)$$

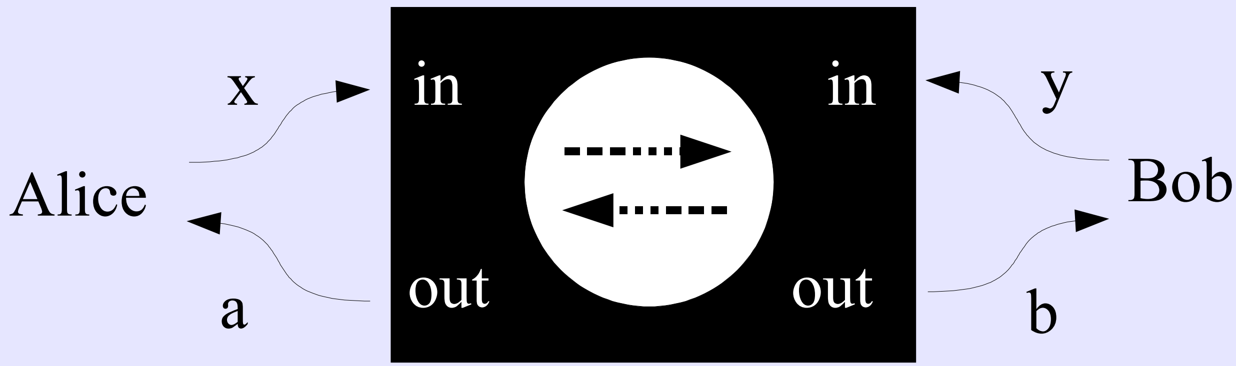
$$\leqslant 4 \qquad \text{algebraic limit}$$

- Quantum mechanics is non-local (even if "non-maximally") but respects causality (no-signalling): statistics of local measurments determined by reduced state

$$|\psi^0_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|00_{AB}\rangle + |11_{AB}\rangle\right) \quad \longrightarrow \quad \rho_A = \rho_B = \frac{1}{2}$$

Maximal (quantum) correlation, non-locality but no superluminal signalling

(Classical) boxes



Example (*Popescu-Rohrlich box*):

$a, b, x, y \in \{0, 1\}$

a, b *locally random* but *correlated*: $a \oplus b = x \cdot y$

defining $a_{0,1} = a_{x=0,1} \quad A_{0,1} = (-1)^{a_{0,1}}$

$b_{0,1} = b_{y=0,1} \quad B_{0,1} = (-1)^{b_{0,1}}$

we have $\langle A_0 B_0 \rangle = \langle (-1)^{a_0 \oplus b_0} \rangle \langle (-1)^0 \rangle = 1$ etc.

so that $\langle Bell \rangle = 4$

x	y	$a \quad b$
0	0	$\left\{\begin{matrix}0 & 0\\ 1 & 1\end{matrix}\right\}$
0	1	$\left\{\begin{matrix}0 & 0\\ 1 & 1\end{matrix}\right\}$
1	0	$\left\{\begin{matrix}0 & 0\\ 1 & 1\end{matrix}\right\}$
1	1	$\left\{\begin{matrix}0 & 1\\ 1 & 0\end{matrix}\right\}$

Maximal violation of non-locality but still preservation of causality

"Quantization" of the classical PR box

- **Coherent version** (using entangled ancilla)

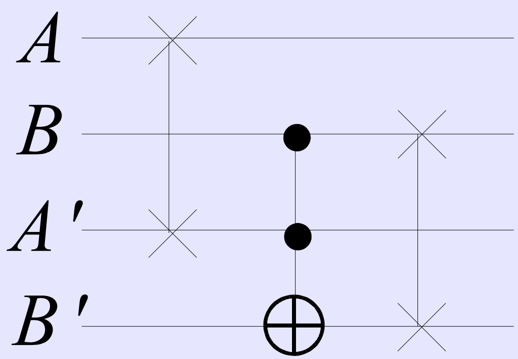
$$\rho'_{AB} = \Lambda^{E}[\rho_{AB}] = (1-p)\,P_0 + p\,P_1$$

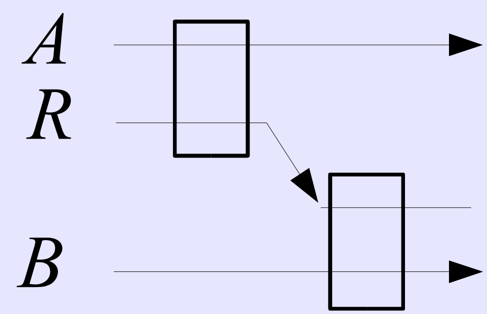
$$p = \langle 11|\rho_{AB}|11\rangle$$

$$P_i = |\psi_i\rangle\langle\psi_i| \qquad |\psi_0\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \qquad |\psi_1\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

input $|\phi_{AB}\rangle$

ancilla $|\psi_0^{A'B'}\rangle$





- Action on the *computational basis*

$|00\rangle, |01\rangle, |10\rangle \longrightarrow |\psi_0\rangle$

$|11\rangle \longrightarrow |\psi_1\rangle$

(x, y: input)

local measurements: output a, b

$$a \oplus b = x \cdot y$$

with

$$\rho'_A = \rho'_B = \frac{1}{2}$$

- **Incoherent version** (using correlated but separable ancillae)

$$\Lambda^S[\rho_{AB}] = (1-p)Q_0 + pQ_1 \qquad p = \langle 11|\rho_{AB}|11\rangle$$

$$Q_0 = \frac{|00\rangle\langle 00| + |11\rangle\langle 11|}{2} \qquad Q_1 = \frac{|01\rangle\langle 01| + |10\rangle\langle 10|}{2}$$

- For both versions we can define a family of maps

$$\Lambda^E_\alpha[\rho_{AB}] = (1-\alpha)P_0 + \alpha\Lambda^E[\rho_{AB}] \qquad 0 \leqslant \alpha \leqslant 1$$

$$\Lambda^S_\alpha[\rho_{AB}] = (1-\alpha)Q_0 + \alpha\Lambda^S[\rho_{AB}]$$

$\alpha = 0$ constant map
maximal standard quantum non-locality

$\alpha = 1$ "original" map $\Lambda^{E,S}$ (maximal non-locality...)

- To perform a map $\Lambda^{E,S}_\alpha$ we need at most α times the communication needed for $\Lambda^{E,S}$

“Experimental” procedure to test non-locality

$$\langle Bell \rangle = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

Dichotomic experiments $A_i \equiv \left(|i\rangle, \vec{a}_i \cdot \vec{\sigma} \right) \quad B_j \equiv \left(|j\rangle, \vec{b}_j \cdot \vec{\sigma} \right)$

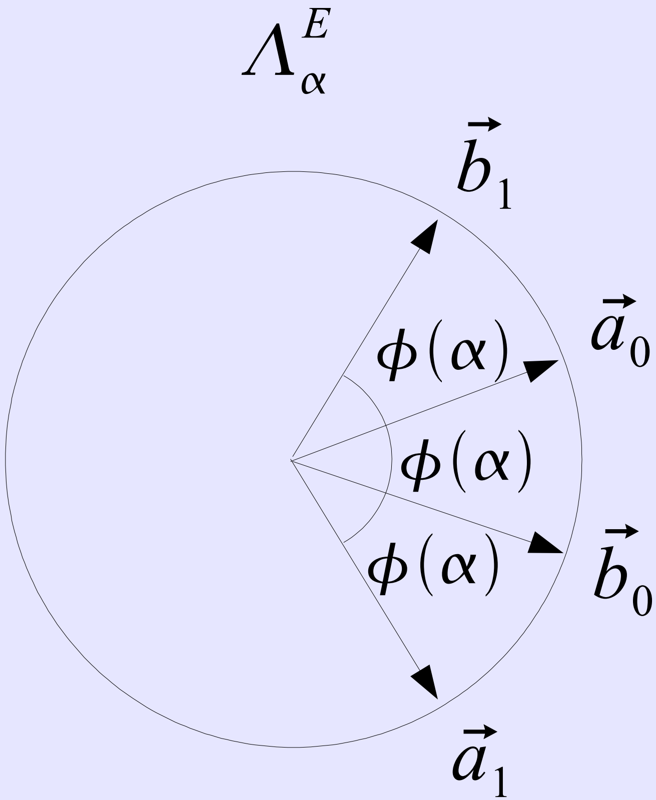
with $$\langle A_i B_j \rangle = Tr\left(\Lambda\left[|i\rangle_A \langle i| \otimes |j\rangle_B \langle j| \right] \left(\vec{a}_i \cdot \vec{\sigma} \otimes \vec{b}_j \cdot \vec{\sigma} \right) \right)$$

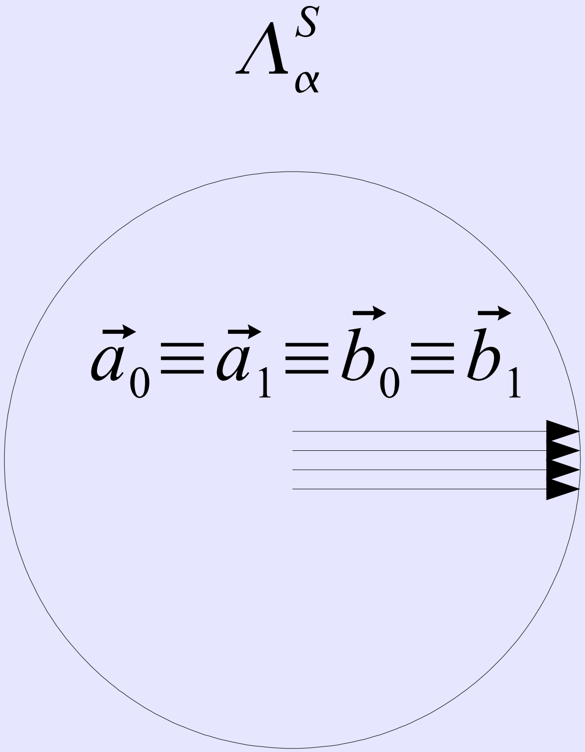
input state

output state

The quantum state with respect to which the observables are measured depends on the observables themselves!

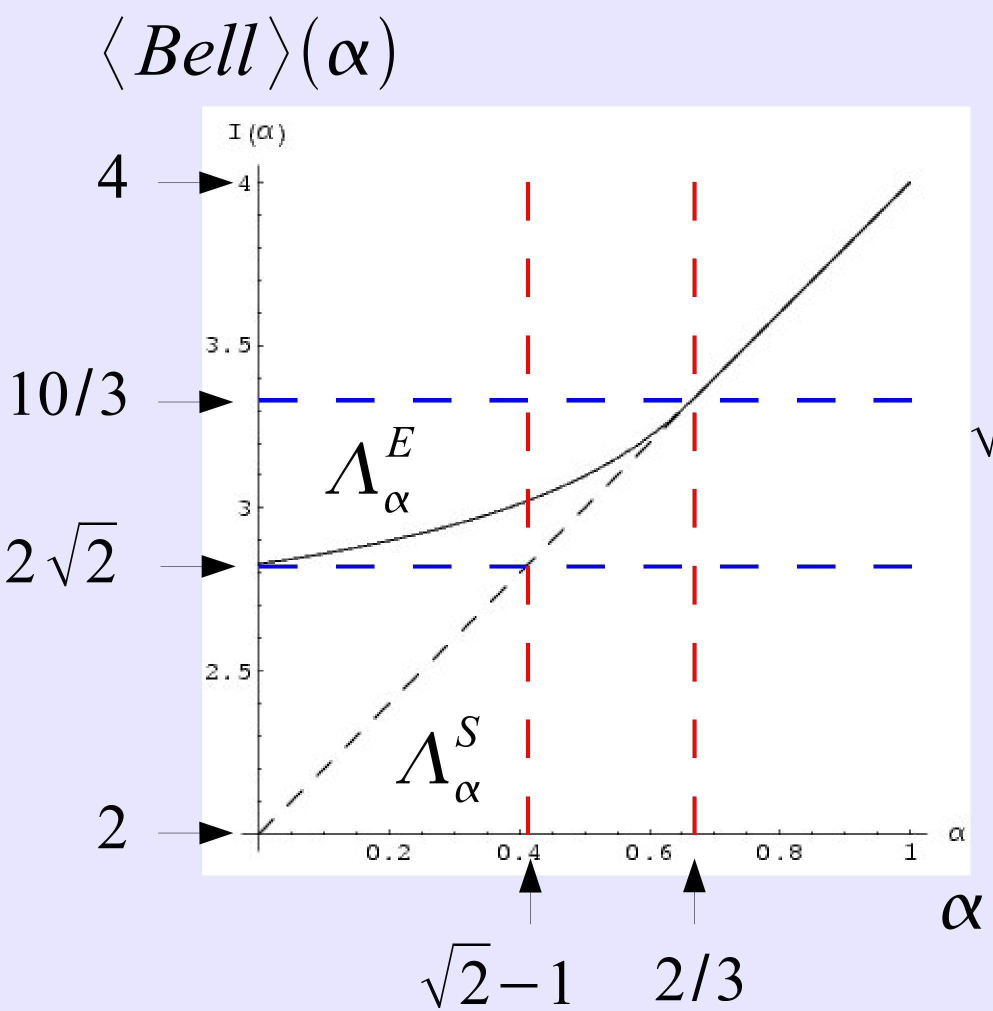
Parameters for maximal $\langle Bell \rangle$

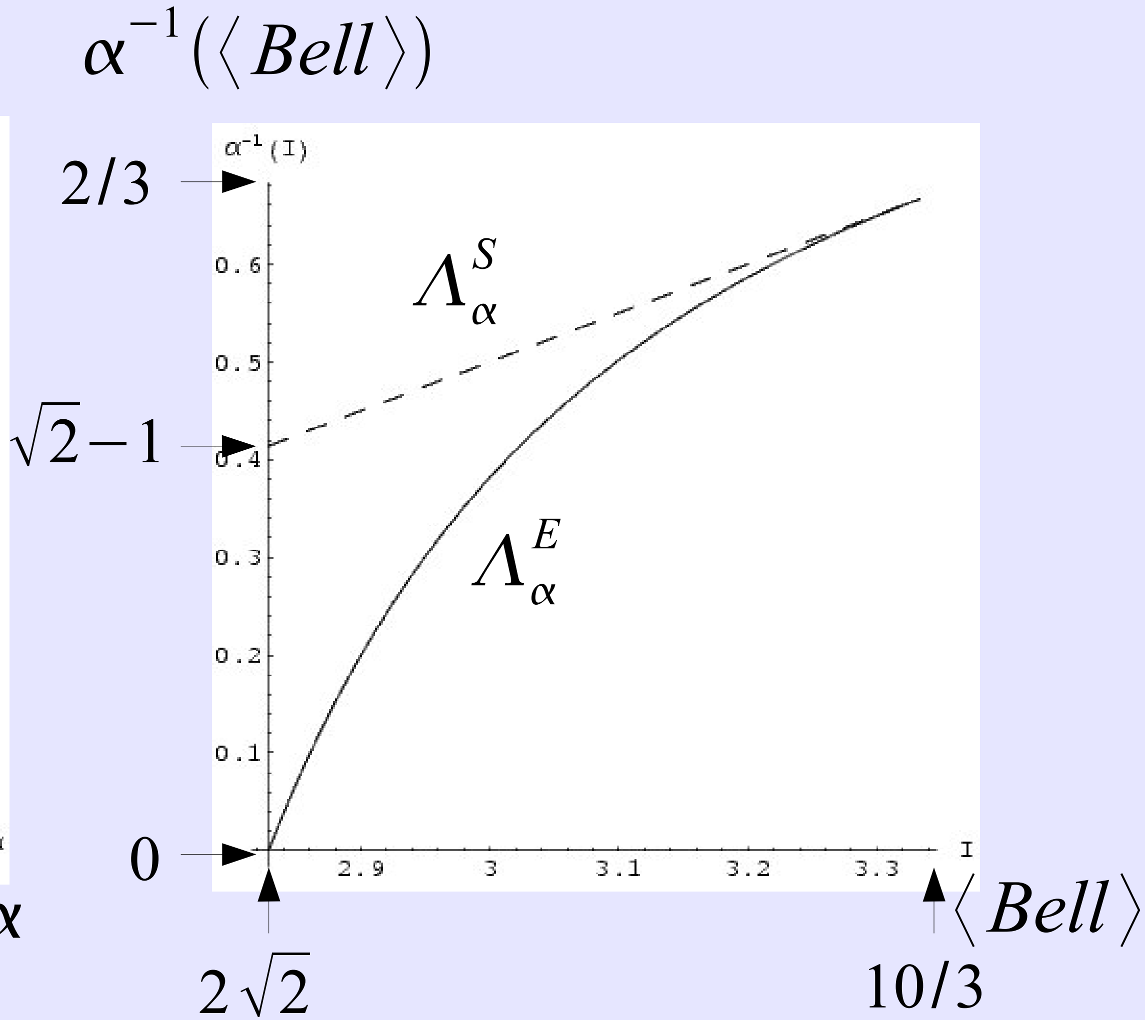




- For Λ_α^E : $\langle Bell \rangle = \begin{cases} \sqrt{\dfrac{(2-\alpha)^3}{1-\alpha}} + \alpha & 0 \le \alpha \le 2/3 \\ 2(1+\alpha) & 2/3 < \alpha \le 1 \end{cases}$

- For Λ_α^S : $\langle Bell \rangle = 2(1+\alpha) \qquad 0 \le \alpha \le 1$





Entanglement seems not to help to obtain maximal non-locality

Non-signalling maps

DEF We say that a bipartite operation is *non-signalling* (or *causal*) if it can not be exploited to send information from one party to the other

D. Beckman et al., Phys. Rev. A 64, 052309 (2001)

$$\rho'_B = Tr_A\left(\Lambda_{AB}[\rho_{AB}]\right) \equiv Tr_A\left(\Lambda_{AB}\left[(\Gamma_A \otimes 1_B)[\rho_{AB}]\right]\right)$$

$$\forall \Gamma_A, \forall \rho_{AB} \in S_{AB}$$

- Alice can not send a signal to Bob using Λ_{AB} iff

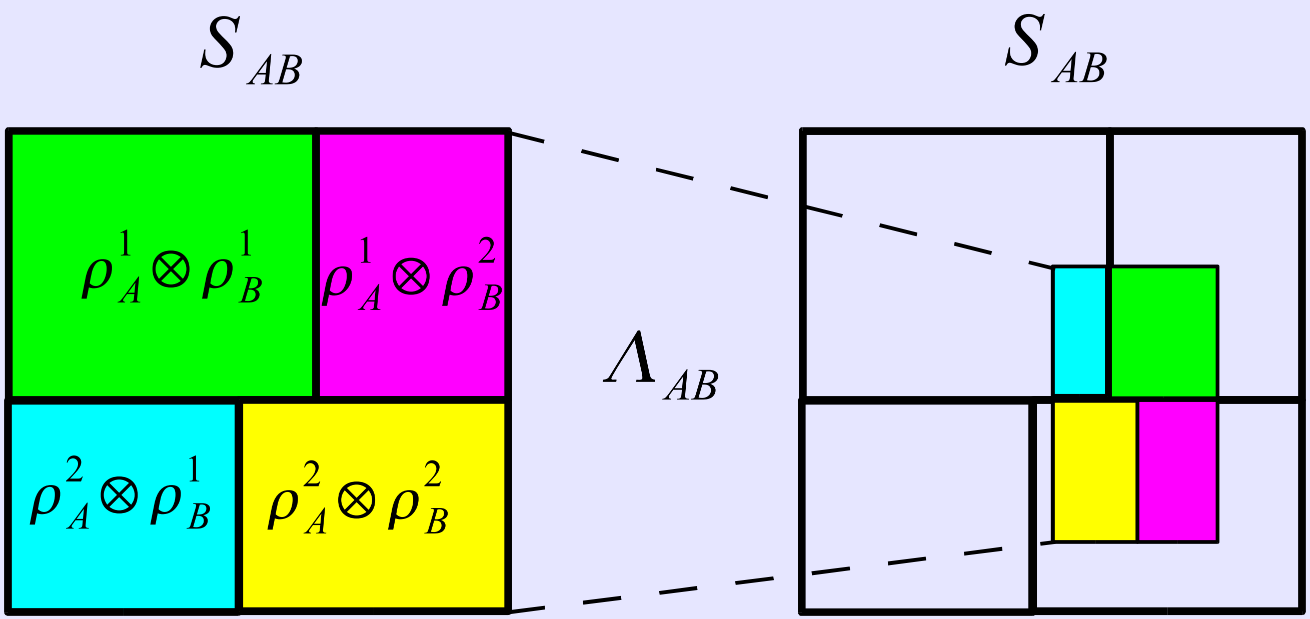
$$(D_A \otimes id_B) \circ \Lambda_{AB} \circ (D_A \otimes id_B) = (D_A \otimes id_B) \circ \Lambda_{AB}$$

with $D_A[X] = Tr(X)\frac{1}{d_A}$ the totally depolarizing channel

- Defining **equivalence classes of states with same reductions**

$$[[\rho_{AB}]]_{AB}=\{\sigma_{AB}|\sigma_{A}=\rho_{A}\wedge\sigma_{B}=\rho_{B}\}\equiv[[\rho_{A}\otimes\rho_{B}]]_{AB}$$

Λ_{AB} is causal iff $\Lambda_{AB}([[\rho_{AB}]]_{AB})\subseteq[[\Lambda_{AB}(\rho_{AB})]]_{AB}$



- For causal maps we can define **reduced maps** and **equivalence classes of maps with the same reductions**

$$[[\Lambda_{A}\otimes\Lambda_{B}]]_{AB}\equiv\{\Gamma_{AB}|\Gamma_{A}=\Lambda_{A}\wedge\Gamma_{B}=\Lambda_{B}\}$$

“Implicit” definition of causal maps

- Consider:

$\{\rho_i^A\}_{i=1}^{d_A^2}$, $\{\rho_j^B\}_{j=1}^{d_B^2}$ bases for S_A , S_B made of states

$\Lambda_{A,B}: S_{A,B} \rightarrow S_{A,B}$ local operations

- We define ***implicitly*** a bipartite causal operation $\Lambda_{AB}: S_{AB} \rightarrow S_{AB}$

$$\Lambda_{AB}[\rho_i^A \otimes \rho_j^B] \equiv \rho'^{AB}_{ij} \in [[\Lambda_A(\rho_i^A) \otimes \Lambda_B(\rho_j^B)]]_{AB}$$

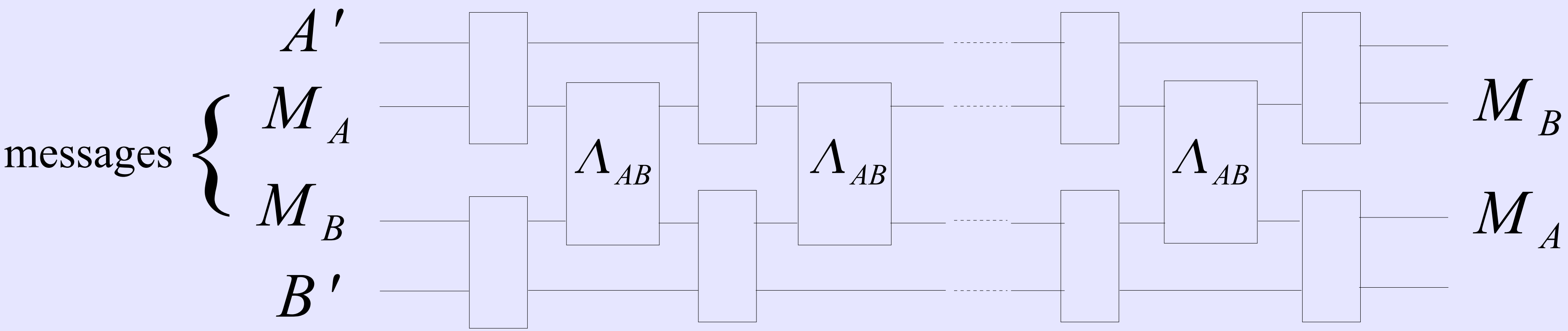
- Varying ρ'^{AB}_{ij} we obtain all possible causal maps in the **equivalence class** $[[\Lambda_A \otimes \Lambda_B]]_{AB} \equiv \{\Gamma_{AB} | \Gamma_A = \Lambda_A \wedge \Gamma_B = \Lambda_B\}$

WARNING: we must check/impose complete positivity!!!

- Classification of causal maps is related to classification of states with the same local reductions

Communication "cost" and "distillable" communication of bipartite maps

- Given local operations and *entanglement* as free resources, what is the (classical) communication needed to perform a map?
- What is the (classical) communication allowed by a map? *(A. M. Childs et al., quant-ph/0506039)*



Two-way channel ⟶ Two rates: $R_{A\to B}, R_{B\to A}$

- Maps which require communication (non-localizable) but do not allow it (causal) imply non-reversibility, i.e. **bound communication**
- Is it possible to recover reversibility allowing as a free resource some standard class of causal maps?

Conclusions

- There are bipartite quantum operations which require communication to be performed (non-localizable) but do not allow it (causal)
- The non-locality exhibited by such operations is due only partially to entanglement; bound communication is important as well
- It is possible to define and study communication cost and distillable communication of maps; is there reversibility under the assumption of having for free some causal map?

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