Evolving Networks with Multispecies nodes and Spread in the Number of Initial Links

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Abstract
We consider models for growing networks incorporating two effects not previously considered: (i) different species of nodes, with each species having different properties (such as different attachment probabilities to other node species) and (ii) when a new node is created, its number of links to old node is random with a given probability distribution. Our numerical simulation show good agreement with analytic solutions. As an application of our model, we investigate the movie-actor network with movies considered as nodes and actors as links.

Introduction
• Growing networks
  - World Wide Web, Citation index, Collaboration, etc.
  - Power-law distribution: \( n_k \sim k^{-\alpha} \) (\( \alpha \sim 3.0 \)).
  - Prefered attachment: \( A_0 \sim k^\alpha \) (\( \alpha \sim 1.0 \)).
• Need improvements
  - Multiple species nodes exist in many networks.
  - A new node can connect initial links to existing nodes.

Model Equation
• Improved Network Model:
  \[
  \frac{dN_i^n(t)}{dt} = -\sum_{j=1}^{n} Q_{ij}^{(n)} A_{ij}^{(n)} N_i^n(t) + \sum_{j=1}^{n} Q_{ji}^{(n)} A_{ji}^{(n)} N_j^{n}(t) + Q_{rk}^{(n)} P_{rk}^{(n)}
  \]
  \( N_i^n(t) \): Number of species \( n \) nodes with \( k \) links.
  \( Q_{ij}^{(n)} \): Probability of adding species \( j \) nodes at each time step.
  \( A_{ij}^{(n)} \): Prefered attachment of connecting a species \( j \) new node to a species \( i \) old node.
  \( P_{rk}^{(n)} \): Initial link probability of a species \( r \) new node making \( k \) connections to old nodes when it is created.
  \( S \): Total number of species nodes.
  \( E_i^{(n)} = \sum_j n_j^{(n)} \): Average number of new links which a species \( j \) new node may create

Formal solutions:
- \( N_i^n(t) = n_i^n \)
- \( \frac{dN_k^{(n)}}{dt} = \frac{\sum Q_{ik}^{(n)} A_{ik}^{(n)} N_i^{(n)}}{\sum_j A_{ji}^{(n)} n_j^{(n)}} + \sum \frac{Q_{jk}^{(n)} P_{jk}^{(n)}}{E_i^{(n)}} \)

Effect of Multiple Species Nodes
- Assumption 1: Two species and \( P_{rk}^{(2)} = \delta_{rk} \)
- Assumption 2: \( A_{ij}^{(2)} = A_{ij}^{(1)} = ak + bh \)
- Assumption 3: \( Q_{ij}^{(1)} = Q_{ij}^{(2)} = 0.5 \)
- Solutions: \( n_{1k}^{(1)} \sim k^{-2} \), \( n_{2k}^{(1)} \sim k^{-3} \)

Effect of Initial Link Probabilities
- Assumption 1: Single species
- Assumption 2: \( A_{ij}^{(1)} = A_{ij}^{(2)} = ak + bh \)
- Assumption 3: \( p_i = p_i(k + c)^{-d} \)
- Solutions:
  - \( n_i \sim p_i \) (if \( \beta < 3 + c/k \))
  - \( n_i \sim k^{-l+c/k} \) (if \( \beta > 3 + c/k \))
  - \( n_i \sim \frac{(2 + c/k)Q_i(k + c)^{(1+2/c)}}{\ln[(k + c)]} \) (if \( \beta = 3 + c/k \))
  - \( n_i \) (open circles) determined by \( p_i \) (solid lines)

Movie-Actor Network: Numerical Results
• Numerical Simulation Procedure:
  - Add a new movie at each time step with probability \( Q_{ij}^{(1)} \)
  - With \( P_{rk}^{(1)} \), choose the number of connections to make.
  - With \( A_{ij}^{(1)} \), connect the new \( j \) movie to old species \( i \) movies.
  - Repeat the above until tacking 100,000 new movies.
  - Calculate \( n_i^{(1)} \), \( n_i^{(2)} \).

The probability distributions \( n_i^{(1)} \)

Movie-Actor Network: Observations
• Internet Movie Database: (http://www.imdb.com)
  - Two species nodes: 226,395 theatrical movies (\( N_i^{(1)} \))
  - and 24,965 made-for-TV movies (\( N_i^{(2)} \))
  - \( Q_{ij}^{(1)} = 0.83 \) and \( Q_{ij}^{(2)} = 0.17 \)
• 555,967 links: actors and actresses
  - Observed \( p_i^{(1)} \) and \( p_i^{(2)} \)

Summary
• Suggest an improved evolving network models incorporating multispecies nodes and spread in the number of initial links.
• Analyze our model in two extreme cases.
  - \( n_k \) determined by \( p_k \) when \( p_k \) decreases slowly, while it shows the growing network dynamics when \( p_k \) decreases sufficiently rapidly.
  - \( A_{ij}^{(1)} \) gives the behavior of \( n_i^{(1)} \)
• As an example, investigate the movie-actor network.

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Effect of Initial Link Probabilities

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Summary

Acknowledgments