Experimental Investigation of the role of unstable periodic orbits in the pattern formation of turbulent motion

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CONTENT

• Motivation and background
• Experimental methodology
• Some results
• Discussion and conclusions
Turbulence

- Major unsolved problem of physics

1452 – 1519
Statistical picture

Kolmogorov's Energy cascade (1941)

Irregular motion \rightarrow Random dynamics

- Integral scale or energy-extracting range
- Taylor scale or Inertial subrange
- Kolmogorov scale or Dissipation range

\log E(\kappa) vs. \log \kappa
Richardson


  "Big whorls have little whorls,
  Which feed on their velocity;
  And little whorls have lesser whorls,
  And so on to viscosity
  (in the molecular sense)."
Deterministic viewpoint

Irregular motion → Deterministic properties of solutions

Navier-Stokes Equation (1845)

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla P}{\rho} + \nu \nabla^2 \mathbf{u}
\]

\[\nabla \cdot \mathbf{u} = 0\]

\(\mathbf{u}\): velocity field
\(P\): pressure
Unstable solutions

• All solutions to NSE at high Re numbers (in turbulent regime) are unstable!
• The coherent structures in the velocity field result from close passes to unstable equilibrium solutions of Navier-Stokes.
• These solutions and their unstable manifolds impart a rigid structure to state space, which organizes the turbulent dynamics.

• Waleffe et al, 1995, 1997
• Christiansen, Cvitanovic, et al. 1997
• Gibson, Halcrow and Cvitanović, 2008, 2009
Coherent structures

- Coherent patterns recur!
- Experimentally observed for decades.
- Recent theory: Special Navier-Stokes solutions (Cvitanovic)
Plane Couette flow

Gibson, Halcrow and Cvitanović, 2008, 2009
Experimental PIV Setup

- 2D turbulent flow
- Electrolytic cell
- Taken away from equilibrium by Electromagnetic forcing.

\[ F = J \times B \]
Analytical MHD Equations?

\[
\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_j \frac{\partial \bar{U}_i}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{U}_i}{\partial x_j} - \tau_{ij} \right) - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + F^L_i \\
\tau_{ij} = \frac{1}{3} \tau_{kk} \delta_{ij} - v_t S_{ij}, \\
F^L_i = \frac{1}{\rho} \epsilon_{ijk} E_j B_k
\]

\[
\nu_t = \nu_s \exp \left[ -\frac{\sigma}{\rho} \left( C_m \Delta \right)^2 B^2 / \nu_s \right], \\
\nu_s = \left( C_s \Delta \right)^2 (S_{ij} S_{ij})^{1/2}, S_{ij} = \frac{1}{2} \left( \frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i} \right), \\
\frac{\partial \bar{U}_i}{\partial x_i} = 0, \quad \frac{\partial \bar{B}_i}{\partial x_i} = 0, \quad \frac{\partial \bar{E}_i}{\partial x_i} = 0
\]

Shimomura (1991); Kenjereš and Hanjalic' (2000, 2004), Kenjereš et al. (2004):
Recurrence Plots (RP) Analysis

- RP is a technique of statistical nonlinear data analysis.
- The dots correspond to times at which a state of a dynamical system recurs.

\[ R_{i,j} = \Theta (\| x_i - x_j \| - th) \cdot \]


N. Marwan http://www.recurrence-plot.tk/
Non-thresholded RP

Chaotic forced pendulum

periodic forced pendulum
Measuring the experimental velocity field

PIV

- Correlation-based analysis.
- Obtain time-dependent velocity field.
Higher-dimensional RPs

• Compute the “distance” between the velocity fields at time $t$ and $\tau$.
• The distance is computed with a normed energy, e.g. Euclidean norm.

$$\sum_{\text{All Velocity Vectors}} \Delta \vec{v} \cdot \Delta \vec{v} ; \quad \Delta \vec{v}(t, \eta) \equiv \vec{v}(t) - \vec{v}(t+\eta)$$
Some results

2500 mA / high density

2500 mA / low density
RP vs velocity fields
Transitions to turbulence

1000 mA / low density

1500 mA / low density
Conclusions

- Recent turbulent theory: Coherent structures and unstable exact solutions
- Experimental test needed: 2D Electromagnetic flows may provide a good test
- RP: Nonlinear time series analysis is helpful in finding periodicities in phase space (even for infinite dimensions).
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A deterministic mechanism for randomness

The following function can produce random sequences [J. Gonzalez et al 2001, J.J. Suarez et al 2004]

\[ X_n = \sin^2(\theta \pi Z^n) \quad (1) \]

For \( z \) integer the eq. (1) can be the solution to chaotic maps
- For \( z \) rational and irrational numbers, the function produces a sequence of deterministically independent values.
Deterministic independent values
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