Ultrashort Optical Pulse Propagators

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Lecture I. From Maxwell to Nonlinear Schrödinger

Lecture II. A Unidirectional Maxwell Propagator

Lecture III. Relation to other Propagators and Applications
An Artists Rendering

- New Scientist, February 19, 2000

Breaking the light barrier
From Linear to Extreme NLO

Table 1.1. Light intensities $I$ (in units of $\text{W/cm}^2$) from the very dim to the extremely bright.

<table>
<thead>
<tr>
<th>$10^{30}$</th>
<th>generation of real electron–positron pairs from vacuum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{28}$</td>
<td>electron acceleration by light comparable to edge of black hole</td>
</tr>
<tr>
<td>$10^{26}$</td>
<td></td>
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<tr>
<td>$10^{24}$</td>
<td>nonlinear optics of the vacuum?</td>
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<tr>
<td>$10^{22}$</td>
<td></td>
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<tr>
<td>$10^{20}$</td>
<td>photonuclear fission – light splits nuclei</td>
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<tr>
<td>$10^{18}$</td>
<td>relativistic nonlinear optics of vacuum electrons</td>
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<tr>
<td>$10^{16}$</td>
<td></td>
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<tr>
<td>$10^{14}$</td>
<td>electrostatic tunneling of electrons from atoms</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>Rabi flopping in semiconductors becomes optical</td>
</tr>
<tr>
<td>$10^{10}$</td>
<td></td>
</tr>
<tr>
<td>$10^{8}$</td>
<td></td>
</tr>
<tr>
<td>$10^{6}$</td>
<td>laser intensity in the first experiment on nonlinear optics in 1961</td>
</tr>
<tr>
<td>$10^{4}$</td>
<td></td>
</tr>
<tr>
<td>$10^{2}$</td>
<td>a continuous-wave laser of that intensity hurts</td>
</tr>
<tr>
<td>$1$</td>
<td>total intensity of the sun on the earth’s surface ($10^{-1}$ W/cm$^2$)</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>thermal radiation from a human</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td></td>
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<tr>
<td>$10^{-8}$</td>
<td></td>
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<tr>
<td>$10^{-10}$</td>
<td>total intensity of the cosmic 2.8 K background radiation</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td></td>
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<tr>
<td>$10^{-14}$</td>
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<td>$10^{-16}$</td>
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<td>$10^{-18}$</td>
<td></td>
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<tr>
<td>$10^{-20}$</td>
<td></td>
</tr>
<tr>
<td>$10^{-22}$</td>
<td>visible intensity in a “dark” room at 300 K ($10^{-23}$ W/cm$^2$)</td>
</tr>
</tbody>
</table>

Light string propagation in air.
Maxwell Equations I

Maxwell’s Equations

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \quad \nabla \cdot \vec{D} = \rho \]

\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j}; \quad \nabla \cdot \vec{B} = 0 \]

Constitutive Relations

\[ \vec{B} = \mu_0 \vec{H}; \quad \vec{D} = \varepsilon_0 \vec{E} + \vec{P} \]

Polarization \( \vec{P}(\vec{r},t) \) couples light to matter.
Maxwell Equations II

Vector Wave Equation

\[ \nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} - \nabla \left( \nabla \cdot \vec{E} \right) = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2 \vec{P}}{\partial t^2} \]

Note: Most pulse propagation models derived from Maxwell assume that \( \nabla \left( \nabla \cdot \vec{E} \right) = 0 \). Coupling between field components can only occur via nonlinear polarization i.e. nonlinear birefringence.

\[ \vec{P}(\vec{r}, t) = \vec{P}_L(\vec{r}, t) + \vec{P}_{NL}(\vec{r}, t) \]
Maxwell Equations III

Linear and Nonlinear Polarization Models

\[ \tilde{P}_L(\vec{r},t) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi^{(1)}(\vec{r},t-\tau) \tilde{E}(\vec{r},\tau) d\tau \leftrightarrow \tilde{P}(\vec{r},\omega) = \varepsilon_0 \chi(\vec{r},\omega) \tilde{E}(\vec{r},\omega) \]

\[ \tilde{P}_{NL}(\vec{r},t) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi^{(3)}(\vec{r},t-\tau_1,t-\tau_2,t-\tau_3) \tilde{E}(\vec{r},\tau_1) \tilde{E}(\vec{r},\tau_2) \tilde{E}(\vec{r},\tau_3) d\tau_1 d\tau_2 d\tau_3 \]

\[ \Rightarrow \]

\[ \tilde{P}_{NL}(\vec{r},\omega) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi^{(3)}(\vec{r},\omega_1,\omega_2,\omega_3) \tilde{E}(\vec{r},\omega_1) \tilde{E}(\vec{r},\omega_2) \tilde{E}(\vec{r},\omega_3) \]

\[ \delta(\omega_1 + \omega_2 + \omega_3 - \omega) d\omega_1 d\omega_2 d\omega_3 \]

Practically, the linear polarization response is treated at some approximate level – nonlinear polarization is assumed to be instantaneous (Kerr) with a possible delayed (Raman) term.

For ultra wide bandwidth pulses even linear dispersion is nontrivial to describe!
Maxwell Equations III

Some nonlinear polarization models:

Induced nonlinear refractive index

$$\vec{P}_{NL} = \varepsilon_0 \Delta \chi \vec{E} = 2n_b n_2 \left[ (1 - f) I + f \int_0^\infty R(t) I(t - \tau) d\tau \right]$$

Coupling to Plasma

$$\partial_t \rho = a I \rho + b(I) - c^2$$

$$\vec{j}_t = \frac{e^2}{m_e} \rho(t) \vec{E}(t) - \vec{j}(t) / \tau_c$$
Formal Derivation of NLS I

- asymptotic expansion in a small parameter $\mu$

$$L\tilde{E} \equiv \nabla^2 \tilde{E} - \nabla \left( \nabla \cdot \tilde{E} \right) - \frac{1}{c^2} \frac{\partial^2 \tilde{E}}{\partial t^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int \chi\left(t - \tau\right) \tilde{E}(\tau) d\tau = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2 \tilde{P}_{NL}}{\partial t^2}$$

Seek solutions in the form of a perturbation expansion

$$\tilde{E} = \mu \tilde{E}_0 + \mu^2 \tilde{E}_1 + \mu^3 \tilde{E}_2 + \mu^4 \tilde{E}_3 + \cdots$$

where

$$\tilde{E}_0 = \tilde{e} \left( B \left( X = \mu x, Y = \mu y, Z = \mu z, T = \mu t \right) e^{i(kz - \omega t)} + \left( \ast \right) \right)$$

is a wavepacket linearly polarized perpendicular to the direction of propagation $z$. 
Formal Derivation of NLS II

\[ L = \begin{bmatrix}
\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left( 1 + \chi^{(1)} \right) & -\frac{\partial^2}{\partial x \partial y} & -\frac{\partial^2}{\partial x \partial z} \\
-\frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left( 1 + \chi^{(1)} \right) & -\frac{\partial^2}{\partial y \partial z} \\
-\frac{\partial^2}{\partial x \partial z} & -\frac{\partial^2}{\partial y \partial z} & \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left( 1 + \chi^{(1)} \right)
\end{bmatrix} \]

Assume slow variation of B

\[ \frac{\partial B}{\partial Z} = \frac{\partial B}{\partial Z_1} + \mu \frac{\partial B}{\partial Z_2} + \cdots \]

to remove secular terms proportional to z in \( \tilde{E}_1, \tilde{E}_2 \), etc.
Formal Derivation of NLS III

Can write $\mathbf{L} \mathbf{E}_0 = e^{i(kz-\omega t)} \left( \mu L_0 B \mathbf{e} + \mu^2 L_1 B \mathbf{e} + \mu^3 L_2 B \mathbf{e} + \cdots \right) + (*)$

$L_0 = \begin{bmatrix} -k^2 + k^2 (\omega) & 0 & 0 \\ 0 & -k^2 + k^2 (\omega) & 0 \\ 0 & 0 & k^2 (\omega) \end{bmatrix}$

$L_1 = \begin{bmatrix} 2ik \frac{\partial}{\partial Z_1} + 2ikk' \frac{\partial}{\partial T} & 0 & -ik \frac{\partial}{\partial X} \\ 0 & 2ik \frac{\partial}{\partial Z_1} + 2ikk' \frac{\partial}{\partial T} & 0 \\ -ik \frac{\partial}{\partial X} & -ik \frac{\partial}{\partial T} & 2ikk' \frac{\partial}{\partial T} \end{bmatrix}$

$L_2 = \begin{bmatrix} \frac{\partial^2}{\partial X^2} + 2ik \frac{\partial}{\partial Z_2} \\ \frac{\partial^2}{\partial Z_1^2} - (kk'' + kk^{12}) \frac{\partial^2}{\partial T^2} \\ -\frac{\partial^2}{\partial X \partial Y} \\ -\frac{\partial^2}{\partial Y \partial Z_1} \\ -\frac{\partial^2}{\partial Z_1 \partial Z_1} \\ \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \\ \frac{\partial^2}{\partial Z_1 \partial Z_1} - (kk'' + kk^{12}) \frac{\partial^2}{\partial T^2} \end{bmatrix}$
Formal Derivation of NLS IV

Solvability condition at order $\mu$ yields linear dispersion relation

$$O(\mu): \quad k = k(\omega)$$

Solvability condition at order $\mu^2$

$$O(\mu^2): \quad \frac{\partial B}{\partial Z_1} + k' \frac{\partial B}{\partial T} = 0$$

reflecting the fact that, to leading order, the wavepacket travels with the group velocity.
Formal Derivation of NLS V

Solvability condition at order $\mu^3$ yields NLSE

$$O(\mu^3): \frac{\partial B}{\partial Z} = \frac{i}{2k} \left( \frac{\partial^2 B}{\partial X^2} + \frac{\partial^2 B}{\partial Y^2} \right) - i \frac{k''}{2} \frac{\partial^2 B}{\partial T^2} + ik \frac{n_2}{n} |B|^2 \frac{\partial B}{\partial Z}$$

Adding terms at $O(\mu^2)$ and $O(\mu^3)$

$$\frac{\partial B}{\partial Z} = -ik' \frac{\partial B}{\partial T} + i \left( \frac{\partial^2 B}{\partial X^2} + \frac{\partial^2 B}{\partial Y^2} \right) - i \frac{k''}{2} \frac{\partial^2 B}{\partial T^2} + ik \frac{n_2}{n} |B|^2 \frac{\partial B}{\partial T}$$

Higher order correction terms such as 3rd GVD, Raman and Envelope shock terms appear at next order!
Heuristic Derivation of NLSE

NLSE is most easily derived from the general dispersion relation:

\[ \omega = \omega(\vec{k}, |\Psi|^2) \]

Expand in a multi-variable Taylor series about \((\vec{k}_0, 0)\)

\[
\omega = \omega_0 + \sum_{j=1}^{d} \left[ \frac{\partial \omega}{\partial k_j} \right]_{(k_0, \omega_0, 0)} (k_j - k_{0j}) + \left[ \frac{\partial \omega}{\partial |\Psi|^2} \right]_{(k_0, \omega_0, 0)} |\Psi|^2 \\
+ \frac{1}{2} \sum_{j,l=1}^{d} \left[ \frac{\partial^2 \omega}{\partial k_j \partial k_l} \right]_{(k_0, \omega_0, 0)} (k_j - k_{0j})(k_l - k_{0l}) + \text{hot}
\]

Generic description of weakly nonlinear dispersive waves \(\rightarrow\) invert Fourier transform \(\rightarrow\)
D-Dimensional Nonlinear Schrödinger Equation

Canonical description of weakly nonlinear dispersive waves

\[ i \frac{\partial \Psi}{\partial t} + \sum_{j=1}^{D} \left( \frac{\partial \omega}{\partial \xi_j} \right) \frac{\partial \Psi}{\partial \xi_j} + \sum_{j,l=1}^{D} \left( \frac{\partial^2 \omega}{\partial \xi_j \partial \xi_l} \right) \frac{\partial^2 \Psi}{\partial \xi_j \partial \xi_l} + \left( \frac{\partial \omega}{\partial |\Psi|^2} \right) |\Psi|^2 \Psi = 0 \]

Group Velocity  Group Velocity Dispersion  Nonlinearity

\[ D=1 - \text{Integrable} \quad D=2,3 - \text{Blowup singularity in finite time} \]

Strong Turbulence Theory and Higher Dimensional NLS

Applications:

• Deep Water Waves

• Langmuir Turbulence in Plasmas

• Relativistic beam plasma turbulence

• Bose Einstein Condensates

Types of Dispersion in NLO

**Diffraction:** Narrow waist beams of initial width $w_0$ spread over characteristic length scales $L_D \approx \frac{2\pi}{\lambda} w_0^2$

**Dispersion:** Short laser pulses of duration $\tau_p$ spread as they propagate over distances $L_{Disp} = \frac{\tau_p^2}{|k''|}$ where $k''$ is the group velocity dispersion.
Self-Phase Modulation

SPM (Kerr)  SPM (Delayed Nonlinearity)

SPM with Normal GVD

Anti-Stokes  Stokes

Dispersive shock (Wave breaking)
Plasma-Induced Spectral Blue Shift

Rae and Burnett, PRA 46, p1084 (1992)

Optical field induced ionization can cause a rapid increase in electron density and, hence a decrease in refractive index - end result is a strong blue shift in the generated spectrum

Spectral Blue Shift

\[ \Delta \lambda = -\frac{e^2 N_i \lambda^3 L}{8\pi^2 \varepsilon_0 m_e c^3} \frac{dZ}{dt} \]
Ray Optics Estimation of Critical Power

Circular Aperture

Incident Plane Wave

Critical Angle: \( \theta_c = \sqrt{\frac{n_2 E_0^2}{n_0}} \)

Diffraction Angle: \( \theta_D = \frac{0.61 \lambda_0}{2an_0} \)

Critical Power: \( P_c = \frac{(1.22)^2 \lambda_0^2 c}{128n_2} \) (Air \( P_c = 3-4 \text{ GW} \))
Role of Dimension in NLS

\[ iA_z + \nabla^2_D A + |A|^s A = 0 \]

**Critical collapse boundary (sD = 4)**

**Region of supercritical collapse (sD > 4)**

**Summary**

<table>
<thead>
<tr>
<th>s</th>
<th>D</th>
<th>NLO Manifestation</th>
</tr>
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<tbody>
<tr>
<td>A:</td>
<td>1</td>
<td>0+1 cw spatial soliton</td>
</tr>
<tr>
<td></td>
<td>1+0</td>
<td>temporal soliton</td>
</tr>
<tr>
<td>B:</td>
<td>2</td>
<td>0+2 cw beam critical focus</td>
</tr>
<tr>
<td></td>
<td>1+1</td>
<td>simultaneous compression in space and time</td>
</tr>
<tr>
<td>C:</td>
<td>3</td>
<td>1+2 3D collapse in bulk</td>
</tr>
</tbody>
</table>

**Note:** Supercritical collapse can occur in optics in anomalous GVD regime - when \( k'' < 0 \) Laplacian operator is positive definite.
Normal GVD Arrests Collapse

3D NLS: \[ i A_z = a \nabla^2 T A - k'' A_{tt} + |A|^2 A \]

- Anomalous GVD - \( k'' < 0 \)
  - 3D Collapse (simultaneous space-time compression)

- Normal GVD - \( k'' > 0 \)
  - 2D Collapse with pulse splitting in time

Review Articles:
ODE Description near Collapse Manifold

Self-similar solution: 
where \( g(\tau) \propto 1/\sqrt{z(t) - z_0(t)} \)

\[
A(\tau, \zeta, t) = g(\tau)\chi(\zeta)e^{-i\zeta^2 + i\tau}
\]

Phase Portraits

Fixed point

Small parameter:
\( \varepsilon = k''z''_0(t) \)

Luther, Newell and Moloney, Physica D, Vol. 74, p59 (1994)
Modulational Instability I

- A modulational instability across the wide beam front causes growth of collapsing filaments – 2D analog of 1D MI in a fiber
- When the instability growth spatial wavelength is much less than the transverse dimension of the beam, one can analyze it as a perturbation on a plane wave

Starting Point is the NLSE

\[ iA_z + \gamma \nabla_T^2 A + pN(|A|^2)A = 0 \]

Where \( A(\vec{x}, z) \) is the complex slowly-varying envelope of the electric field

Key Idea: Linearize about an infinite plane wave solution

\[ A(\vec{x}, z) = g e^{ipN(gg^*)z} \]

i.e \( N(|A|^2) \) is an exact integral of NLSE for a plane wave
Assume \[ A(\tilde{x}, z) = \left(g + y(\tilde{x}, z)\right)e^{ipN(1)z}\]

where \(y(\tilde{x}, z)\) is a small perturbation on the plane wave.

Substitute into NLSE

\[ iy_z - pN(I_0)(g + y(\tilde{x}, z)) + \gamma \nabla_T^2 y(\tilde{x}, z) + pN(I_0)(g + y(\tilde{x}, z)) \]

\[ + p\left(g^*y(\tilde{x}, z) + gy^*(\tilde{x}, z)\right)(g + y(\tilde{x}, z))\frac{\partial N}{\partial I}\bigg|_0 = 0 \]

Canceling second and fourth terms and retaining terms first-order in \(y\)

\[ iy_z + \gamma \nabla_T^2 y + pI_0N'y = -pg^2N'y^* \]

\[ -iy_z^* + \gamma \nabla_T^2 y^* + pI_0N'y^* = -pg^{*2}N'y \]
Modulational Instability III

To determine stability, seek perturbations with transverse spatial structure that grow with $z$ i.e.

$$y(\vec{x},z) = e^{\sigma z} \left( a e^{i\vec{K} \cdot \vec{x}} + b e^{-i\vec{K} \cdot \vec{x}} \right) + e^{-\sigma z} \left( c e^{i\vec{K} \cdot \vec{x}} + d e^{-i\vec{K} \cdot \vec{x}} \right)$$

$$y^*(\vec{x},z) = e^{\sigma^* z} \left( a^* e^{-i\vec{K} \cdot \vec{x}} + b^* e^{i\vec{K} \cdot \vec{x}} \right) + e^{-\sigma^* z} \left( c^* e^{-i\vec{K} \cdot \vec{x}} + d^* e^{i\vec{K} \cdot \vec{x}} \right)$$

After straightforward algebra coupled system of linear pde’s reduce to the solution of a matrix problem:

For $\sigma$ real:

$$\begin{bmatrix}
  i\sigma - \gamma K^2 + pI_0 N' & -pg^{*2} N' \\
  -pg^2 N' & -i\sigma - \gamma K^2 + pI_0 N'
\end{bmatrix}
\begin{bmatrix}
  a \\
  b^*
\end{bmatrix} = 0$$

For $\sigma=iv$, pure imaginary

$$\begin{bmatrix}
  -v - \gamma K^2 + pI_0 N' & -pg^{*2} N' \\
  -pg^2 N' & v - \gamma K^2 + pI_0 N'
\end{bmatrix}
\begin{bmatrix}
  a \\
  b^*
\end{bmatrix} = 0$$
Modulational Instability III

Summary:

1. Transverse perturbations will grow if $\text{Re}(\sigma)>0$
2. Instability growth determined by the condition

$$\sigma^2 = 2pI_0 N' \gamma K^2 - \gamma^2 K^4 > 0$$

3. Critical wavenumber: fastest growing mode

$$K_c = \sqrt{\frac{2pI_0 N'}{\gamma}} = \frac{2\pi}{\lambda_c}$$
Saturable Forced-Damped 2D NLS
- a paradigm for optical turbulence

Movie first shown at Optical Bistability Meeting, Aussois, 1984

Filamentation of Intense Laser Pulse in Air

NRL Laser Parameters

- Pulse energy = 1.425 J
- Pulse length = 400 fsec
- Peak power = 3.56 TW
- Propagation distance = 10 m
Optical Breakdown

• **Avalanche Photo-Ionization**
  - free impurity or seed electrons (produced by multi-photon ionization) are accelerated by light field and multiply exponentially.

• **Multi-Photon Ionization**
  - high intensity laser field ionizes electrons essentially instantaneously at a rate proportional to the light intensity $I^N \equiv |A|^{2N}$ where $N$ is the number of quanta needed to reach the ionization continuum.
Mathematical Model

\[ \frac{\partial A}{\partial z} = \frac{i}{2k} \nabla_T^2 A - i \frac{k}{2} \frac{\partial^2 A}{\partial t^2} + i(1 - f_R) kn_2 |A|^2 A - \frac{\sigma}{2} (1 + i \omega \tau) \rho A \]

\[- \frac{\beta^{(N)}}{2} |A|^{2N-2} A + if_R kn_2 \int_{-\infty}^{t} R(t') |A(t-t')|^2 \, dt' A \]

\[ \frac{\partial \rho}{\partial t} = \frac{1}{n_b^2 E_g} \rho \left| A \right|^2 + \frac{\beta^{(N)}}{N \hbar \omega} \left| A \right|^{2N} - a \rho^2 \]

- Plasma Drude Model

Avalanche generation  Multi-photon generation  Plasma recombination

Diffraction  GVD  Kerr Nonlinearity  Plasma Absorption/refraction

Multi-photon absorption  Delayed Raman Response
Nonlinear Spatial Replenishment
- multiple recurrence of collapse within pulse

Single Filament - Radial symmetry
Experimental Verification

Detected radiation indicating recurrence of collapse.

Optically Turbulent Atmospheric Light Guide

What happens when the femtosecond laser pulse is wide and contains tens to hundreds of critical powers?

A wide laser beam can undergo a modulational instability whereby an initially smooth beam profile breaks up into multiple collapsing light filaments.

Strong Turbulence Scenario!
4D Strong Turbulence Simulation

Computational Challenges:

• Severe compression of multiple interacting light filaments in space and time requires adaptive mesh refinement and parallel computation.

• Data Representation:
  - surface plot of filaments as slider moves from front to back of pulse fixed propagation distance
  - isosurface representation of filament distribution over pulse as a function of z.

Simulation
- full (x,y,z,t) resolution
- field intensity plots

Single Time Slice

Adaptive Mesh Parallel Solver

Dynamic regridding in space and time
Turbulent Interaction and Nucleation of Light Filaments

Movie sweeps from front to back of laser pulse at a fixed propagation distance $z=9.0 \, \text{m}$. Filaments collapse on front end, decay and recur at random. Collapse singularity arrested by strong defocusing which conserves energy. Dissipation is very weak in air.
Optical Turbulence

- Chaotic filament creation within propagating pulse

- 3D box contains main part of pulse

Leading Edge  

Trailing Edge

Diagram showing a 3D box with chaotic filaments located within propagating pulse.
Summary of NLSE Model

- Captures qualitative behavior of light string creation and plasma generation
- Severe time (and space) compression within violently focusing pulse may invalidate NLSE description
- Simple Drüde plasma model with multiphoton ionization source captures lowest order plasma effects – improved model planned with Becker and Faisal.
- Beyond NLSE – resolve optical carrier while propagating over meter distances: UPPE model