Self organized chaos and tipping transitions in adapting dynamical networks

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Motivation

Polyhomeostatic adaptation occurs when evolving systems try to achieve a target distribution function for certain dynamical parameters, a generalization of the notion of **homeostatis**. Here we consider a single rate encoding leaky integrator neuron model driven by white noise, adapting slowly its **internal parameters**, the threshold and the gain, in order to achieve a given **target distribution** for its time-average firing rate. For the case of sparse encoding, when the target firing-rated distribution is bimodal, we observe the occurrence of spontaneous quasi-periodic **adaptive oscillations** resulting from fast transition between two quasi-stationary attractors. We interpret this behavior as self-organized **stochastic tipping**, with noise driving the escape from the quasi-stationary attractors.

Model: Leaky Integrator

- leaky integrator neuron:

$$\dot{x}(t) = -\Gamma x(t) + \xi(t)$$

- random noise input:

$$\langle \xi(t)\xi(t')\rangle = Q\delta(t-t')$$

- sigmoidal transfer function:

$$g(x) = \frac{1}{1+e^{-a(x-b)}}$$

$x \in \mathbb{R}$
membrane potential

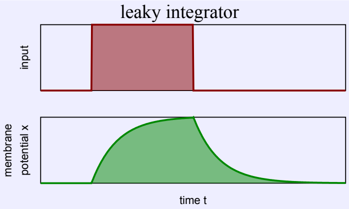
$y \in [0,1)$
firing rate as a result of the transfer function, $y = g(x)$

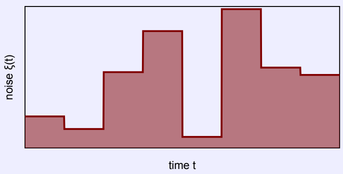
$\Gamma \in \mathbb{R}^+$
leak reducing the membrane potential exponentially

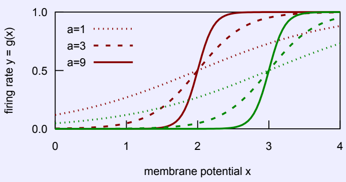
$\xi \in [\Xi_1, \Xi_2]$
random noise (white, Brownian), piecewise constant

$a \in \mathbb{R}^+$
transfer function gain (slope $= \frac{a}{4}$)

$b \in \mathbb{R}$
transfer function threshold (maximum sensitivity)







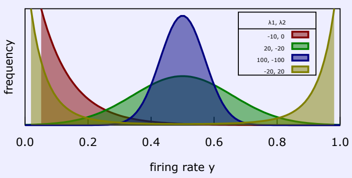
Target Firing Rate Distribution

- Shannon entropy:

$$H(q) = -\int \mathrm{d}y\, q(y) \ln q(y)$$

- maximizing with respect to a specified mean and standard derivation (variational calculus)
- Gaussian distribution:

$$q(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) \propto \exp\left(\lambda_1 y + \lambda_2 y^2\right)$$



Stochastic Adaptation Rules

- Relative entropy (Kullback-Leibler divergence):

$$D_{\mathrm{KL}}(p,q) = \int \mathrm{d}y\, p(y) \ln \frac{p(y)}{q(y)}$$

- minimizing the relative entropy:

$$\frac{\partial D_{\mathrm{KL}}}{\partial \theta} = \int \mathrm{d}x\, \rho(x)\left[-\frac{1}{g'}\frac{\partial g'}{\partial \theta} - \frac{q'}{q}\frac{\partial g}{\partial \theta}\right] \equiv \int \mathrm{d}x\, \rho(x)\, \frac{\partial d}{\partial \theta}$$

- transfer function gain adaptation rule:

$$\delta a = \epsilon_a \left[\frac{1}{a} + (x-b)\Big[1 - 2y + (\lambda_1 + 2\lambda_2 y)(1-y)y\Big]\right]$$

- transfer function threshold adaptation rule:

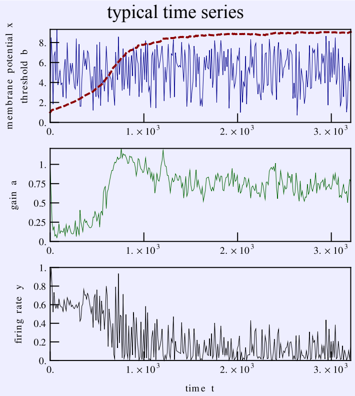
$$\delta b = \epsilon_b \left[-a\Big(1 - 2y + (\lambda_1 + 2\lambda_2 y)(1-y)y\Big)\right]$$

Methods

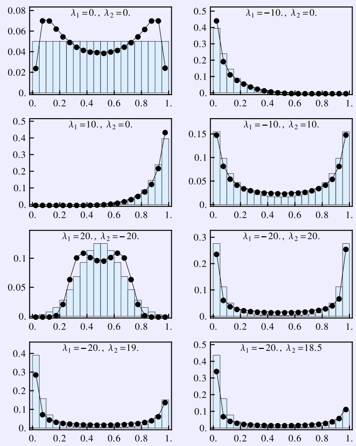
- direct simulation using numerical iteration:

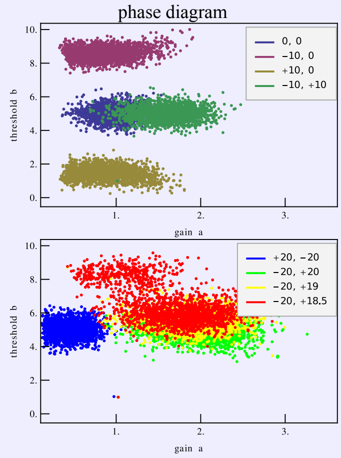
$$x(t_{n+1}) = x(t_n) + \dot{x}(t_n)\Delta t$$

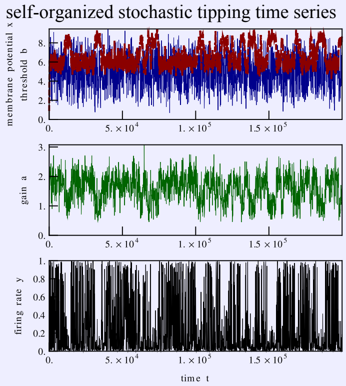
- slow-fast dynamical system
- randomly choosen initial values $x(0)$, $a(0)$, $b(0)$ for different trials to overcome local minima and to find fixed points
- simulation parameters:
  - $\Delta t = 10^{-1}$
  - $\Gamma = 1$
  - $\varepsilon_a = 10^{-2}$
  - $\varepsilon_b = 10^{-2}$
- different ε_a and ε_b for learning rate dependent relative entropies and tipping

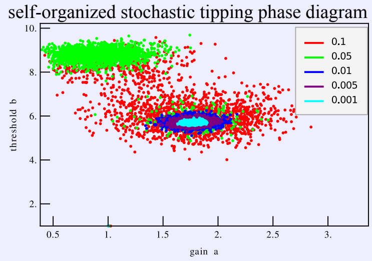


Results









relative entropies (various target distributions)

λ_1	λ_2	shape	D_{KL}
0	0	uniform	0.043
-10	0	left dominant	0.034
+10	0	right dominant	0.028
-10	+10	left/right dominant	0.018
+20	-20	hill	0.076
-20	+20	left/right, symmetric	0.175
-20	+19	left/right, left skewed	0.244
-20	+18.5	left/right, left skewed	0.283

- good target firing rate distribution approximation (not perfect due to only two degrees of freedom)
- well minimized relative entropy
- robust against replacing the transfer function (erf, tanh, arctan, other sigmoidal functions)
- target firing rate distribution fingerprint in phase space
- learning rate dependent behavior changes
- no bimodal firing rate distribution as a steady state time-average
- rate-induced tipping transitions between quasi-stationary fixed points

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