Directedness of information flow in mobile phone communication networks

Fernando Peruani

In collaboration with:
Lionel Tabourier

References:
FP and Sibona, PRL (2008)
FP and L. Tabourier, PLoS ONE 6, e28860 (2011)
L. Tabourier, A. Stoica, FP, ComsNets 12
Synchronization in a spatial time-varying network

Kuramoto oscillators moving in the space...

At a given time $t$, the state of the system can be represented as

Equation of motion of the oscillators:

$$\dot{\theta}_i(t) = -\gamma \left[ \frac{1}{n_i} \sum_{|x_i-x_j|<r} \sin(\theta_i - \theta_j) \right] + \sqrt{2C}\xi_{\theta,i}(t)$$

$$\dot{x}_i(t) = \sqrt{2D}\xi_{x,i}(t)$$

Noise
Synchronization in a spatial time-varying network

Kuramoto oscillators moving in the space...

Equation of motion of the oscillators:

\[
\dot{\theta}_i(t) = -\gamma \left[ \frac{1}{n_i} \sum_{|x_i - x_j| < r} \sin(\theta_i - \theta_j) \right] + \sqrt{2C} \xi_{\theta,i}(t)
\]

\[
\dot{x}_i(t) = \sqrt{2D} \xi_{x,i}(t)
\]
Synchronization in a spatial time-varying network

Any finite amount of noise is sufficient to prevent consensus in large enough systems of mobile autonomous agents randomly moving in 1D or 2D.

\[
\dot{\theta}_i(t) = -\gamma \left[ \frac{1}{n_i} \sum_{|x_i-x_j|<r} \sin(\theta_i - \theta_j) \right] + \sqrt{2C}\xi_{\theta,i}(t)
\]

\[
\dot{x}_i(t) = \sqrt{2D}\xi_{x,i}(t)
\]

[Remember A. Diaz-Guilera’s talk: if there no noise, then global synchronization is possible]

Peruani, Nicola, Morelli, NJP (2010)
Synchronization in a spatial time-varying network

Langevin Eqs.:
\[
\dot{\theta}_i(t) = -\gamma \left[ \frac{1}{n_i} \sum_{|x_i-x_j|<r} \sin(\theta_i - \theta_j) \right] + \sqrt{2C} \xi_{\theta,i}(t)
\]
\[
\dot{x}_i(t) = \sqrt{2D} \xi_{x,i}(t)
\]

Associated (non linear!) Fokker-Planck Eq.:
\[
\partial_t \rho(x, \theta, t) = D \partial_{xx} \rho(x, \theta, t) + C \partial_{\theta \theta} \rho(x, \theta, t)
\]
\[
+ \frac{\gamma}{n(x)} \partial_\theta \left[ \int_0^L dx' \int_0^{2\pi} d\theta' g(x-x') \sin(\theta - \theta') \rho(x', \theta', t) \rho(x, \theta, t) \right]
\]

Solution:
\[
\rho_s(x, \theta) = \mathcal{N} \exp \left[ \frac{\gamma R}{D_{\text{eff}}} \cos(\theta - kx) \right]
\]

Multiple solutions!!!

All these solutions exist and are linearly stable for an infinite system!!!

Peruani, Nicola, Morelli, NJP (2010)
Any finite amount of noise is sufficient to prevent consensus in large enough systems of mobile autonomous agents randomly moving in 1D or 2D.

If agents do not move randomly, global synchronization is possible!

Add a coupling between the phase and the direction of motion of the agents, and synchronization will occur in 2D (1D is more tricky!):

Synchronization = collective motion

Vicsek et al., PRL (1995)
Gregoire & Chaté, PRL (2004) showed that this transition is first order.
Spatial time-varying network & moving agent systems

Three classical approaches for information spreading

Lattice models

Classical MF models

Network models

(strong correlations in space)

\[
\frac{N_i(t)}{N} \sim t^{-0.45}
\]
(at the critical point)

\[
\frac{N_i(t)}{N} \sim t^{-1}
\]
(well-mixed)
(at the critical point)

FP and Sibona, PRL (2008)
Moving agent systems allow us to bridge the gap between lattice to MF models, passing through time-varying networks.

\[
\dot{x}_i(t) = \frac{F_i}{\zeta} + \frac{1}{\zeta} \sum_{j \neq i} \nabla U(x_i(t), x_j(t))
\]

\[
U(x, x') = \begin{cases} 
\gamma |x - x'|^{-\beta} - (2r)^{-\beta} & \text{if } |x - x'| < 2r \\
0 & \text{if } |x - x'| \geq 2r 
\end{cases}
\]

FP and Sibona, PRL (2008)
Motivation

Information flow in mobile phone networks

FP and L. Tabourier, PLoS ONE 6, e28860 (2011)
L. Tabourier, A. Stoica, FP, ComsNets 12
Motivation

Are there causality effects?
Is there an intentional flow of information?

L. Tabourier, A. Stoica, FP, *ComsNets* 12
**Motivation**

What is the question here?

A phone call certainly involves information exchange between two users.

But is there information spreading beyond the two users involved in a communication?

We focus on intentional transmission of information.

What are the statistical features of such information propagation process?

Strategy:

Construct causality trees and study their statistical properties.
Motivation

Unintentional transmission of information

A phone call implies an undirected link!

Here phone calls are not intended to transmit the information
Intentional transmission of information

A phone call implies a directed link!

Here phone calls could be intended to transmit the information!
Motivation

Intentional transmission of information

A is informed $\rightarrow$ A calls $\rightarrow$ B

B calls $\rightarrow$ C $\rightarrow$ C is informed

A info flow $\rightarrow$ C

C info flow $\rightarrow$ A

The temporal sequence of events does not allow it.
Transmission of information

A is informed $\rightarrow$ A calls B $\rightarrow$ B

C calls B $\rightarrow$ after some time $\tau$ C is informed

This defines a valid flow of information, however the fact that C calls B cannot be associated to the fact that B got informed previously. There is no intentionality and would not involve “causality”. 
The social static network (cumulative network)

1-month activity of a European mobile phone network operator

1.127.658 users
14.388.440 phone calls

Looking at the “cumulative” network…

Log-normal distribution!

\[
\langle k_i \rangle = \langle k_o \rangle = 2.86
\]

\[
\langle k_{\text{undir}} \rangle \sim 17
\]

In-out degree correlations:

* Pearson coeff. \( \sim 0.58 \)
* There are super “senders” and “receivers” which are not necessary both.
Definitions

The social static network (cumulative network)

In-out degree correlations:
* Pearson coeff. ~0.58
* There are super "senders" and "receivers" which are not necessary both.
### Definitions

#### Construction of causality trees

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### Definitions

#### Construction of causality trees

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Definitions

Example of a real causality tree

Main features of a tree: size \( s = 11 \) & depth \( d = 5 \)
Causality trees and S-I-R spreading dynamics

If an informed user calls


SIR assuming that mobile phone net is undirected


Disease spreading and mobile phone nets:
Causality trees and S-I-R spreading dynamics

- Inoculate the disease in a node
- Let the disease spread (on the social network)
  (the spreading is dictated by the calling activity of users!)
- We get a tree after extinction of the disease

The probability of finding a causality tree of size s and depth d, is equivalent to the probability that a disease outbreak got extinguished after infecting s users at depth d.
Phase transition associated with $\mathcal{T}$.

The average tree size exhibits a remarkable increase after $\sim30$ hours. We will see that this implies arbitrarily large cascades after 30 hours.
How can we understand that there is a phase transition associated to $\mathcal{T}$?

\[
\dot{S} = -\langle \rho \rangle \langle k_o \rangle_\infty SI, \quad \dot{I} = \langle \rho \rangle \langle k_o \rangle_\infty SI - \frac{I}{\tau}, \quad \dot{R} = \frac{I}{\tau}
\]

- $\rho_{i \to j}$ → phone call rate from user $i$ to $j$
- $\langle \rho \rangle$ → average of this quantity
- $\langle k_o \rangle_\infty$ → average out degree of the “cumulative” network
  [network that contains all connection that occur in the database]

The infection rate of an informed user $\langle \rho \rangle \langle k_o \rangle_\infty$ competes with the time the user remains active $\mathcal{T}$.
How can we understand that there is a phase transition associated to $T$?

$$\dot{S} = -\langle \rho \rangle \langle k_o \rangle \infty SI, \quad \dot{I} = \langle \rho \rangle \langle k_o \rangle \infty SI - \frac{I}{\tau}, \quad \dot{R} = \frac{I}{\tau}$$

$\langle s \rangle$ → average tree size → directed related to $R(t \to \infty)$

$$\langle s \rangle / N = 1 - \exp \left[ -\langle \rho \rangle \langle k_o \rangle \infty \tau \langle s \rangle / N \right]$$

$R_0 = \tau \langle \rho \rangle \langle k_o \rangle \infty > 1$ → Critical condition according to the MF
Theory & results

How can we understand that there is a phase transition associated to $\mathcal{T}$?

\[ \dot{S} = -\langle \rho \rangle \langle k_o \rangle_\infty S I, \quad \dot{I} = \langle \rho \rangle \langle k_o \rangle_\infty S I - \frac{I}{\tau}, \quad \dot{R} = \frac{I}{\tau} \]

\[ \langle s \rangle \rightarrow \text{average tree size} \rightarrow \text{directed related to} \rightarrow R(t \rightarrow \infty) \]

\[ \frac{\langle s \rangle}{N} = 1 - \exp \left[ -\langle \rho \rangle \langle k_o \rangle_\infty \tau \langle s \rangle/N \right] \]

\[ R_0 = \tau \langle \rho \rangle \langle k_o \rangle_\infty > 1 \]

\[ \tau_c = 1/\left( \langle \tilde{\rho} \rangle \langle k \rangle_\infty^{-1} \right) \]

\[ \tilde{\rho}_{i,j} = \rho_{i\to j} + \rho_{j\to i} \rightarrow \langle \tilde{\rho} \rangle \]

Av. undir. degree
Taking into account the network topology

[in-out degree distribution – node-node correlations neglected]

"cumulative" (=social) network

\[ p_\infty(k_i, k_o) = p(k_i, k_o; \tau \to \infty) \]

\[
p(k_i, k_o; \tau) = \sum_{k'_i = k_i; k'_o = k_o}^\infty p_\infty(k'_i, k'_o)
\times \binom{k'_i}{k_i} T_i(k'_i, \tau)^{k_i} (1 - T_i(k'_i, \tau))^{k'_i - k_i}
\times \binom{k'_o}{k_o} T_o(k'_o, \tau)^{k_o} (1 - T_o(k'_o, \tau))^{k'_o - k_o}
\]

\[ T_i(k'_i, \tau) \rightarrow \text{Probability that an edge that arrives at a node of in-degree } k'_i \text{ is used during } \tau \]

\[ T_o(k'_o, \tau) \rightarrow \text{Probability that an edge that emerges from a node of out-degree } k'_o \text{ is used during } \tau \]
Simplifying the transmission process

\[ T_{j \to k} = 1 - \exp(-\rho_{j \to k} \tau) \]

\[ T(\tau) = T_i(k, \tau) = T_o(k, \tau) = \int d\rho p(\rho) \left( 1 - e^{-\rho \tau} \right) \]

distribution of activity of an edge

Under all these assumption, the generating function:

\[ g(x, y, \tau) = \sum_{k_i', k_o'} p_\infty(k_i', k_o') \left( 1 + (x - 1)T(\tau) \right)^{k_i'} \]
\[ \times \left( 1 + (y - 1)T(\tau) \right)^{k_o'} \]
Condition for arbitrarily large trees

Percolation threshold for a static directed network \[\langle k_i k_0 \rangle / \langle k_0 \rangle \geq 1\]

From previously computed generating function we get:

\[\langle k_i \rangle(\tau) = \langle k_0 \rangle(\tau) = x \frac{\partial^2 g}{\partial x} |_{x,y=1}\]

\[\langle k_i k_0 \rangle(\tau) = x y \frac{\partial^2 g}{\partial xy} |_{x,y=1}\]

Condition for arbitrarily large trees:

\[T(\tau) \geq \frac{\langle k_0 \rangle_\infty}{\langle k_i k_0 \rangle_\infty}\]

where

\[\langle k_0 \rangle_\infty = \sum_{k_0} k_0 p_\infty(k_0)\]

\[\langle k_i k_0 \rangle_\infty = \sum_{k_i,k_0} k_i k_0 p_\infty(k_i,k_0)\]
**Theory & results**

**Condition for arbitrarily large trees**

To obtain the critical $\tau$ value we simplify the problem by assuming:

$$\rho \tau_c \ll 1 \quad \Rightarrow \quad T(\tau) \sim \langle \rho \rangle \tau \quad \text{for} \quad \tau \leq \tau_c$$

Two extreme situations:

- **fully in-out correlated**

  \[ k_i = k_o \]

  \[ \tau_c = \langle k_o \rangle_\infty / \left( \langle \rho \rangle \langle k_o^2 \rangle_\infty \right) \]

- **fully in-out uncorrelated**

  \[ p_\infty(k_i, k_o) = p_\infty(k_i) p_\infty(k_o) \]

  \[ \tau_c = 1 / \langle \rho \rangle \langle k_o \rangle_\infty \]
**Condition for arbitrarily large trees**

To obtain the critical $\tau$ value we simplify the problem by assuming:

$$\rho \Gamma_c \ll 1 \quad \rightarrow \quad T(\tau) \sim \langle \rho \rangle \tau \quad \text{for} \quad \tau \leq \tau_c$$

**Two extreme situations:**

- **fully in-out correlated**
  $$\kappa_i = \kappa_o$$
  $$\tau_c = \langle \kappa_o \rangle_\infty / \left( \langle \rho \rangle \langle \kappa_o^2 \rangle_\infty \right)$$

- **fully in-out uncorrelated**
  $$p_\infty(k_i, \kappa_o) = p_\infty(k_i) p_\infty(\kappa_o)$$
  $$\tau_c = 1 / \langle \rho \rangle \langle \kappa_o \rangle_\infty$$

**UNDIRECTED**

$$p(k; \tau) = \sum_{k' = -k}^{\infty} p_\infty(k') \left( \begin{array}{c} k' \\ k \end{array} \right) \tilde{T}(k', \tau)^k(1 - \tilde{T}(k', \tau))^{k' - k}$$

$$\tilde{g}(x, \tau) = \sum_{k'} p_\infty(k') \left( 1 + (x - 1) \tilde{T}(\tau) \right)^{k'}$$

$$\tilde{T}(\tau) \sim \langle \tilde{\rho} \rangle \tau \quad \rightarrow \quad \tau_c = \frac{\langle k \rangle_\infty}{\langle \tilde{\rho} \rangle \left( \langle \kappa^2 \rangle_\infty - \langle k \rangle_\infty \right)} \sim 12Hs!$$
Theory & results

Size of trees

[ We look for the tree size distribution \( p(s, \tau) \) ]

\[
G(z, \tau) = \sum_{s=1} p(s, \tau) z^s
\]

The tree size distribution generating function obeys:

\[
G(z; \tau) = z \cdot g(1, G(z; \tau); \tau)
\]

\[
g(1, y; \tau) = \sum_{k_o} p(k_o, \tau) y^{k_o}
\]

We are describing tree the evolution process as a Galton-Watson process fully determined by \( p(k_o, t) \)!

Reasoning:

\[
p(s = 1; \tau) = p(k_o = 0; \tau)
\]

\[
p(s = 2; \tau) = p(k_o = 1; \tau) \cdot p(s = 1; \tau)
\]

in general:

\[
p(s; \tau) \longrightarrow \text{related to } p(s'; \tau)
\]

with \( s' < s \)
Theory & results

Size of trees
Theory & results

Depth of trees
[ We look for the tree depth distribution $p(d, t)$ ]

$E_d(\tau) \rightarrow$ Probability that a tree get extinguished at depth $d$ or less

This probability obeys:

$$E_d(\tau) = p(0, \tau) + p(1, \tau)E_{d-1}(\tau) + p(2, \tau)E_{d-1}(\tau)^2 + \ldots + p(k_0, \tau)E_{d-1}(\tau)^{k_0} + \ldots$$

$$E_d(\tau) = g_1(E_{d-1}(\tau); \tau) \rightarrow$$

Using the fact that

$$E_1(\tau) = p(k_0 = 0, \tau)$$

$$E_d(\tau) = g_d(0; \tau)$$

Tree depth distribution is then simply:

$$p(d; \tau) = g_d(0; \tau) - g_{d-1}(0; \tau)$$

where

$$g_{n+1}(y; \tau) = g_1(g_n(y; \tau); \tau)$$

We can expect the proposed tree theory to work before the emergence of arbitrarily large cascades.
Theory & results

Depth of trees

The diagram illustrates the depth distribution $p(d; \tau)$ for different values of $\tau$: 0.5, 5, and 20, as well as data with and without random walk (RT) correction. The lines represent the theoretical predictions from Equation (11), while the data points are experimental observations.
Theory & results

Depth of trees

- Randomization of data:

User 1

- U17
- U54
- U89

- t11
- t12
- t13

User 2

- U25
- U19
- U67

- t21
- t22
- t23

RT data

- same topology
- "global" mixing of time-stamps
Absence of time-correlation effects
[at the level of size and depth distribution]

The underlying social network in data and data w/ RT is the same. The only difference is due to the absence of time-correlations in data w/ RT.

While for short “time-scales” (tau-values) we can neglect node-node (topological) correlations, above 30 hours these correlations becomes dominant and the simple theory proposed fails to describe the data.
At short time scale the matching between RT and original data is good but not perfect – why?

Is this due to the bursty activity of user (neglected in RT data)?

Is this related to sender-receiver time correlations (also neglected in RT data)?
At short time scale the matching between RT and original is good but not perfect – why?

Answer: **The difference is due to the bursty activity of users [sender-receiver time correlation play no role!]**

---

**RC data**

- same topology
- “user” mixing of time-stamps
## Theory & results

### Looking at different motifs

<table>
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<th>$\delta$</th>
<th>$P_{\sigma,\delta}^{\text{real}}(\tau)$</th>
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- $\tau = 30\text{min}$
- $\tau = 3\text{h}$
- $\tau = 12\text{h}$
Theory & results

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Fig. 4. $R_{\sigma,\delta}(\tau) = \frac{P_{\sigma,\delta}^{\text{real}}}{P_{\sigma,\delta}^{\text{comm}}}$ as a function of $\tau$. Red circles: $R_{2,2}(\tau)$, green diamonds: $R_{3,2}(\tau)$, blue triangles: $R_{3,3}(\tau)$, pink circles: $R_{4,4}(\tau)$. 

\[ \]
Local flow of information

Are there causality/information loops?

\[ t_2 - t_1 < \tau \]

Reciprocity coefficient

\[ C_R = \frac{A \leftrightarrow B}{A \rightarrow B} \]

Dynamical cluster coefficient

\[ C_C = \frac{A \rightarrow B, C}{A \rightarrow B, C \rightarrow D} \]
Local flow of information

Are there causality/information loops?

Data has time-correlations while in data w/ RT time-correlations were washed out.

Time correlations induce larger causality loops for small values of tau!!!
**Summary**

1. Representation of mobile phone data as a directed network shows the presence of **super spreaders** and **super receivers**.

2. **Intentional** information spreading is extremely sensitive to **in-out degree correlations**! These correlations help in the intentional info spreading!

3. At short time scales the tree statistics can be described by a **GW process** – We can neglect topological and temporal correlations!!!

4. Small effect on the tree statistics is observed due to **bursty user activity**, which helps to the info spreading!

5. **At long time** scale, topological **node-node correlation** dominate the dynamics! We can completely ignore temporal correlations!

6. **Macroscopic** intentional spreading is only achieved if user retransmit for more than **30 hours!!!** (For unintentional spreading this is 12 hours!)

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7. **Time-correlations** play a crucial role at very short time-scales in the form of **information cycles** (that do not contribute to info spreading).

8. The idea that there is information spreading beyond nearest and second-nearest neighbors, i.e., beyond a small vicinity, is called into question!

L. Tabourier, A. Stoica, FP, ComsNets 12
Thanks for your attention!

References:
FP and Sibona, PRL (2008)
FP and L. Tabourier, PLoS ONE 6, e28860 (2011)
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