Continuous configuration-interaction for condensates in a ring

or

Interacting bosons with definite total momentum

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Motivation

Ground state of attractive condensates in a 1D ring:

- GP equation $\longrightarrow$ Symmetry-broken solution [1,2].
- Diagonalization $\longrightarrow$ fragmentation [2].
- Exact solution for $N = 2$ [3], $N = 3$ [4] only.

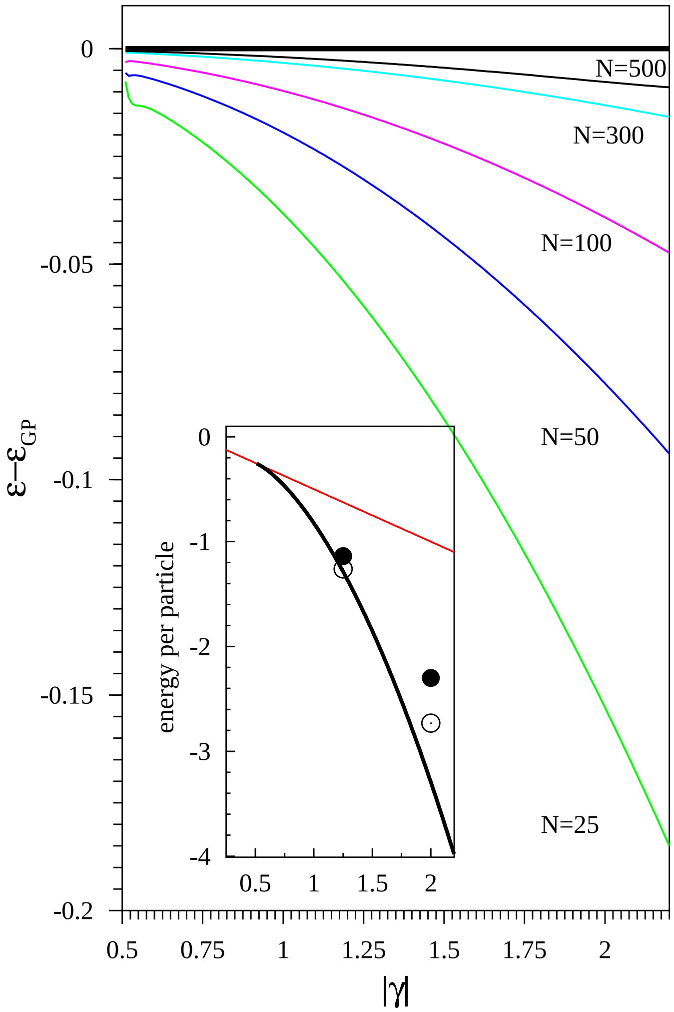
[1] L. D. Carr, C. W. Clark, and W. P. Reinhardt, Phys. Rev. A **62**, 063611 (2000).

[2] R. Kanamoto, H. Saito, and M. Ueda, Phys. Rev. A **67**, 013608 (2003).

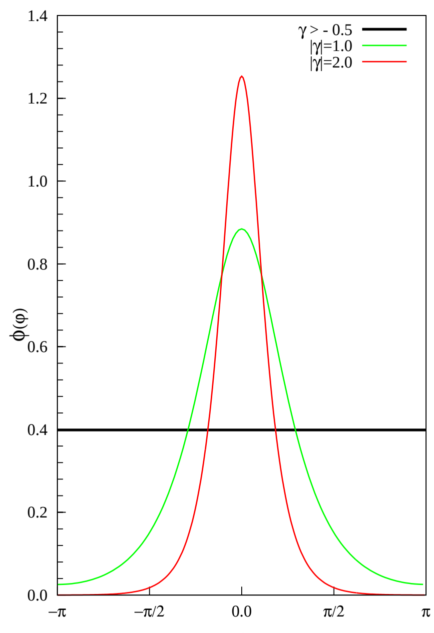
[3] E. H. Lieb and W. Liniger, Phys. Rev. **130**, 1605 (1963).

[4] J. G. Muga and R. F. Snider, Phys. Rev. A **57**, 3317 (1998).

Ground-state energy per particle



GP symmetry-broken, symmetry-preserving solution



How to go beyond GP on the ring?

[OEA, A. Streltsov, K. Sakmann, and L. S. Cederbaum, Europhys. Lett. **67**, 8 (2004)]

N-body Hamiltonian:

$$\hat{H} = -\sum_i \frac{\hat{\partial}^2}{\partial \varphi_i^2} + \frac{U_0}{2} \sum_{i \neq j} \delta(\varphi_i - \varphi_j), \ \ \gamma = \frac{U_0(N-1)}{2\pi}$$

Employ GP symmetry-broken orbital, wavefunction:

$$\phi(\varphi) \longrightarrow \Phi(\vec{\varphi}) = \Pi_i \, \phi(\varphi_i)$$

Continuous configuration-interaction (CCI) ansatz:

$$\Psi(\vec{\varphi}) = \int_0^{2\pi} d\theta \, C(\theta) \, \Phi(\vec{\varphi} - \theta),$$

Minimize the CCI energy:

$$\varepsilon[\phi] = \frac{1}{N} \frac{\left\langle \Psi(\vec{\varphi}) | \hat{H} | \Psi(\vec{\varphi}) \right\rangle}{\langle \Psi(\vec{\varphi}) | \Psi(\vec{\varphi}) \rangle}$$

Variational solution:

$$C(\theta) = e^{+iL\theta}, \ \ L = 0, \pm 1, \pm 2, \ldots$$

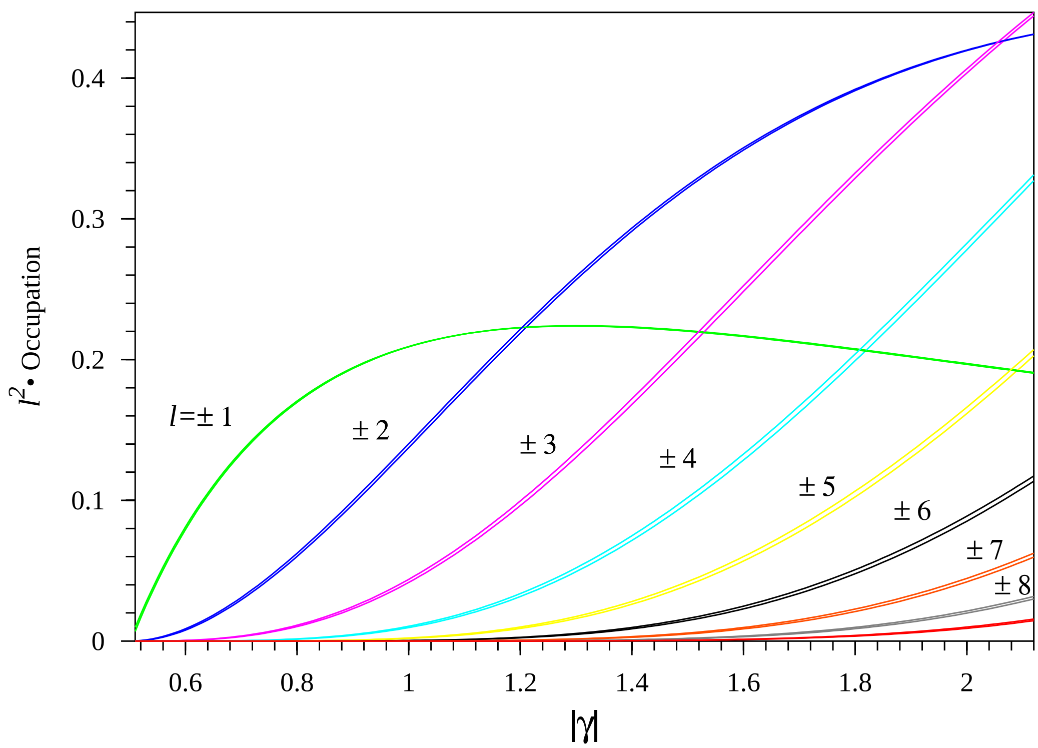
Fragmentation from GP orbital?

GP approach cannot describe fragmentation of condensates *in principle*,

but our new extended mean-field approach does $\longrightarrow$ Goto **Alexej Streltsov** Poster.

Reduced one-particle density:

$$\rho(\varphi,\varphi')[\Psi(\varphi)] = \frac{1}{2\pi}\sum_l p_l\, e^{+il(\varphi-\varphi')}$$



Why diagonalization is so "difficult"?

$N = 25$, $|\gamma| = 2.0 \longrightarrow l \leq 8$ ...

$$> 1{,}000{,}000{,}000 \ \text{configurations!}$$

Interacting bosons with definite total momentum

[When the mean-field solution *is not* symmetry-broken]

[OEA, A. Streltsov, and L. S. Cederbaum, quant-ph/0406095]

"Any *exact* eigenstate with a definite momentum of a many-body Hamiltonian can be written as an integral over a *symmetry-broken* function Φ":

$$\Psi(\vec{\varphi}) = \int_0^{2\pi} d\theta \, C(\theta) \, \Phi(\vec{\varphi} - \theta)$$

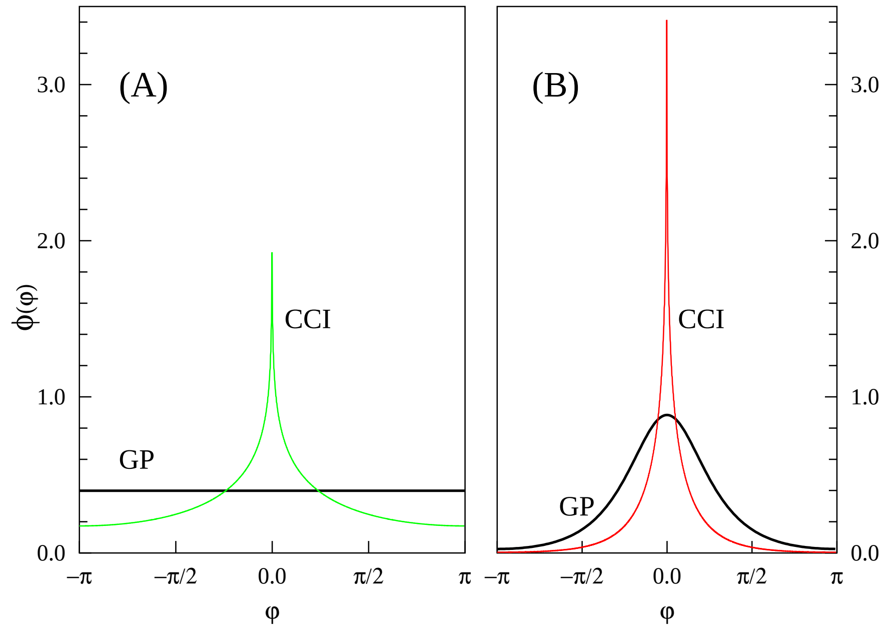
Simplest case: take for Φ the Hartree product:

$$\Phi(\vec{\varphi}) = \Pi_i \, \phi(\varphi_i)$$

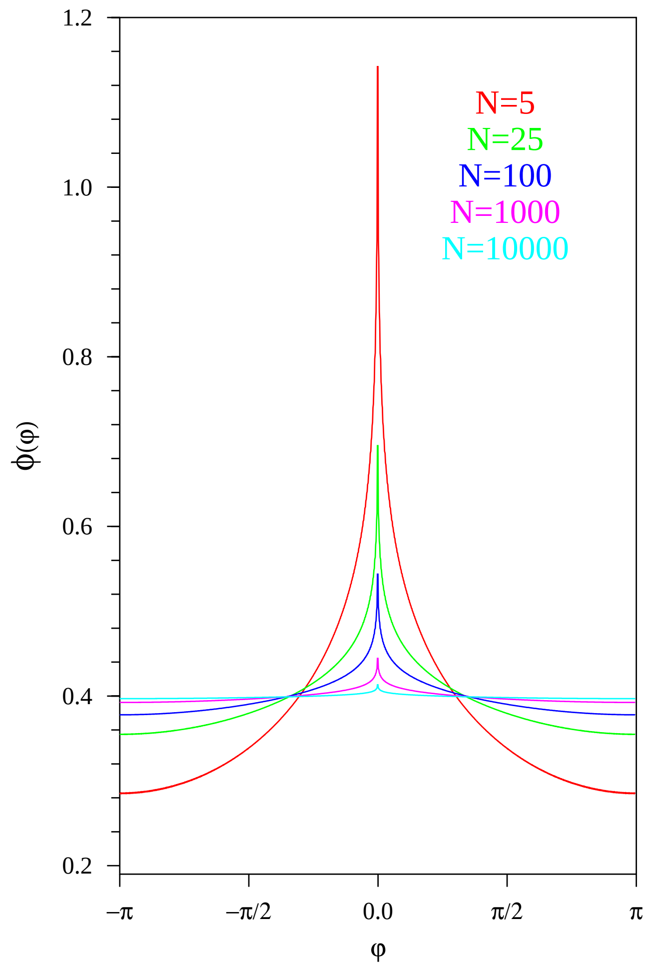
Question: What is the *optimal orbital* $\phi(\varphi)$?

[it is not the GP (mean-field) orbital]

CCI orbital for 2 bosons is **exact**: (A) $|\gamma| = 0.2$; (B) $|\gamma| = 1.0$



CCI orbital for $N > 2$ bosons: $|\gamma| = 0.2$



Summary and outlook

Main message of this talk:

- The CCI ansatz allows us to obtain many-boson wavefunctions with definite momentum and lower energy than that provided by GP.

- The numerical implementation is straightforward (DVR) and the computational effort does not depend on the number of bosons N.

Outlook:

- Angular momentum (excitations).

- Fermions.

- Corrugation (asymmetry).

Epilog:

We know now to solve, numerically exact (> 20 digits), the problem of up to 50 bosons in a ring!

[K. Sakmann, A. Streltsov, OEA, and L. S. Cederbaum, in preparation]

Acknowledgments

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