HOW TO DETECT THE CONDENSATION OF PAIRS IN A SPIN 1/2 FERMI GAS ?

I. Carusotto, BEC-INFM, Trento

Y. Castin, LKB-ENS, Paris

OUTLINE

- Expected phase diagram for a spin 1/2 Fermi gas
  - Quantum Monte Carlo results in 1D
- How to detect the phase transition ?
- Our proposal: interferometric detection
  - case of the bosons
  - extension to paired fermions
  - set up 1: with a matter wave beam splitter
  - set up 2: without a matter wave beam splitter
- The unitary quantum gas
  - an exact time dependent solution

EXPECTED PHASE DIAGRAM FOR A SPIN 1/2 FERMI GAS

Good news from current experiments:

- scattering length a changed at will with Feshbach resonance
- Spin 1/2 Fermi gas remains stable

Current theories:

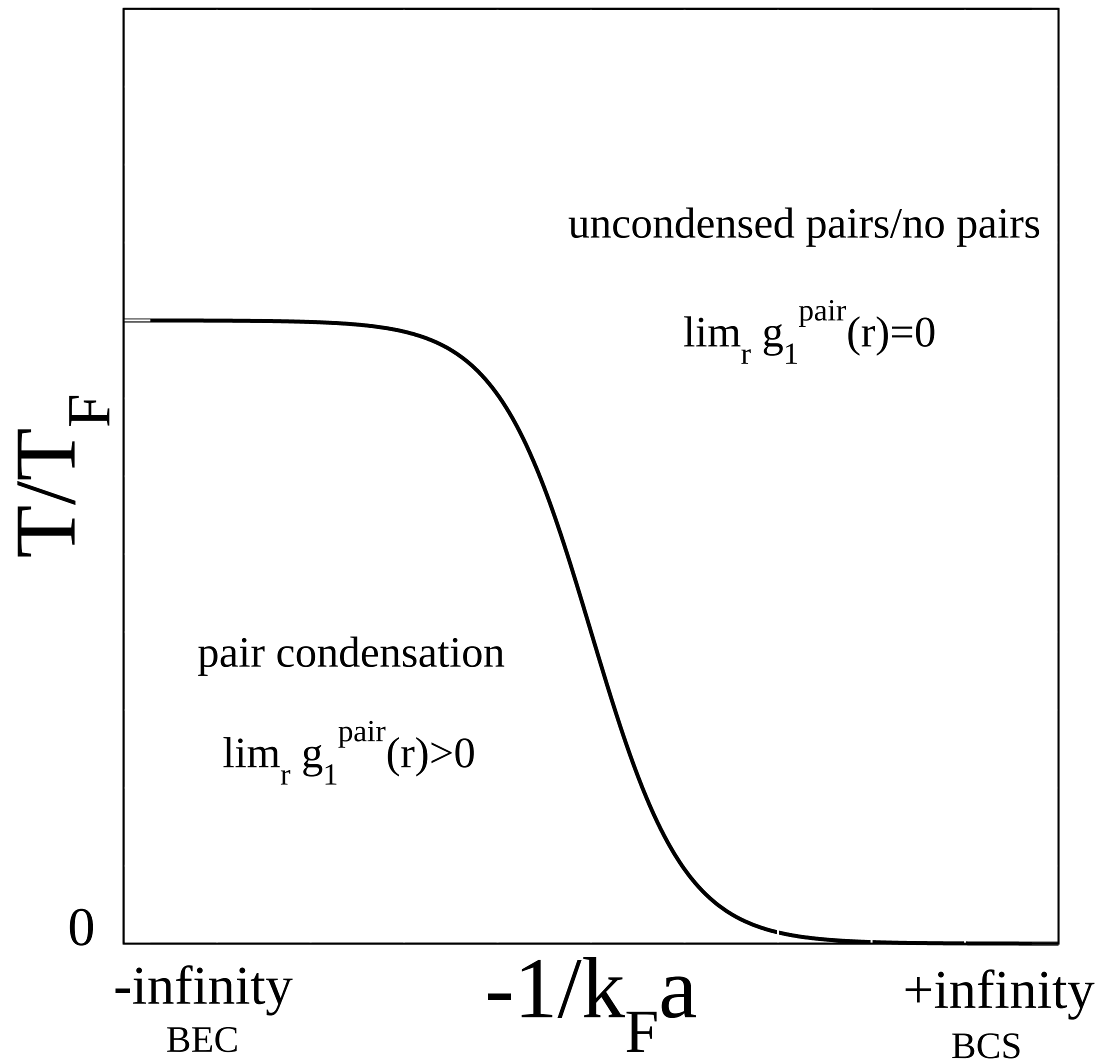
- Condensation of pairs governed by the order parameter

$$\lim_{r\to+\infty} g_1^{\text{pair}}(\mathbf{r}) > 0$$

with pair first order coherence function

$$g_1^{\text{pair}}(\mathbf{r}) \equiv \langle \hat{\psi}_\uparrow^\dagger(\mathbf{r})\hat{\psi}_\downarrow^\dagger(\mathbf{r})\hat{\psi}_\downarrow(\mathbf{0})\hat{\psi}_\uparrow(\mathbf{0})\rangle.$$

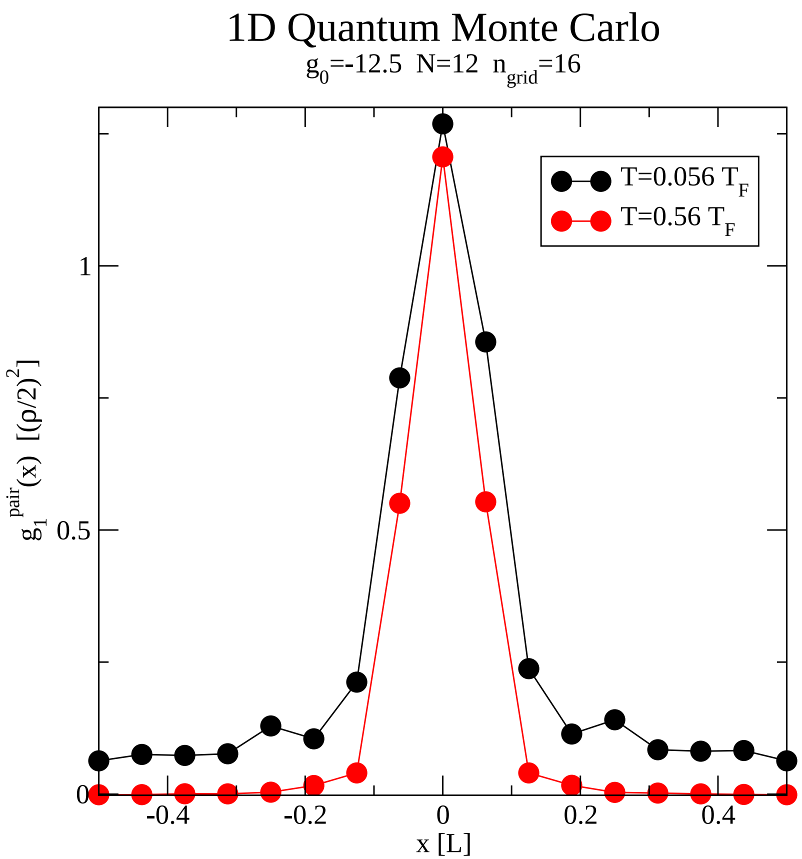
EXPECTED PHASE DIAGRAM

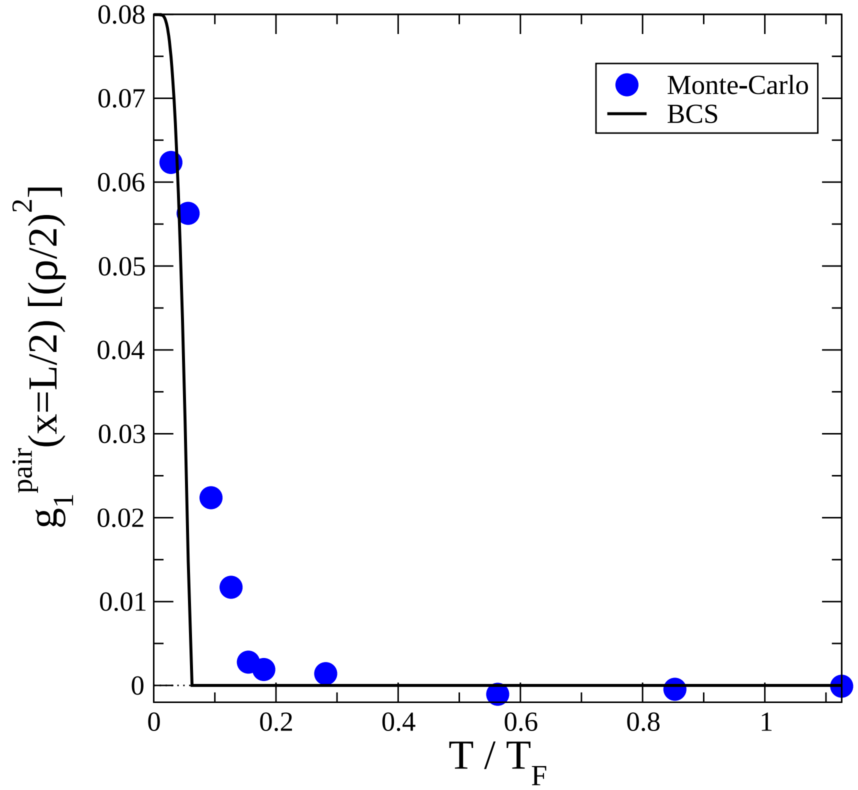


QUANTUM MONTE CARLO IN 1D

Stochastic Hartree-Fock ansatz of Juillet and Chomaz (2002)

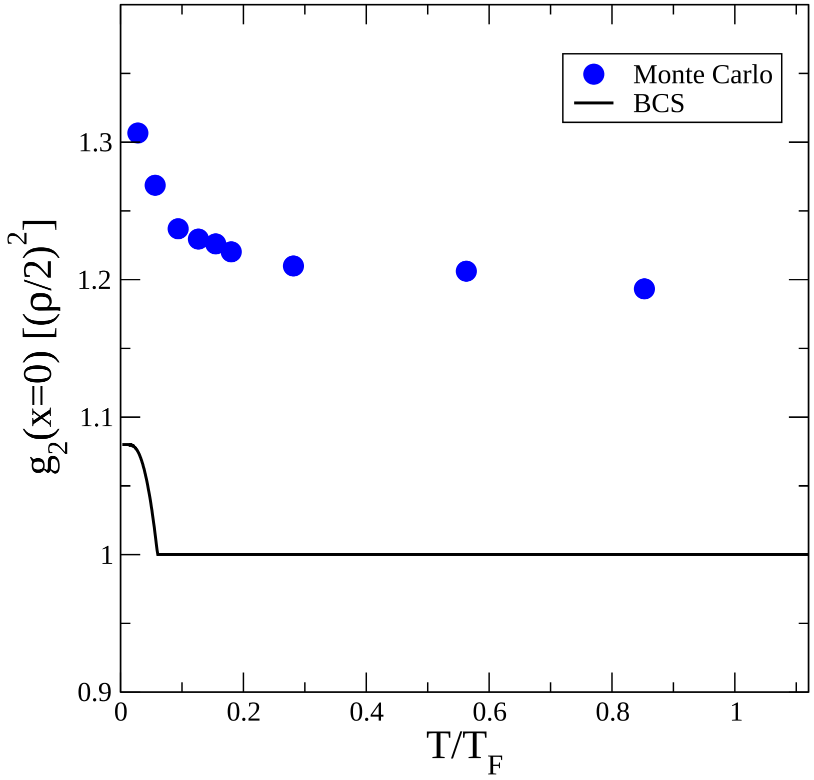
In between weak/strong interactions: $\hbar^2|g|/m\rho \sim 1$





I. Carusotto, Y. Castin, physics/0404025

OPPOSITE SPIN DENSITY-DENSITY CORRELATIONS



- not an indicator of long range order
- not well predicted by BCS theory

HOW TO DETECT THE TRANSITION ? (1)

1. **Pairing:**

- **measure the gap:** Zoller-Törma, expt: Grimm
- **pairing in momentum space:** Lukin

2. **Superfluidity:**

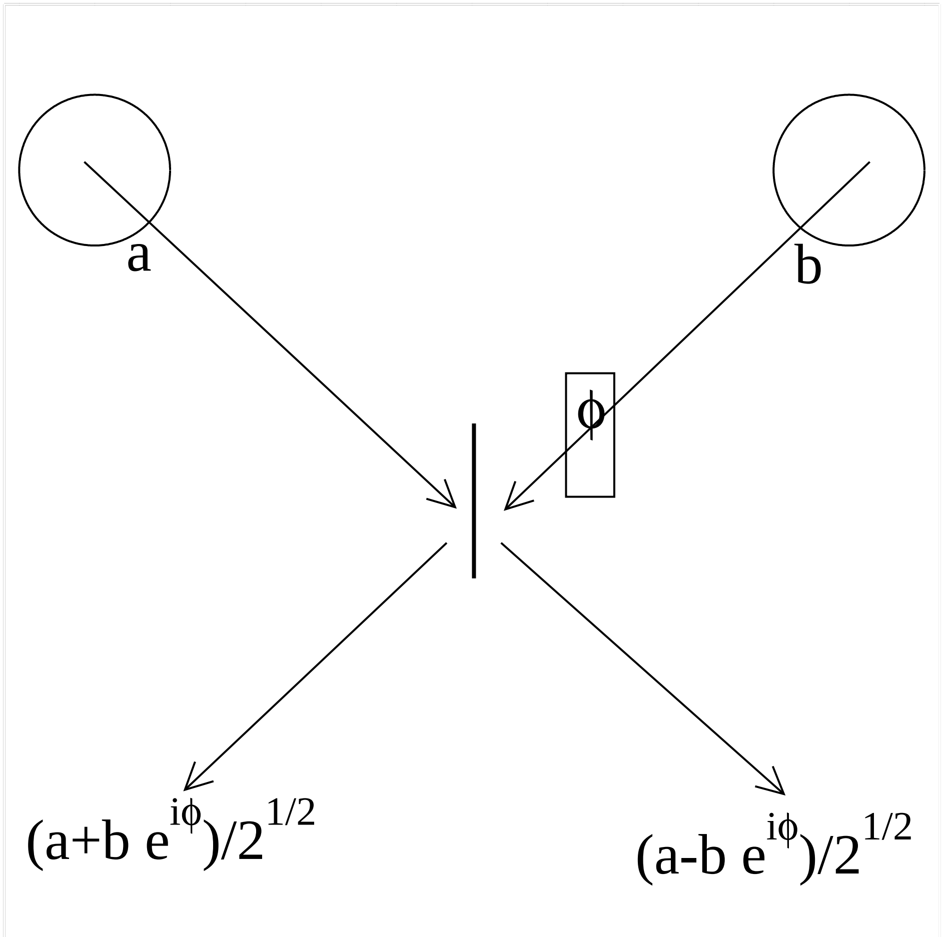
- **anisotropy in ballistic expansion:** Stringari-Menotti, expt: Thomas
- **frequency and damping of collective modes:** Baranov, Bruun, Stringari, expt: Grimm
- **linear response to rotation:** Schuck, Stringari **; quantized vortices**
- **damping of motion of test particle:** Minguzzi-Castin

HOW TO DETECT THE TRANSITION ? (2)

3. Condensation of pairs:

- **BEC limit: as for bosons, expt: Jin, Grimm, Ketterle, Salomon**
- $\boldsymbol{a < 0}$ **side: mapping to BEC side by change of** $\boldsymbol{a}$**, expt: Jin, Ketterle**
- **our proposal: direct interferometric measurement of** g_1^{pair}

INTERFEROMETRIC DETECTION FOR BOSONS (1)



INTERFEROMETRIC DETECTION FOR BOSONS (2)

Operators number of atoms in each output channel:

$$\hat{N}_{\pm} = \left(\frac{\hat{a}^{\dagger} \pm \hat{b}^{\dagger} e^{-i\phi}}{\sqrt{2}}\right)\left(\frac{\hat{a} \pm \hat{b}\, e^{i\phi}}{\sqrt{2}}\right)$$

$$\hat{N}_{\pm} = \frac{1}{2}\left[\hat{a}^{\dagger}\hat{a} + \hat{b}^{\dagger}\hat{b} \pm \left(\hat{a}^{\dagger}\hat{b}\, e^{i\phi} + \text{h.c.}\right)\right]$$

Measure mean atom number difference between the two channels:

$$\langle \hat{N}_{+} - \hat{N}_{-} \rangle = \langle \hat{a}^{\dagger}\hat{b} \rangle e^{i\phi} + \text{c.c.}$$

Y. Castin, J. Dalibard, PRA 55, 4330 (1997).

EXTENSION TO SPIN 1/2 FERMIONS

Now two spin components:

$$\hat{N}_+ - \hat{N}_- \rightarrow \hat{N}_+^{\uparrow} - \hat{N}_-^{\uparrow} \quad \text{and} \quad \hat{N}_+^{\downarrow} - \hat{N}_-^{\downarrow}.$$

$$\hat{N}_+^{\sigma} - \hat{N}_-^{\sigma} = \hat{a}_{\sigma}^{\dagger}\hat{b}_{\sigma}e^{i\phi} + \text{h.c.}$$

Coherence between pairs:

- **no long distance first order coherence:**

$$\langle \hat{a}_{\sigma}^{\dagger}\hat{b}_{\sigma}\rangle \rightarrow 0$$

- **consider correlation function**

$$C = \langle (\hat{N}_+^{\uparrow} - \hat{N}_-^{\uparrow})(\hat{N}_+^{\downarrow} - \hat{N}_-^{\downarrow})\rangle \langle \hat{N}_{\uparrow}\rangle^{-1}$$

$$C = \left[e^{2i\phi}\langle \hat{a}_{\uparrow}^{\dagger}\hat{a}_{\downarrow}^{\dagger}\hat{b}_{\downarrow}\hat{b}_{\uparrow}\rangle + \langle \hat{a}_{\uparrow}^{\dagger}\hat{b}_{\downarrow}^{\dagger}\hat{a}_{\downarrow}\hat{b}_{\uparrow}\rangle + \text{h.c.}\right]\langle \hat{N}_{\uparrow}\rangle^{-1}$$

MORE REALISTIC SET-UP (1)

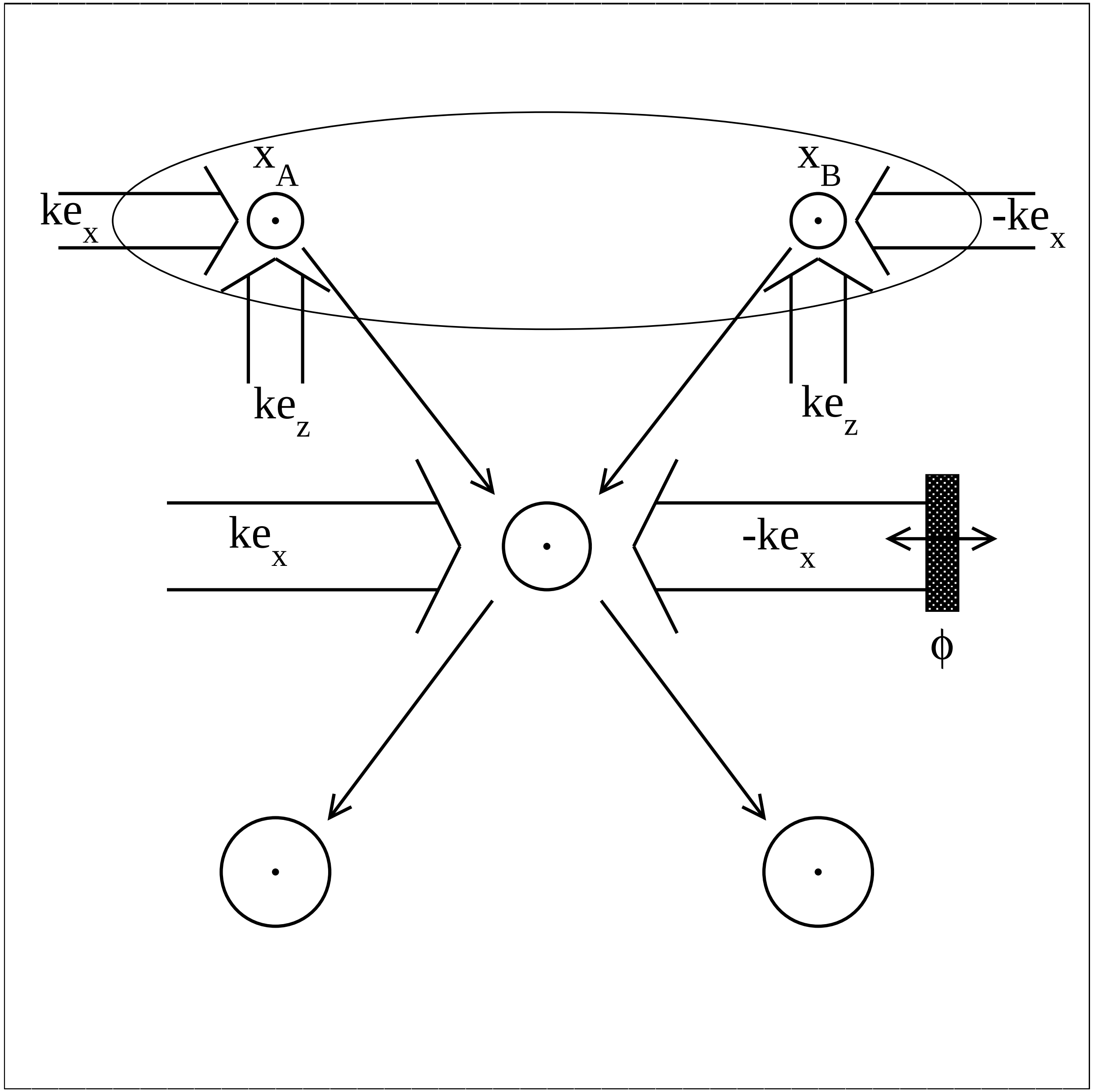
Experimental procedure:

- prepare a trapped Fermi gas at thermal equilibrium
- $t = 0$: switch off trap and set $a = 0$
- $t = 0^+$: Bragg extraction of two wavepackets

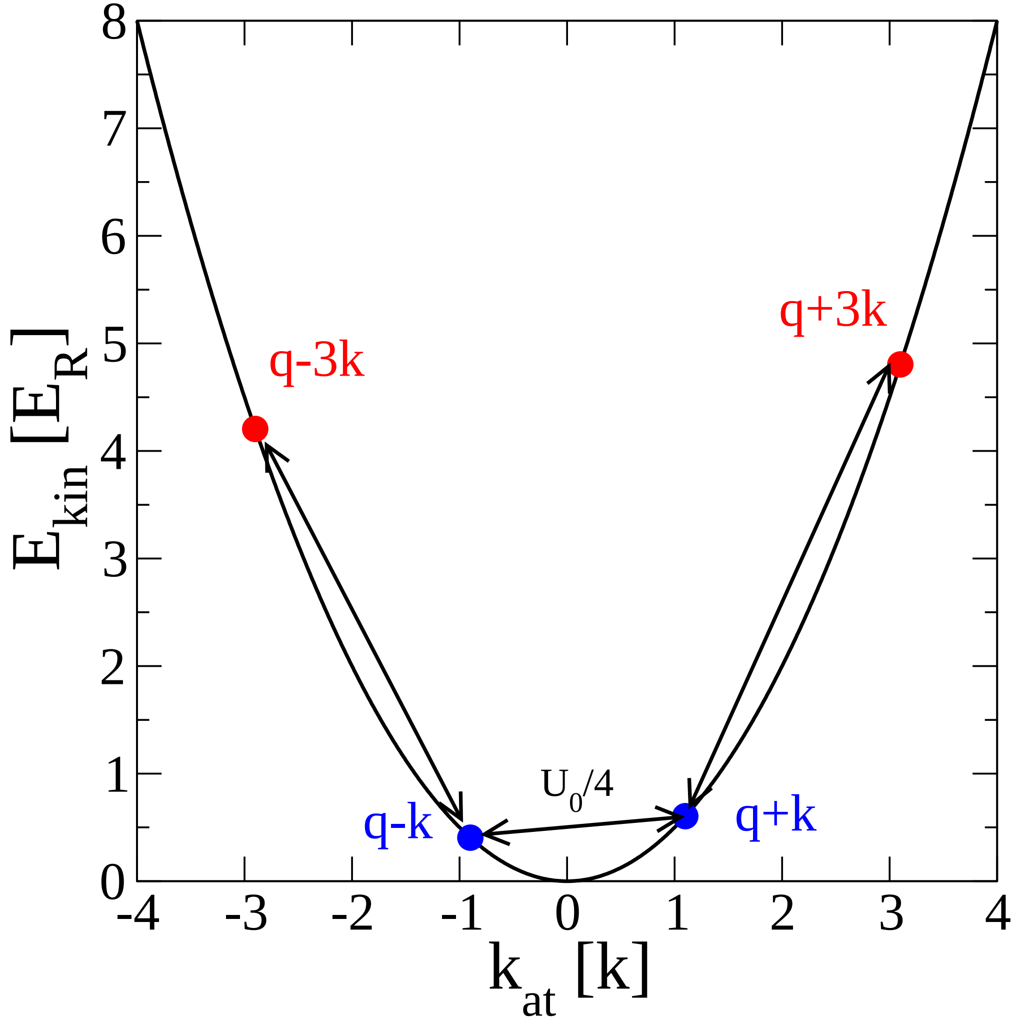
$$\Delta k_{\mathrm{at}} \simeq k_F \ll k$$

- at overlap time: apply matter wave beam splitter
- measure atom numbers in each output channel and spin component

MORE REALISTIC SET-UP (2)



THE MATTER WAVE BEAM SPLITTER



RESULTS OF REALISTIC SET-UP

Correlation function:

$$C = \langle \hat{N}_\sigma \rangle^{-1} \int d\mathbf{r} \int d\mathbf{r}' \, |u(\mathbf{r})|^2 |u(\mathbf{r}')|^2$$

$$\times \Big[-e^{2i\phi} \langle \hat{\psi}^\dagger_\uparrow(\mathbf{r}_A + \mathbf{r}) \hat{\psi}^\dagger_\downarrow(\mathbf{r}_A + \mathbf{r}') \hat{\psi}_\downarrow(\mathbf{r}_B + \mathbf{r}') \hat{\psi}_\uparrow(\mathbf{r}_B + \mathbf{r}) \rangle$$

$$+ \langle \hat{\psi}^\dagger_\uparrow(\mathbf{r}_A + \mathbf{r}) \hat{\psi}^\dagger_\downarrow(\mathbf{r}_B + \mathbf{r}') \hat{\psi}_\downarrow(\mathbf{r}_A + \mathbf{r}') \hat{\psi}_\uparrow(\mathbf{r}_B + \mathbf{r}) \rangle + \text{h.c.} \Big]$$

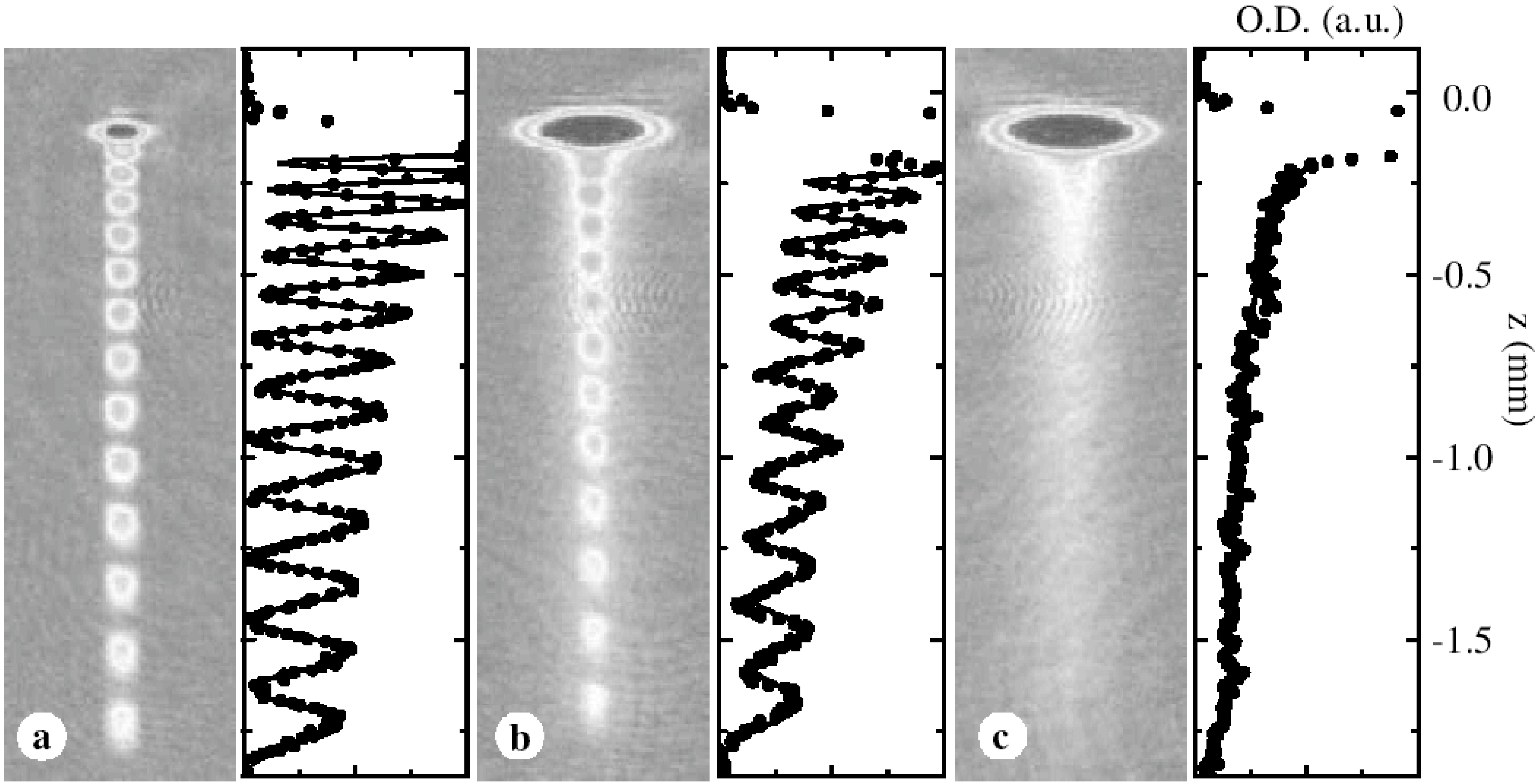
where the Bragg extraction functions are $u(\mathbf{r} - \mathbf{r}_{A,B})$.

In the BCS state for a large extraction region:

$$C = \frac{3\pi}{8\sqrt{2}} \cos 2\phi \, |u|^2_{\max} \frac{\Delta}{E_F} \quad \text{in BCS limit}$$

$$C = \frac{1}{\sqrt{2}} \cos 2\phi \, |u|^2_{\max} \quad \text{in BEC limit}$$

BEC: NO MATTER WAVE BEAM SPLITTER REQUIRED



T. Esslinger, I. Bloch, T. Hänsch, Jour. of Modern Optics 47, 2725 (2000)

ALTERNATIVE SET-UP

What was done for bosons:

- measure the mean density profile in the overlap region

Extension to paired fermions:

- opposite spin density-density correlation function:

$$g_2(\mathrm{r},\mathrm{r}';t_{\mathrm{ov}}) = \langle \hat{\psi}^\dagger_\uparrow(\mathrm{r},t_{\mathrm{ov}})\hat{\psi}^\dagger_\downarrow(\mathrm{r}',t_{\mathrm{ov}})\hat{\psi}_\downarrow(\mathrm{r}',t_{\mathrm{ov}})\hat{\psi}_\uparrow(\mathrm{r},t_{\mathrm{ov}})\rangle$$

- In BCS theory:

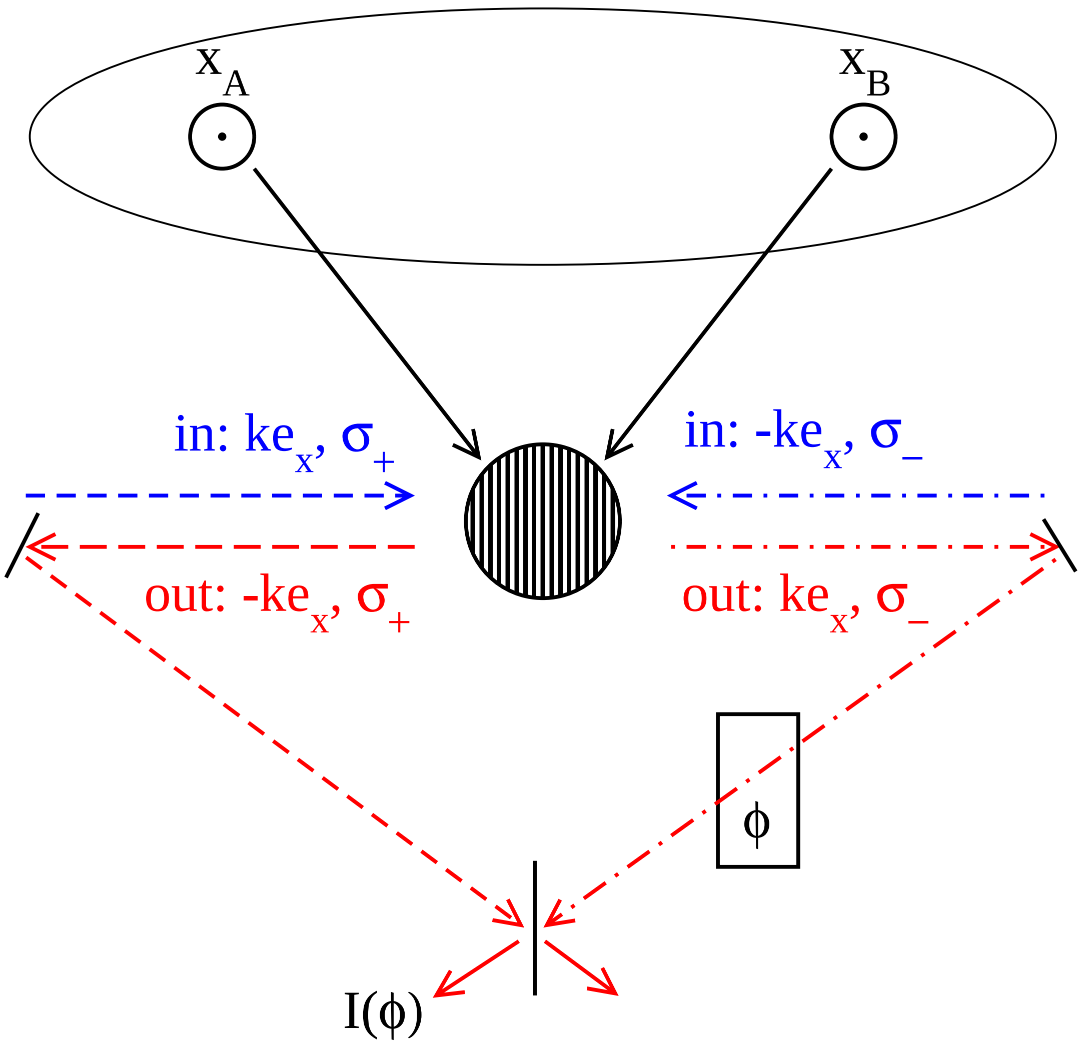
$$g_2(\mathrm{r},\mathrm{r}';t_{\mathrm{ov}}) = |u(\mathrm{r}-\mathrm{r}_{\mathrm{ov}})|^2|u(\mathrm{r}'-\mathrm{r}_{\mathrm{ov}})|^2 \times \left\{\rho^2 + \cos^2[k(x+x'-2x_{\mathrm{ov}})]|\mathcal{A}(\mathrm{r}'-\mathrm{r})|^2\right\}$$

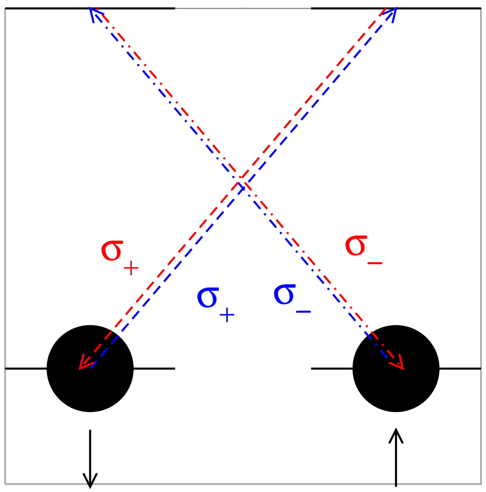
with anomalous average

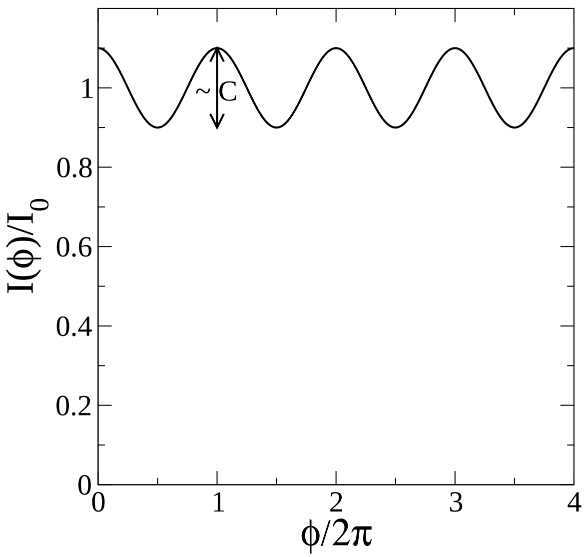
$$\mathcal{A}(\mathrm{u}) = \langle \hat{\psi}_\downarrow(\mathrm{r}_{\mathrm{ov}})\hat{\psi}_\uparrow(\mathrm{r}_{\mathrm{ov}}+\mathrm{u})\rangle_{\mathrm{bcs}}$$

$\rightarrow$ fringes on centroid position with wave vector $2k$.

LIGHT SCATTERING OFF THE MATTER WAVE GRATING







BALLISTIC EXPANSION

A very common experimental procedure:

- prepare trapped gas in steady state
- switch off trapping potential, let gas expand
- laser beam absorption imaging gives integrated density:

$$\mathrm{signal}(x,y) \propto \int dz\, \rho(x,y,z;t)$$

- used as a 'magnifying lens': e.g. to reveal a vortex lattice in a BEC (J. Dalibard)
- Truly a 'magnifying lens' if there is a scaling relation

$$\rho(x,y,z;t) = \frac{1}{\prod_\alpha \lambda_\alpha(t)} \rho_0 \left[\frac{x}{\lambda_1(t)}, \frac{y}{\lambda_2(t)}, \frac{z}{\lambda_3(t)} \right].$$

BRIEF HISTORY OF SCALING SOLUTIONS

- For an ideal gas in a harmonic potential
- For the Boltzmann equation in a harmonic isotropic potential: Boltzmann
- For the Gross-Pitaevskii equation in a harmonic trap:
  - in Thomas-Fermi regime: G. Shlyapnikov, E. Surkov, Yu. Kagan (1996), R. Dum, Y. Castin (1996)
  - in Thomas-Fermi regime for rotating traps: M. Olshanii, P. Storey (2000), Y. Castin, S. Sinha (2001)
  - in 2D in isotropic trap: G. Shlyapnikov, E. Surkov, Yu. Kagan (1996)
- For superfluid hydrodynamics in a harmonic trap with equation of state $\mu \propto \rho^\gamma$: Stringari, Menotti (2002)

- **for N-body Schrödinger equation of 1D gas of impenetrable bosons in harmonic trap (formally equivalent to an ideal Fermi gas): Girardeau**
- **For N-body Schrödinger equation in 2D, isotropic harmonic trap, $1/r_{12}^2$ or $\delta(\vec{r_{12}})$ interaction potential: Pitaevskii, Rosch (1997).**
- **BUT required regularization of δ breaks scaling invariance: Olshanii, Pricoupenko (2002) so Pitaevskii's result applies only to states with no particles at same point**

$$\psi(\ldots, \vec{r_i} = \vec{r_j}, \ldots) = 0 \quad \forall i \neq j$$

like Laughlin state.

SCALING SOLUTION FOR THE 3D UNITARY QUANTUM GAS

The problem in an isotropic trap:

- Free Schrödinger equation over domain $r_{ij} \neq 0$:

$$i\hbar\partial_t\psi = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_{\vec{r_i}} + \frac{1}{2}m\omega^2(t)r_i^2\right]\psi$$

- plus contact conditions:

$$\psi(\vec{r}_1,\ldots,\vec{r_N}) = \frac{A(\vec{R}_{ij},\{\vec{r_k},k\neq i,j\})}{r_{ij}} + o(1).$$

- Initially, stationary state in static trap $\omega = \omega_0$ with energy E.

Ansatz: gauge plus scaling transform:

$$\psi(\vec{r}_1,\ldots,\vec{r_N}) = \frac{e^{-i\theta(t)}}{\lambda^{3N/2}(t)} \exp\left[\frac{im\dot{\lambda}}{2\hbar\lambda}\sum r_j^2\right] \psi_0(\vec{r_1}/\lambda,\ldots,\vec{r_N}/\lambda).$$

- **scaling preserves contact conditions**
- **gauge transform preserves contact conditions:**

$$r_i^2 + r_j^2 = 2R_{ij}^2 + \frac{1}{2}r_{ij}^2.$$

- **solves free Schrödinger equation if**

$$\ddot{\lambda} = \frac{\omega_0^2}{\lambda^3} - \omega^2(t)\lambda$$

$$\theta(t) = E\int_0^t \frac{d\tau}{\hbar\lambda^2(\tau)}.$$

Y.Castin, Comptes Rendus Physique 5, 407 (2004).

CONSEQUENCES OF SCALING SOLUTION

- **Linear response: undamped mode of frequency $2\omega_0$**
- **Existence of lowering/raising operator:**

$$L_{\pm} = -\frac{3N}{2} \mp \frac{E}{\hbar\omega_0} - \sum_{j=1}^{N} \vec{r_j} \cdot \partial_{\vec{r_j}} \pm \frac{m\omega_0}{\hbar} \sum_{j=1}^{N} r_j^2.$$

$L_-|\psi_0\rangle$ **vanishes or has eigenenergy** $E - 2\hbar\omega_0$.

- **Virial theorem, F. Chevy:**

$$E = 2E_{\mathrm{harm}} > 0$$

$\rightarrow$ **spectrum semi-bounded, stability**

NB. For isotropic trap hydrodynamic prediction gives same scaling as exact solution.

CONCLUSION

- Long range order in a spin 1/2 Fermi gas
  - Quantum Monte Carlo results in 1D
- Interferometric detection of the long range order
  - set up 1: with a matter wave beam splitter
  - set up 2: without a matter wave beam splitter
- The unitary quantum gas
  - an exact time dependent solution