The Dynamic Structure Factor of the 1D Bose Gas near the Tonks-Girardeau Limit

A. Yu. Cherny_

Joint Institute for Nuclear Research, Dubna

with Joachim Brand

Max-Planck-Institut für Physik komplexer Systeme, Dresden

Overview

- Lieb-Liniger and Tonks-Girardeau gas and the Landau criterion of superfluidity
- A pseudopotential Hamiltonian
  Fermionization of 1D Bosons – Bosonization of Fermions
- Hartree-Fock theory
- Dynamic Structure factor in RPA

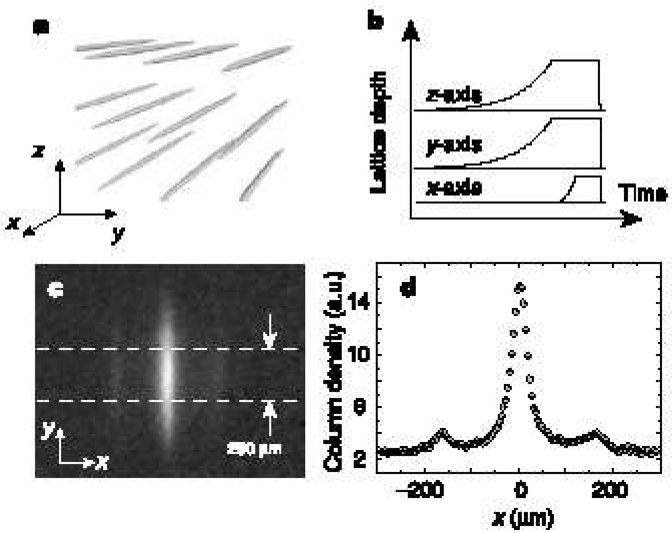
Tonks-gas – Experiments

letters to nature

Tonks–Girardeau gas of ultracold atoms in an optical lattice

Belén Paredes[1], Artur Widera[1,2,3], Valentin Murg[1], Olaf Mandel[1,2,3], Simon Fölling[1,2,3], Ignacio Cirac[1], Gora V. Shlyapnikov[4], Theodor W. Hänsch[1,2] & Immanuel Bloch[1,2,3]

MPQ Garching



up to $\gamma_{eff} \approx 200$

other experiments:

T. Esslinger (Zürich)

W. Phillips (NIST)

D. Weiss (PSU), $\gamma \approx 5.5$

$$\gamma \approx \frac{\text{interaction energy}}{\text{kinetic energy}}$$

$$\gamma \simeq \frac{m}{\hbar} \frac{\omega_\rho}{n_{1D}} a_{3D}$$

1D Bose Gas – Lieb-Liniger model

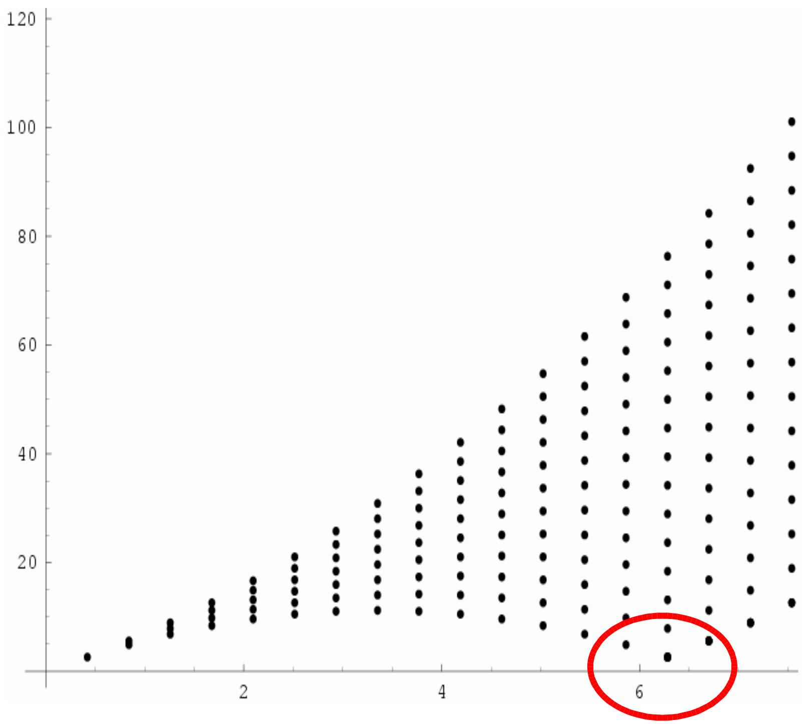
$$H = \sum_i \left[\frac{p_i^2}{2m} + V_{\text{ext}}(x_i) \right] + \frac{\hbar^2}{m} c \sum_{i<j} \delta(x_i - x_j)$$

- 1D Bosons with repulsive δ interactions
- Ground- and excited-state wavefunctions of homogeneous system (V_{ext}=0) are exactly known from Bethe ansatz [Lieb, Liniger 1963]
- Interaction parameter $\gamma = c/n_{1D}$
- Quasicondensate, GP+Bogoliubov for $\gamma \ll 1$
- For $\gamma \to \infty$, problem is mapped exactly to free Fermi gas (Tonks-Girardeau gas) [Girardeau 1960]

Excitation spectrum for the Lieb-Liniger model

$\gamma=\infty$

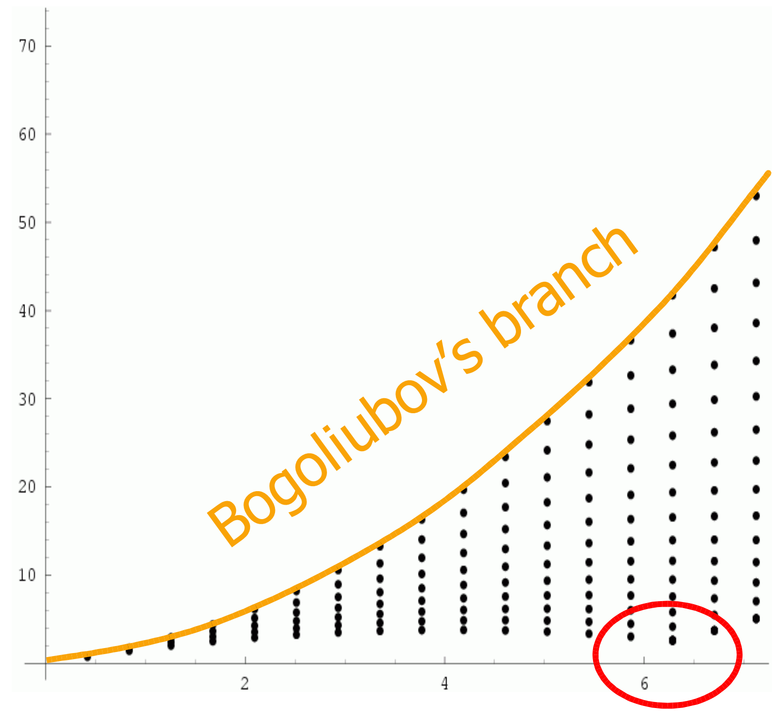
$\hbar\omega\pi^2/\varepsilon_F$



momentum $q\pi/k_F$

$\gamma=1$

$\hbar\omega\pi^2/\varepsilon_F$



umklapp excitation

momentum $q\pi/k_F$

$k_F=\pi n_{1D}$; $\varepsilon_F=\hbar^2 k_F^2/(2m)$

Elementary excitations – Landau superfluidity?

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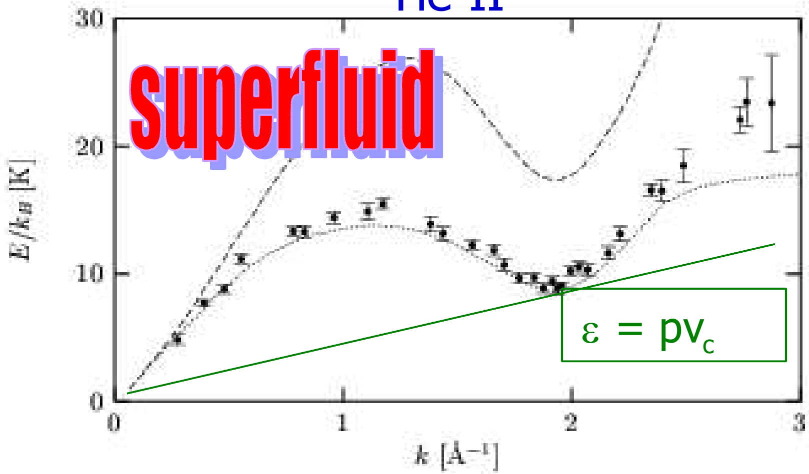
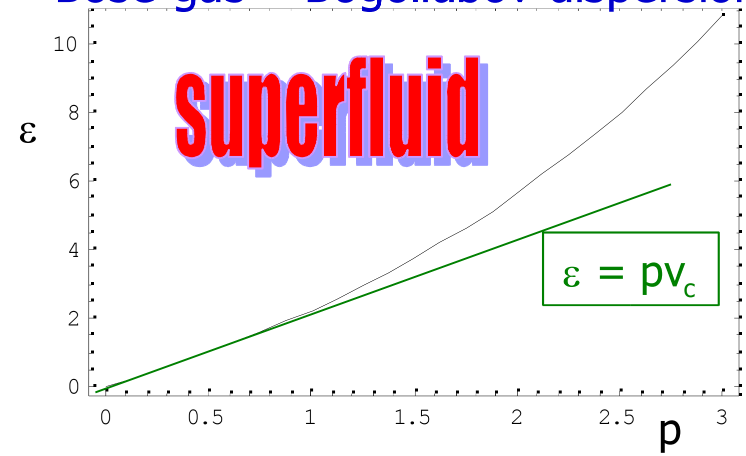
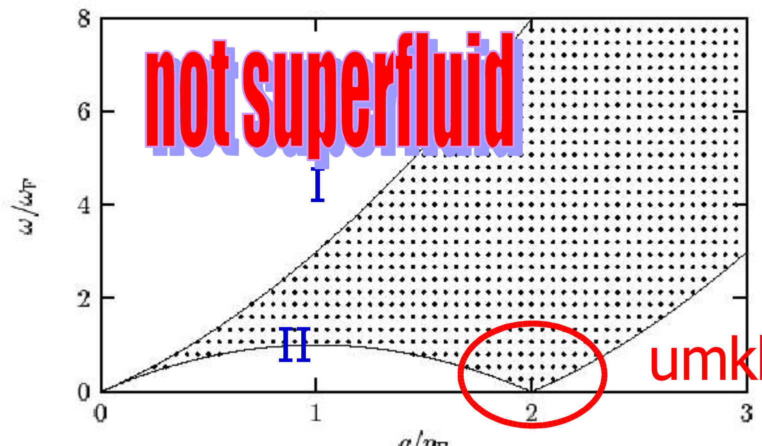


Bose gas – Bogoliubov dispersion



Free Fermi gas – Lieb Liniger



umklapp excitation

$v_c = 0$?

Landau criterion:

Are elementary excitations from roughness of the wall energetically favourable?

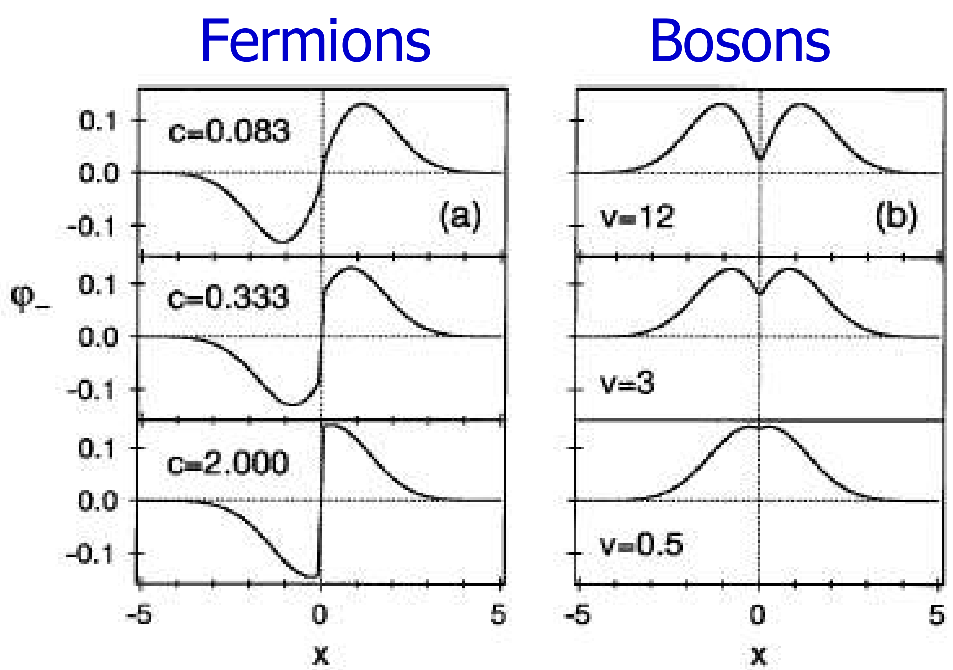
Bose-Fermi mapping

"In 1D, there is no distinction between Bosons and Fermions"

Strong repulsive interactions for bosons have the same effect as the Pauli exlusion principle for fermions.

$$\phi^B = |\phi^F|$$

Bosons with strong but finite interactions map to spinless fermions with weak short-range interactions



Cheon and Shigehara PRL 1999

Pseudopotential in the Fermionic picture

Pseudopotential generates energy levels to first order in $1/\gamma$

$$V(x_1, x_2) = -\frac{2\hbar^2}{mc}\delta''(x_1 - x_2)$$ [D. Sen 1999]

generalization for arbitrary γ:

$$V(x_1, x_2, x_2', x_1') = -\frac{4\hbar^2}{mc}\delta\left(\frac{x_1 + x_2 - x_2' - x_1'}{2}\right)\delta'(x_1 - x_2)\delta'(x_1' - x_2')$$

compare with
Granger and Blume [2004]
and Girardeau and Olshanii [2004]

Hartree-Fock with pseudopotential

Hartree-Fock operator becomes local!

$$\hat{T} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + V_{\text{ext}}(y) + \frac{2\hbar^2}{mc}\rho(y)\frac{\partial^2}{\partial y^2} + \frac{4\hbar^2}{mc}\mathcal{P}(y) i\frac{\partial}{\partial y} - \frac{2\hbar^2}{mc}\Gamma(y)$$

Momentum and kinetic energy densities appear:

$$\begin{aligned} \mathcal{P}(y) &= -i\sum_j n_j {\varphi^*}'_j(y)\varphi_j(y) \\ \Gamma(y) &= \sum_j n_j\left[\varphi_j^*(y)\varphi_j''(y) + 2{\varphi^*}'_j(y)\varphi_j'(y)\right] \end{aligned}$$

For $V_{\text{ext}}(y)=0$, the quasiparticle energy dispersion

$$\varepsilon_q = \frac{\hbar^2 q^2}{2m^*} - \frac{2\hbar^2}{mc}\frac{\pi^2}{3}n^3$$

with renormalized mass $m^* = \dfrac{m}{1-4\gamma^{-1}}.$

The Hartree-Fock ground state energy is correct up to first order in γ^{-1}

Brand and Cherny [cond-mat/0410311]

Elementary excitations: Random-Phase approximation (RPA)

- RPA is the linearized time-dependent Hartree-Fock method:
  calculate linear response of density

$$\chi(q,\omega) = \frac{\delta\rho(q,\omega)}{\delta V_{\text{ext}}(q,\omega)}$$

- can be applied for homogeneous (analytical) and inhomogeneous (numerical) problems
- Dynamic structure factor (DSF) contains information about excitation probabilities

$$S(q,\omega) = \sum_n |\langle 0|\hat{\rho}_q|n\rangle|^2 \delta(\hbar\omega - E_n + E_0)$$

$$= -\frac{\text{Im}\chi(q,-\omega-i\epsilon)}{\pi} \quad (\omega > 0)$$

The Polarizability of the Tonk's gas

$$\chi_1^{(0)}(q,\omega) = \frac{Nm^*}{2\hbar^2 q k_\mathsf{F}} \ln\left|\frac{\omega^2-\omega_-^2(q)}{\omega_+^2(q)-\omega^2}\right|,$$

$$\chi_2^{(0)}(q,\omega) = -\frac{N\pi m^*}{2\hbar^2 q k_\mathsf{F}} \begin{cases} \pm 1, & \omega_- \le \pm\omega \le \omega_+, \\ 0, & \text{else}, \end{cases}$$

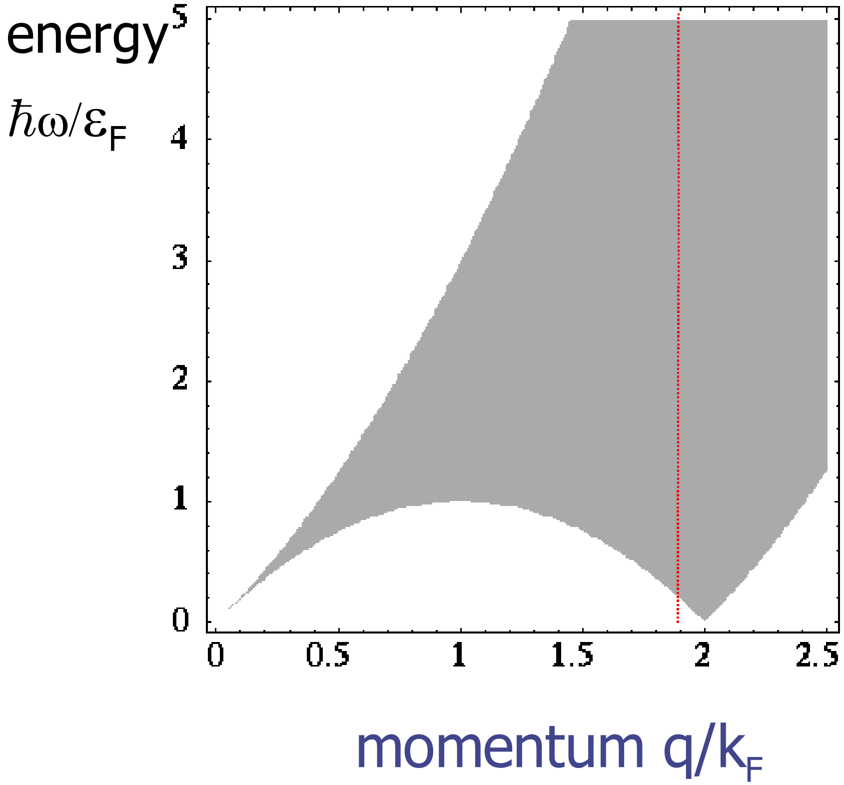
The DSF of the Tonk's gas

$$S(q,\omega) = \frac{Nm^*}{2\hbar^2 q k_\mathsf{F}} \begin{cases} 1, & \omega_- \le \omega \le \omega_+, \\ 0, & \text{else}, \end{cases}$$

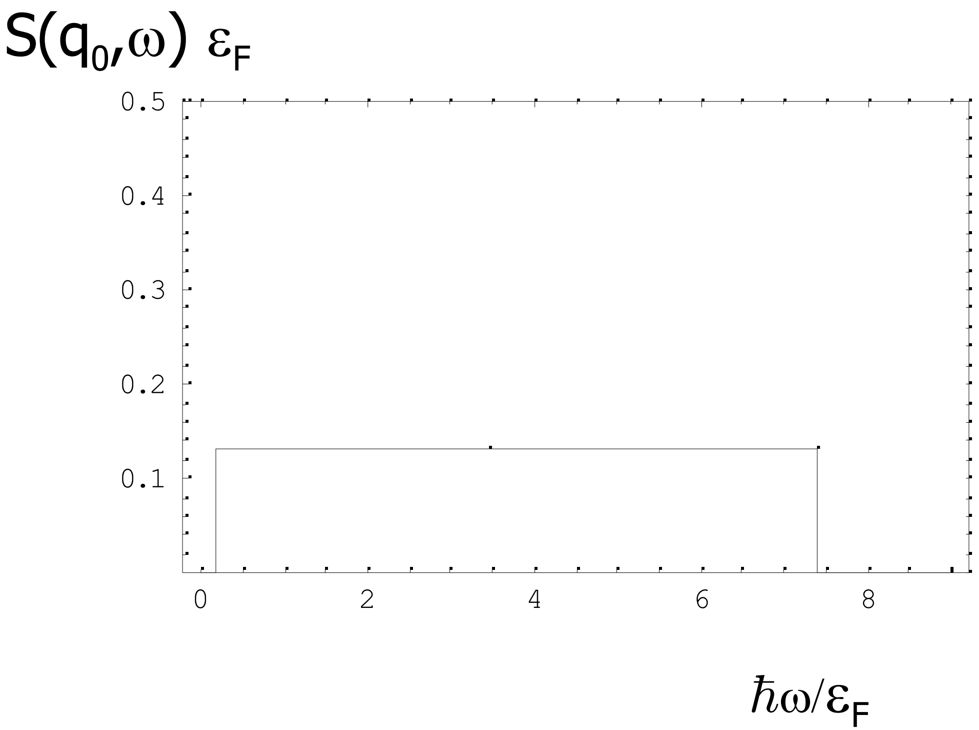
$$\omega_\pm(q) = \frac{\hbar|2k_F q \pm q^2|}{2m^*}; \quad k_F = \pi n_{1D}$$

mpipks

Dynamic Structure Factor for the Tonk's gas



Energy dependence at q_0=1.9



Dynamic Structure Factor in RPA

$$\chi(q,\omega+i\varepsilon)=\frac{\chi^{(0)}(q,\omega+i\varepsilon)}{(1-4\gamma^{-1})[B-D\chi^{(0)}(q,\omega+i\varepsilon)]}$$

$$S(q,\omega)=\frac{-\chi_2^{(0)}(q,\omega)B}{\pi(1-4\,\gamma^{-1})\left[\left(B-D\chi_1^{(0)}\right)^2+\left(D\chi_2^{(0)}\right)^2\right]}$$

$$+\delta[\omega-\omega_0(q)]A(q)$$

$$B=1-4\,(3\,\gamma-16)/(\gamma-4)^3$$

$$D=\frac{4\varepsilon_F}{N}\frac{\gamma}{(\gamma-4)^2}\left\{\frac{q^2}{k_F^2}\frac{2\gamma-9}{2\gamma}-\frac{2}{\gamma}-\left[\frac{\hbar(\omega+i\varepsilon)\,k_F}{\varepsilon_F q}\right]^2\frac{3\gamma-16}{2(\gamma-4)^2}\right\}$$

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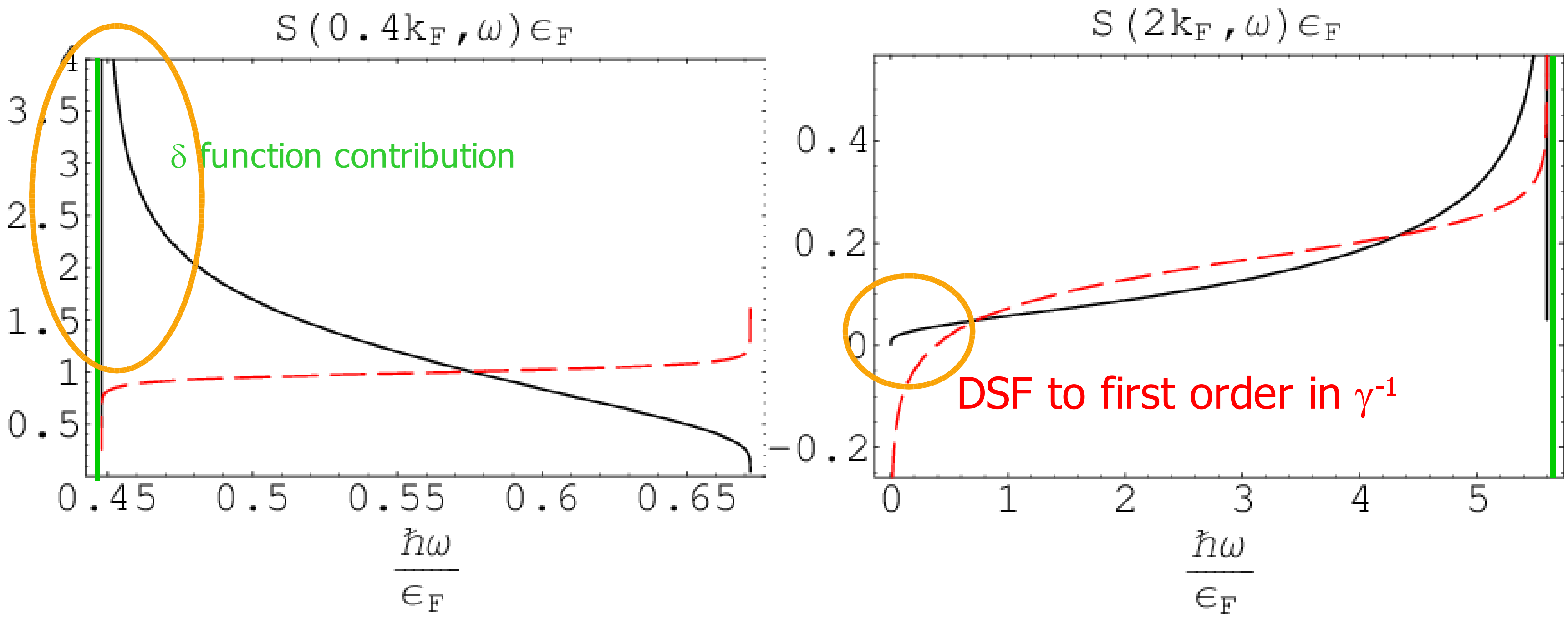
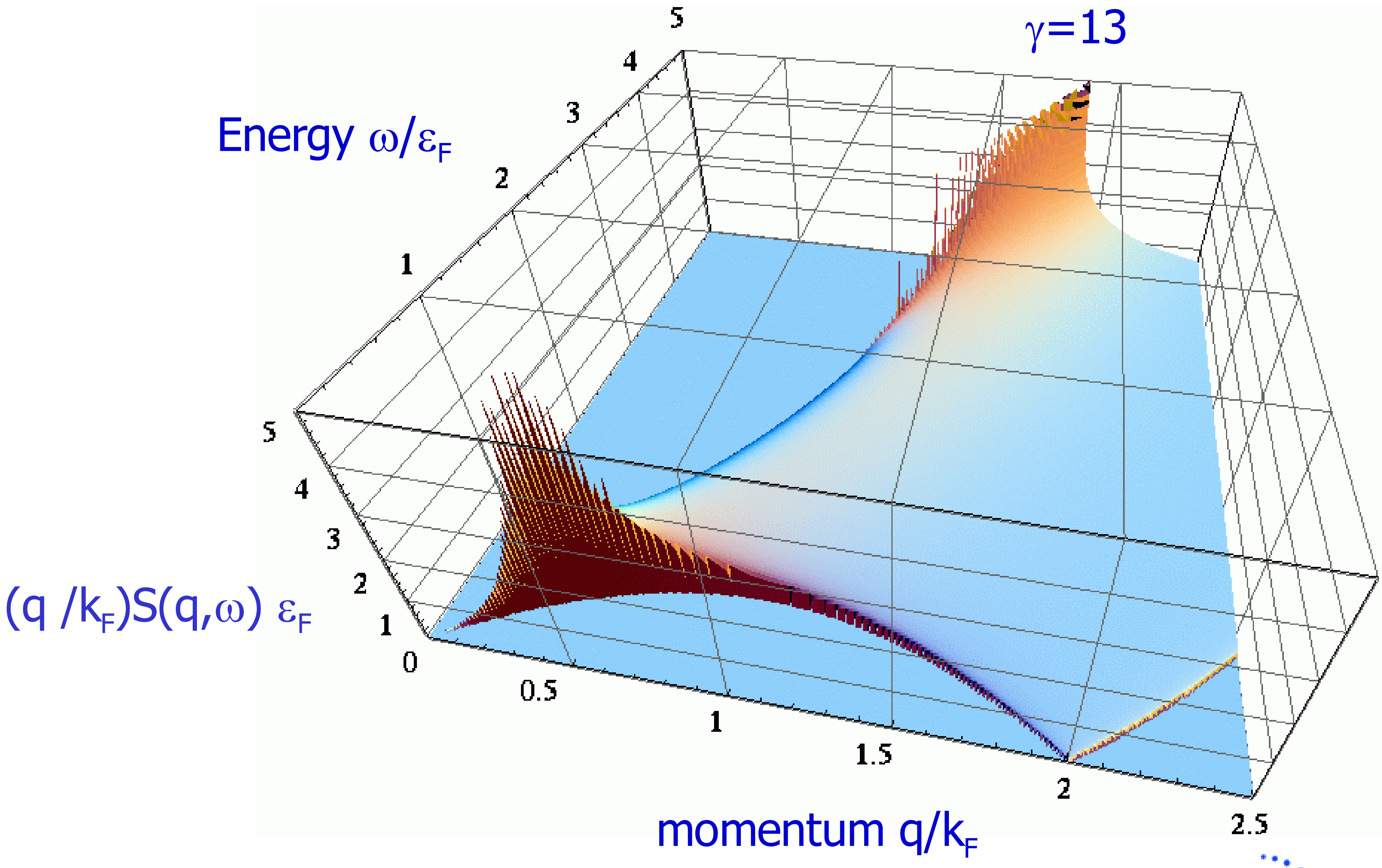
Dynamic Structure Factor in RPA

Energy dependence at q=const, γ=13



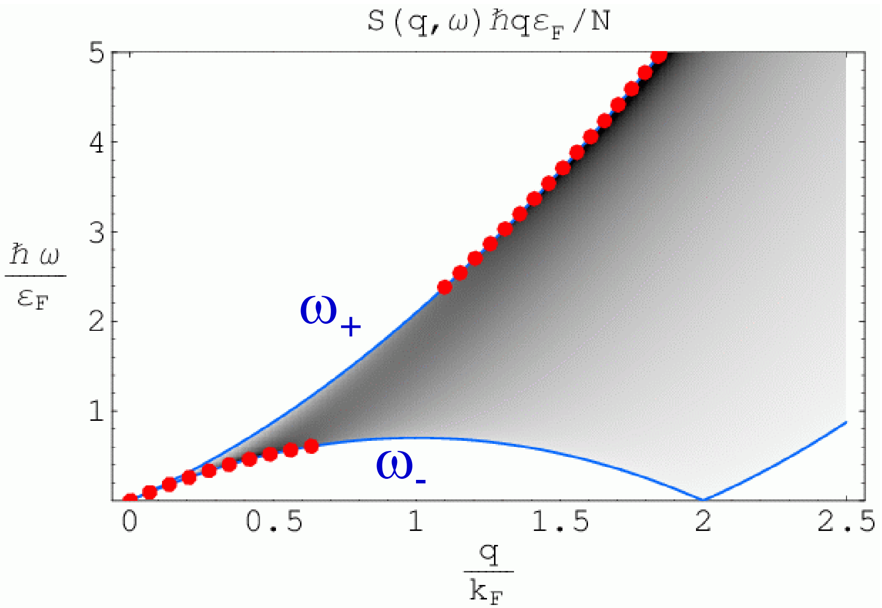
- Significant suppression of umklapp excitation is predicted to occur at $\gamma \gg 1$ where RPA should be applicable
- Peak of DSF near ω_- branch for small momenta

Dynamic Structure Factor in RPA



Dynamic Structure Factor in RPA

γ=13



Red dots - the δ–branch ω_0

what is physical meaning?

$$q = \text{const}, \quad \gamma \to +\infty$$

$$A(q) \simeq 2N\gamma\,\text{exp}(-\gamma q/k_F)$$

$$|\omega_0 - \omega_\pm| \propto \text{exp}(-\gamma q/k_F)$$

$$\gamma = \text{const}, \quad q \to +\infty$$

$$A(q) \simeq N$$

$$\hbar\omega_0(q) \simeq \frac{\hbar^2 q^2}{2m}$$

First-order expansion of DSF

$$S(q) = \hbar \int_0^{+\infty} d\omega \, S(q,\omega)/N$$

$$g(x) = 1 + n^{-1} \int \frac{dx}{2\pi} [S(q) - 1] e^{iqx}$$

$$\varepsilon_F S(q,\omega) = \frac{k_F}{4q} + \frac{2k_F}{\gamma q} + \frac{1}{2\gamma} \ln f(q,\omega) + \mathcal{O}(\gamma^{-2})$$

$$f(q,\omega) = |(\omega^2 - \omega_-^2)/(\omega_+^2 - \omega^2)|$$

$$g(x=0) = \mathcal{O}(\gamma^{-2})$$

$$\omega_\pm(q) = \frac{\hbar |2k_F q \pm q^2|}{2m^*}; \quad k_F = \pi n_{1\mathrm{D}}; \quad \varepsilon_F = q = \frac{k_F^2}{2m}$$

Bragg scattering

The dynamic structure factor could be measured by Bragg spectroscopy. Similar experiments on 3D BECs have been done.

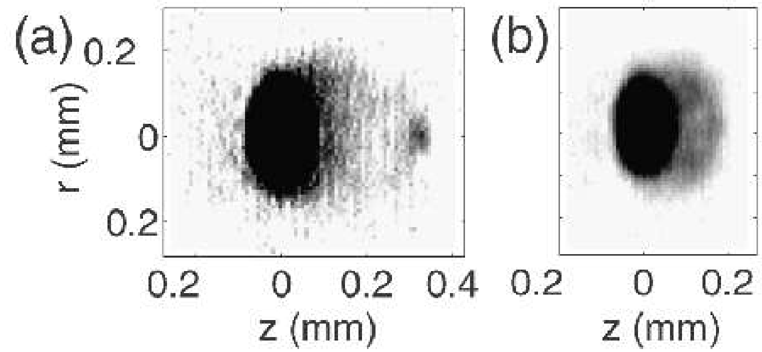


FIG. 2. Absorption TOF images of excited Bose-Einstein condensates. (a) Absorption image for $k = 2.63$, with the large cloud at the origin corresponding to the unperturbed BEC. A clear halo of scattered atoms is visible between the BEC and the cloud of unscattered outcoupled excitations. (b) Absorption image for $k = 1.06$. For this value of k the distinction between scattered and unscattered excitations is not clear, since both types of excitations occupy the same region in space.

From Katz et al. **89** 220401 (2002)

Summary

- RPA scheme has been applied to the pseudopotential, obtained expressions are valid at least up to first order in $1/\gamma$
- Prediction of enhancement of ω_+ and ω_- branches can be tested by experiment
- The response function and DSF in RPA provide information not previously accessible from exact solutions
- Landau superfluidity emerges by suppression of umklapp excitations
- Extensions to finite temperature and inhomogeneous systems are obvious

Brand and Cherny [cond-mat/0410311]