Bose-Einstein condensates in fast rotation

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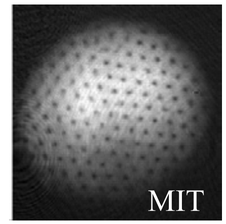
Th: Amandine Aftalion, Xavier Blanc

+ many discussions with Yvan Castin,
Sandro Stringari, Gora Shlyapnikov

Rotating condensates

Rotation at angular frequency Ω

Boulder: evaporative spin-up ; ENS, MIT, Oxford: stirring



Abrikosov lattice of vortices

$$\oint \vec{v}\,.d\vec{r} = \frac{nh}{M}$$ *Feynman, Onsager*

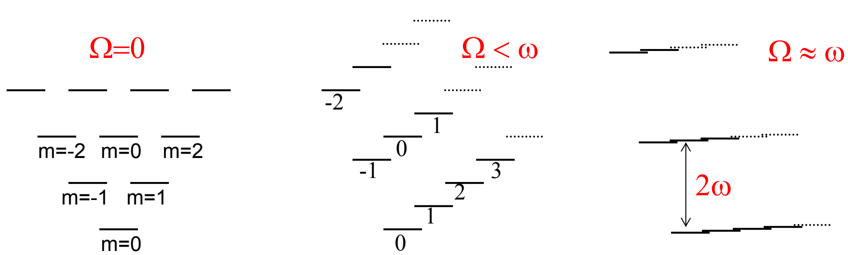
Rigid body rotation $\vec{v} = \vec{\Omega} \times \vec{r}$ for the coarse-grain average of the velocity field if the vortex surface density is:

$$\rho_v = \frac{M\Omega}{\pi\hbar}$$

Landau levels for a rotating gas

Isotropic harmonic trapping in the *xy* plane with frequency ω

Hamiltonian in the rotating frame: $H - \Omega L_z$



When Ω~ω, one approaches a situation with a macroscopically degenerate ground state for the one-body hamiltonian

Same physics as for a charged particle in a magnetic field

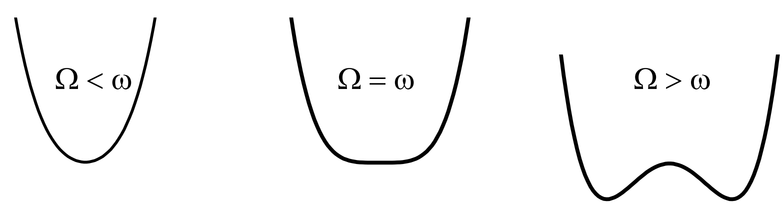
An interesting variant: the r^2+r^4 potential

Harmonic potential superimposed with a small quartic term
(formed using an additional laser beam propagating along the axis of rotation):

$$V(r) = \frac{1}{2} m\omega^2 r^2 + \frac{\gamma}{4} r^4 \quad \text{in the } xy \text{ plane} \quad \left(r^2 = x^2 + y^2\right)$$

Stirring frequency Ω of the gas: $\Omega \sim \omega$

Trapping + centrifugal potential: $\frac{1}{2} m\left(\omega^2 - \Omega^2\right) r^2 + \frac{\gamma}{4} r^4$



Coriolis vs. centrifugal forces

$$\vec{F}_{\text{Coriolis}} = 2M\vec{\Omega}\times\vec{v} \qquad \vec{F}_{\text{centrif.}} = M\Omega^2\vec{r}_\perp$$

$$H = \frac{p^2}{2M} + V(\vec{r}) - \Omega L_z = \frac{(\vec{p}-\vec{A})^2}{2M} + V(\vec{r}) - \frac{1}{2}M\Omega^2 r_\perp^2$$

$$\vec{A} = M\vec{\Omega}\times\vec{r}$$

Is the Coriolis force significant for fast rotating gases?

1. Condensation temperature — NO
2. Equilibrium shape of the condensate — IT DEPENDS
3. Vibration modes of the condensate — YES

1.

Condensation temperature of an ideal rotating gas

two (little) miracles…

Condensation threshold in the semi-classical approximation

$$N = \frac{1}{h^3}\int \frac{d^3r\ d^3p}{\exp(E(\vec{r},\vec{p})/kT)-1} \qquad E(\vec{r},\vec{p}) = \frac{(\vec{p}-\vec{A})^2}{2M} + V(\vec{r}) - \frac{1}{2}M\Omega^2 r_\perp^2$$

Change of variables:

$$p_x' = p_x + M\Omega y$$
$$p_y' = p_y - M\Omega x$$

(Landau-Lifshitz)

$$N = \int \frac{d^3r\ d^3p'}{\exp(\tilde{E}(\vec{r},\vec{p}\,')/kT)-1} \qquad E(\vec{r},\vec{p}\,') = \frac{p'^2}{2M} + V(\vec{r}) - \frac{1}{2}M\Omega^2 r_\perp^2$$

No effect of Coriolis force

Analogous to Bohr- van Leeuven theorem: no classical magnetism

Semi-classical approximation (2)

Purely harmonic potential (Stringari 1999):

$$\omega^2 \longrightarrow \omega^2 - \Omega^2 \qquad N = 1.212\frac{(kT/\hbar)^3}{\left(\omega^2-\Omega^2\right)\omega_z}$$

experimental check: Boulder

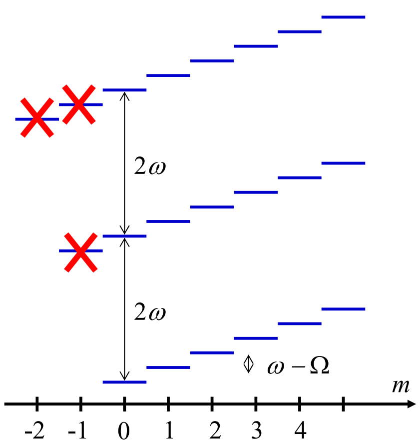
Harmonic + quartic potential exactly at $\Omega = \omega$:

$$V_\perp(r) = \frac{1}{2}M\omega^2 r_\perp^2 + \frac{\gamma}{4}r_\perp^4 - \frac{1}{2}M\Omega^2 r_\perp^2 \longrightarrow \frac{\gamma}{4}r_\perp^4$$

$$N = 1.19\frac{M(kT)^{5/2}}{\hbar^3\sqrt{\gamma}\ \omega_z}$$

Are these results compatible with a quantum description?

Calculation of T_c with Landau levels (harmonic case)



$$N = \sum_{\text{levels}}' \frac{1}{\exp\left[(E-E_0)/kT\right]-1}$$

If $2\omega >> \omega - \Omega$, one can neglect the contribution of the levels with $m < 0$

Same spectrum as a 3D anisotropic oscillator:

$$\omega - \Omega \quad , \quad 2\omega \quad , \quad \omega_z$$

$kT >> \hbar\omega$

$$N^{(\text{class})} = 1.2 \frac{(kT/\hbar)^3}{\left(\omega^2 - \Omega^2\right)\omega_z}$$

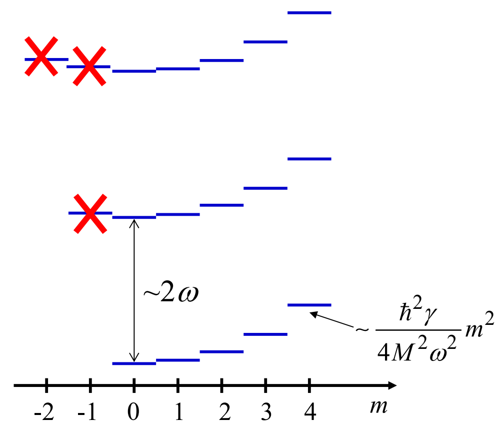
$\omega \simeq \Omega$

$\omega^2 - \Omega^2 \simeq 2\omega(\omega - \Omega)$

$$N = 1.2 \frac{(kT/\hbar)^3}{2\omega\ (\omega - \Omega)\ \omega_z}$$

Calculation of T_c with Landau levels $(r^2 + \gamma r^4/4)$

$\omega = \Omega$



Neglect the levels with $m<0$

Same spectrum as a 3D system with:

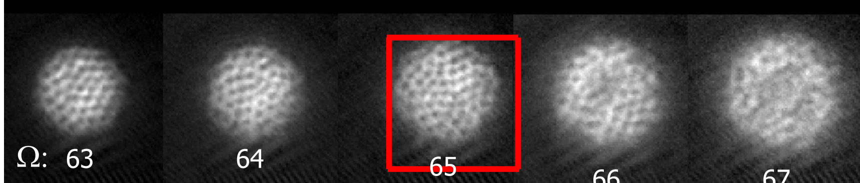
- free motion along 1 direction with an effective mass M_{eff} prop. to $M^2\omega^2/\gamma$
- harmonic trap 2ω
- harmonic trap ω_z

$kT >> \hbar\omega$

$$N^{(\text{class})} \propto \frac{M(kT)^{5/2}}{\sqrt{\gamma}\ \omega_z} \longleftrightarrow N \propto \frac{\sqrt{M_{\text{eff}}}\ (kT)^{5/2}}{2\omega\ \omega_z}$$

Experimental results in quadratic+quartic potential

$\omega_\perp / 2\pi = 65\,\text{Hz}$ $3\ 10^5$ atoms $T = 5 - 10$ nK



For Ω=65-67 Hz, the vortices become less and less visible:

- Not any more a degenerate gas? $T_c = 60$ nK
- Bending of the lines?
- Breakdown of mean-field approximation?

Phys. Rev. Lett. **92**, 050403 (2004)

2.

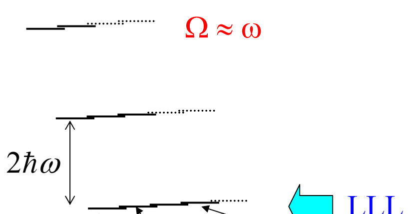
Equilibrium shape of a fast rotating condensate

2D system: the z motion is frozen (gaussian of width $a_z = \sqrt{\hbar/(m\omega_z)}$)

LLL physics, see also:

- Fischer, Baym
- Watanabe, Baym, Pethick
- Komineas, Read, Cooper
- Experiments by the Boulder group

Physics in the lowest Landau level



$e^{-r^2/2a_0^2}$ $(x+iy)e^{-r^2/2a_0^2}$ $(x+iy)^n e^{-r^2/2a_0^2}$

If the chemical potential is much smaller than $2\hbar\omega$, the physics is restricted to the lowest Landau level

$$r^2 = x^2 + y^2$$

$$a_0 = \sqrt{\hbar / m\omega}$$

General one-particle state in the LLL:

$$e^{-r^2/2a_0^2} P(x+iy) \longrightarrow e^{-r^2/2a_0^2} \prod_{j=1}^{n} (u-u_j)$$

Polynomial or analytic function

$u = x + iy$ $\quad u_j$: vortices

Ground state in the LLL approximation

Minimization of $E = E_{\rm kin} + E_{\rm ho} + E_{\rm rot} + E_{\rm int}$ $\qquad \hbar = M = \omega = 1$

$$E_{\rm kin} = \frac{1}{2}\int |\nabla \psi|^2 \qquad E_{\rm ho} = \frac{1}{2}\int r^2 |\psi|^2 \qquad E_{\rm rot} = -\Omega \int \psi^* [L_z \psi]$$

$$E_{\rm int} = \frac{Ng}{2}\int |\psi|^4 \qquad g = \sqrt{8\pi}\, a_s / a_z$$

a_s: scattering length

In the LLL: $E_{\rm kin} = E_{\rm ho} = \frac{1}{2} + \frac{E_{\rm rot}}{2}$

Minimization of: $E = \Omega + (1-\Omega) \int \left(r^2 |\psi|^2 + \frac{\Lambda}{2} |\psi|^4 \right)$

only one parameter: $\Lambda = \frac{Ng}{1-\Omega}$

LLL vs. centrifugal force approximation ($\omega = 1$)

Minimization of $\int \left(r^2 |\psi|^2 + \frac{\Lambda}{2} |\psi|^4 \right)$ $\qquad \Lambda = \frac{Ng}{1-\Omega}$

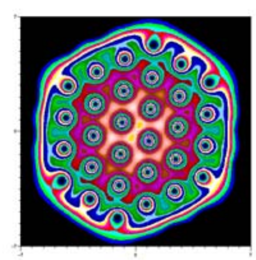
"Thomas-Fermi" approximation: $\bar{\rho}_{\rm atom} \propto R_0^2 - r^2$ $\qquad R_0 \propto \Lambda^{1/4}$

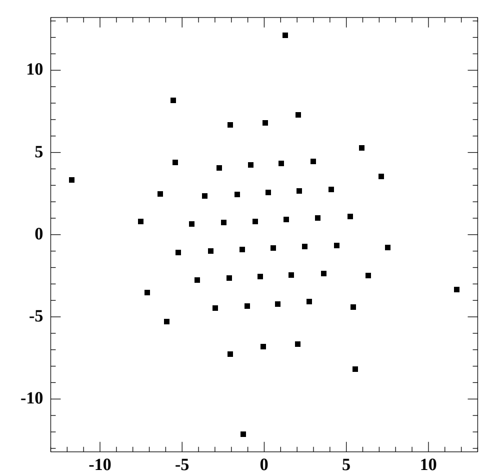
Validity: $Ng(1-\Omega) \ll 1$

"Centrifugal force" approximation: $-\frac{1}{2}\nabla^2\bar{\psi} + \frac{1-\Omega^2}{2} r^2 \bar{\psi} + Ng|\bar{\psi}|^2 \bar{\psi} = \mu \bar{\psi}$

2D $\quad Ng >> 1$: Thomas-Fermi regime, similar to LLL

$Ng << 1$: Gaussian regime, with a width $\propto \left(1-\Omega^2\right)^{1/4} >> R_0$
very different from the LLL prediction





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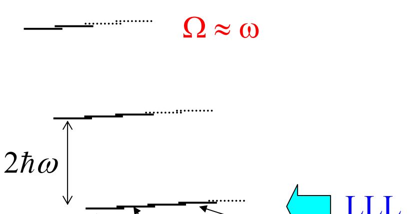

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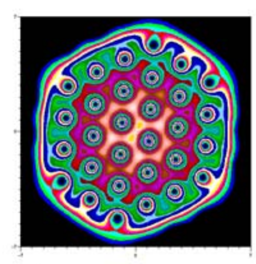
LLL solution for Λ=3000

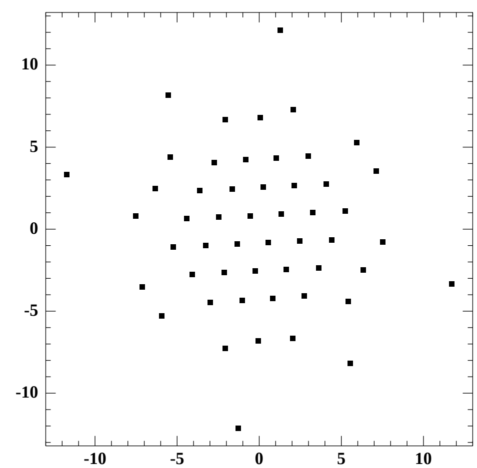
$$\psi(x,y) = e^{-r^2/2a_0^2} \prod_{j=1}^{n} (u-u_j)$$

$u = x + iy$



atomic distribution

1000 Rb atoms, Ω=0.99 ω
$\omega_z / (2\pi)$ = 150 Hz



vortex distribution

strong distortion on the edges
≠ rigid body rotation (T.L. Ho)!

3.

Breathing mode of a rotating condensate

The transverse monopole oscillation

For a non rotating gas in a harmonic trap, this "breathing mode" correspond to a scaling transform of the steady-state distribution

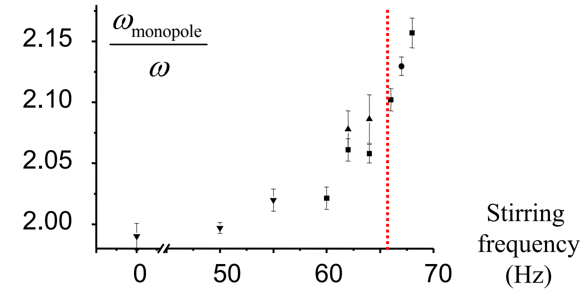
$$\omega_{\text{monopole}} = 2\omega$$

Chevy *et al*, PRL 88, 250402 (2002)
Guilleumas & Pitaevskii, Jackson *et al*, Kagan *et al*

Prediction for a rotating gas in a harmonic trap (Cozzini & Stringari)

$\omega_{\text{monopole}} = 2\omega$ **and not** $\omega_{\text{monopole}} = 2\sqrt{\omega^2 - \Omega^2}$

Experiment in a quadratic+ quartic trap

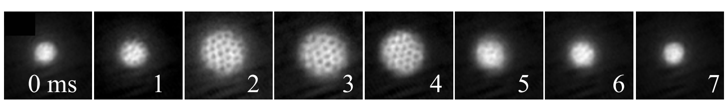


S. Stock et al, Europhys. Lett. **65**, 594 (2004)

The structure of the monopole mode

For relatively low rotation frequencies, usual breathing mode:

60 Hz
$\Omega/\omega \simeq 0.92$

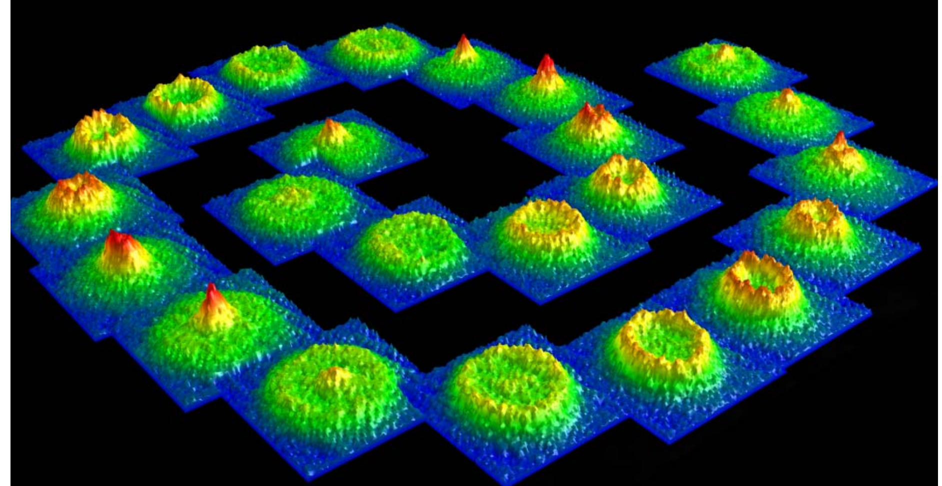


For rotation frequencies above the trap frequency, “strange” structure of the breathing mode…

Excitation of the breathing mode of the fast rotating condensate

One picture every ms

$\Omega/\omega \simeq 1.04$



Th: Stringari, Fetter, Jackson

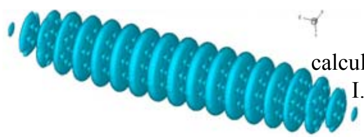
Conclusions

The Coriolis force plays no role for the value of the critical temperature of a rotating gas, if $kT \gg \hbar\omega$ but it is essential to calculate the shape and the modes of the BEC

For fast rotations in the mean-field regime, the LLL basis provides a convenient formalism: *deviation from rigid-body rotation*

Perspectives:

- r^2+r^4: Mexican hat potentials and persistent currents
- Layered rotating superfluid



calculation by I. Danaila

- Reach $N_{\text{vortex}} \sim N_{\text{atomes}}$: correlated states and Fractional Quantum Hall effect