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Dynamical Role of Anyonic Excitation Statistics in Rapidly Rotating Bose Gases

Instability of Bose Gases with $g < 0$

Gas of Bosons Unstable if $g < 0$
Against Collapse

(Meta-)Stable Up to Some $N_c \propto d_{\mathrm{HO}}/|g|$
If Trapped and Dilute
Bradley et al., PRL **78**, 985 (1997)
Ueda and Leggett, PRL **80**, 1576 (1998)

Contrast with Fermi Gases:
Fermi Pressure Prohibits Collapse to a
Singular Distribution

Will Show: 2D Analog "Anyonic" Pressure
Exists in Rapidly Rotating Bose Gases

Bose Gases in Fractional Quantum Hall State

Ginzburg-Landau (GP) Chern-Simons Action

$$S_\Theta = \int d^2\boldsymbol{x}dt \left[\Psi^* i(\partial_t + i\mathcal{A}_0)\Psi - \frac{1}{2m}|\boldsymbol{D}\Psi|^2 - \left(V_{\text{trap}}(\boldsymbol{x}) - \frac{1}{2}m\Omega^2 r^2 \right)|\Psi|^2 - \mathcal{U}(|\Psi|^2) + \frac{\Theta}{4}\epsilon^{\alpha\beta\gamma}\mathcal{A}_\alpha \mathcal{F}_{\beta\gamma} \right]$$

Gauge Covariant Derivative

$$\boldsymbol{D} = \nabla - im\boldsymbol{A} - i\mathcal{A} \qquad (\boldsymbol{A} = \boldsymbol{\Omega} \times \boldsymbol{r})$$

Consequence of Chern-Simons Term: Anyons

Chern-Simons "Gauss" Law

$$\frac{\Theta}{2}\epsilon^{\alpha\mu\nu}\mathcal{F}_{\mu\nu} = J^\alpha$$

Zeroth Component

$$-\Theta\mathcal{B} = |\Psi|^2 = \rho$$

$\Rightarrow$ Statistical Flux attached to particles!

Landau Level Filling Factor $\nu = \pi\rho_0/m\Omega$

$$\Theta = \nu/[2\pi(1-\nu)] \qquad (\nu < 1)$$

Particle-Statistical Flux Composites Have Flux

$$\tilde{\Phi} = \oint d\boldsymbol{x} \cdot [\boldsymbol{A} + \mathcal{A}/m] = \nu\Phi_0$$

Using Bogomol'nyi Decomposition

$$|\boldsymbol{D}\Psi|^2 = |(D_1 \pm iD_2)\Psi|^2 \pm \nabla \times \boldsymbol{J} \pm \mathcal{B}\rho \pm 2m\Omega\rho$$

Hamiltonian Reads

$$\begin{aligned} H &= \int d^2\boldsymbol{x} \left[\frac{1}{2m}|(D_1 \pm iD_2)\Psi|^2 + \mathcal{U}(|\Psi|^2) \right. \\ &\quad \left. + \frac{1}{2}m(\omega_\perp^2 - \Omega^2)r^2|\Psi|^2 \mp \frac{1}{2}\left(\frac{1}{m\Theta}|\Psi|^2 - 2\Omega\right)|\Psi|^2\right] \end{aligned}$$

Potential Expansion

$$\mathcal{U}(|\Psi|^2) = \frac{g}{2}|\Psi|^4 + \frac{\gamma}{6}|\Psi|^6 + \cdots$$

$\Rightarrow$ Effective Interaction Coupling

$$g_{\text{eff}} = g \mp \frac{1}{m\Theta}$$

Choice $g = \pm 1/m\Theta$ $\Rightarrow$ Free Gas of Anyons!

Requirement: *Self-Duality* (Bogomol'nyi) Constraints

$$(D_1 \pm iD_2)\Psi = 0$$

Self-Dual States of Effectively Zero Coupling
Protected by $|\Psi|^6$ of $\mathcal{U}(|\Psi|^2)$ ($\gamma > 0$ for Sixth-Order Stability!)

Self-duality Equations Equivalent to

Liouville Equation

$$\Delta \ln \rho = \pm 2\rho/\Theta \qquad (V(\boldsymbol{x}) = \Omega = 0)$$

(Jackiw and Pi, PRL **64**, 2969 (1990))

Radially Symmetric Vortex Solution $\mathcal{A} \propto 1/r$:

$$\rho(r) = \frac{4\Theta n^2}{r^2}\left[\left(\frac{r_0}{r}\right)^n + \left(\frac{r}{r_0}\right)^n\right]^{-2}$$

r_0 Arbitrary: Scale (Dilation) Invariance

Nonvanishing Trapping and Rotation:
Vortex Soliton Executes Cyclotron Motion and **Core Size Breathes**: $r_0 \to r_0 \cos(\Omega t)$

Soliton Energy Minimized for $r_0 = 0$
$\Rightarrow$ External Force (e.g. Blue Detuned Laser) Creates $r_0 \neq 0$

Non-Abelian Case: Spinor Bose Gases

Non-Abelian Chern-Simons-Gauss Law

$$\mathcal{B} = \frac{i}{\Theta} T^a (\psi^\dagger T^a \psi)$$

T^a: Anti-Hermitian Generators of Lie Group
SO(3): iT^a_{ij} Generate Rotation, are Angular Momentum Operators

Scalar and Vector Interactions Parametrized

$$V_{\text{int}} = \frac{c_0}{2}\psi_i^\dagger\psi_j^\dagger\psi_j\psi_i - \frac{c_2}{2}\psi_i^\dagger\psi_k^\dagger T^a_{ij}T^a_{kl}\psi_l\psi_j$$

$\Rightarrow$ Coefficient of Spin-Spin Interaction

$$(c_2)_{\text{eff}} = c_2 + \frac{1}{m\Theta}$$

$c_0 = 0$: Self-Dual Bogomol'nyi Point Exists
Liouville Equation Generalized to

Toda Equation

Solutions Correspond To
Non-Abelian Fractional Vortices

"Objects possessing no global identity"
Carrying Non-Abelian Flux

Summary

- Bose Gas with Attractive Interaction Coupling Stabilized in Fractional Quantum Hall State of Rotating Gas

- "Free" Anyon Gas at large couplings such that $g = -2\pi\hbar^2(\nu^{-1} - 1)/m$ for given ν

- Physical Interpretation: 2D Analog of Fermi Pressure: "Anyonic" Pressure

- Implementation near Feshbach Resonances, Possibly in Rotating Optical Lattices

Phys. Rev. Lett. **93,** 160403 (2004)