Photoassociation dynamics
in a Bose-Einstein condensate

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Overview

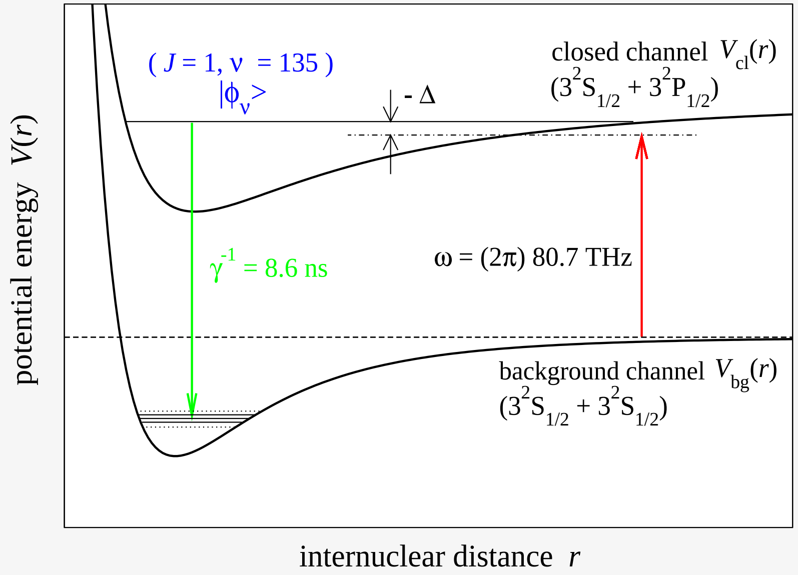
- Microscopic Quantum Dynamics Approach
- Photoassociation — Rate limits
- Two-body dressed states
- Outlook

Funding:
Deutsche Forschungsgemeinschaft
A. v. Humboldt-Foundation

Introduction

Single-colour photoassociation

Closed-channel bound state ϕ_ν coupled to **open channel** by (detuned) laser light.



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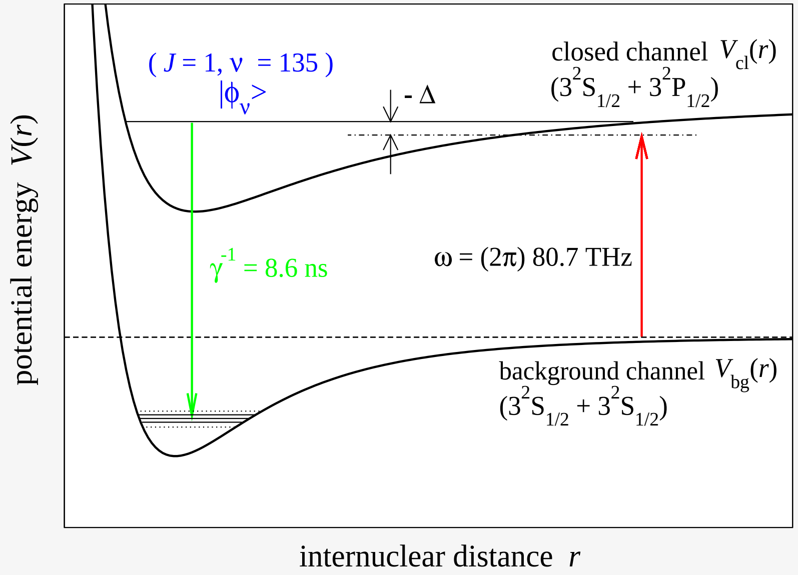
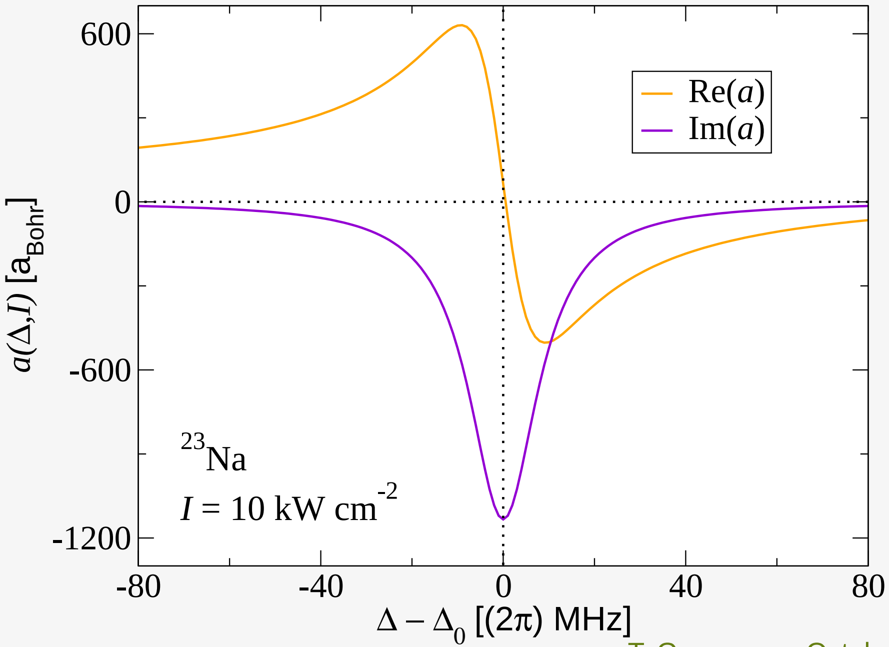


s-wave scattering length: $|a_0(\Delta, I)|$ remains finite if spontaneous electronic decay is present.

$$a(\Delta, I) = a_{\rm bg}\left(1 - \frac{\Gamma(I)}{\Delta - \Delta_0(I) + \frac{i}{2}\gamma}\right)$$



P.O. Fedichev, Yu. Kagan, G.V. Shlyapnikov, J.T.M. Walraven, PRL **77**, 2913 (1996)

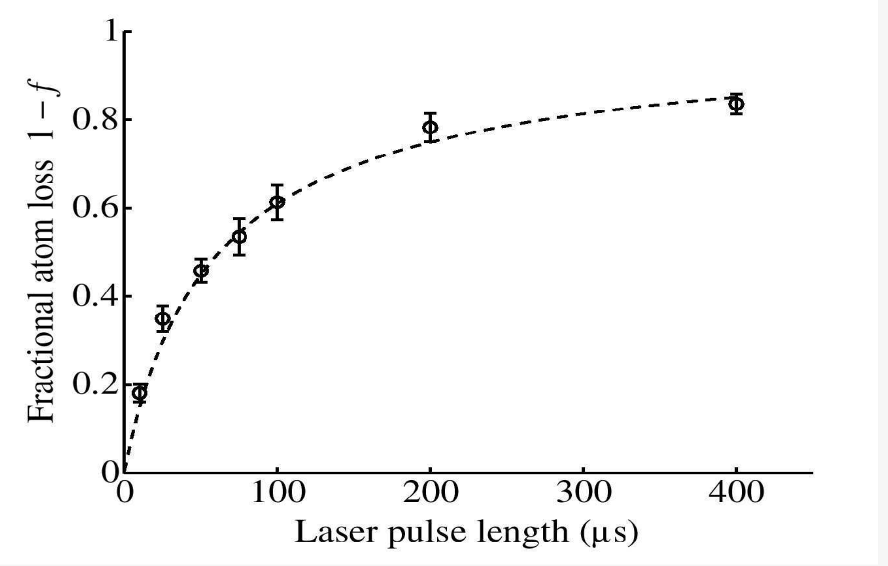
J. Bohn, P.S. Julienne, PRA **54**, R4637 (1996)

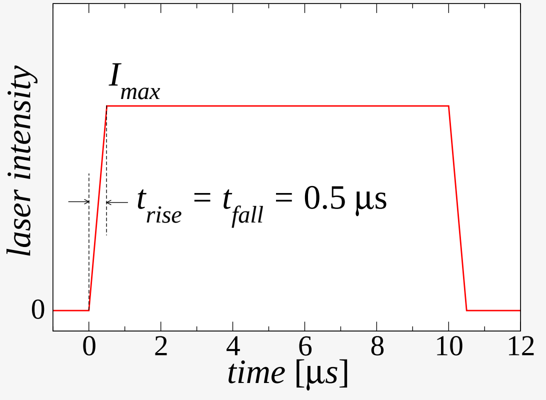
Experiment:
Rate limits?

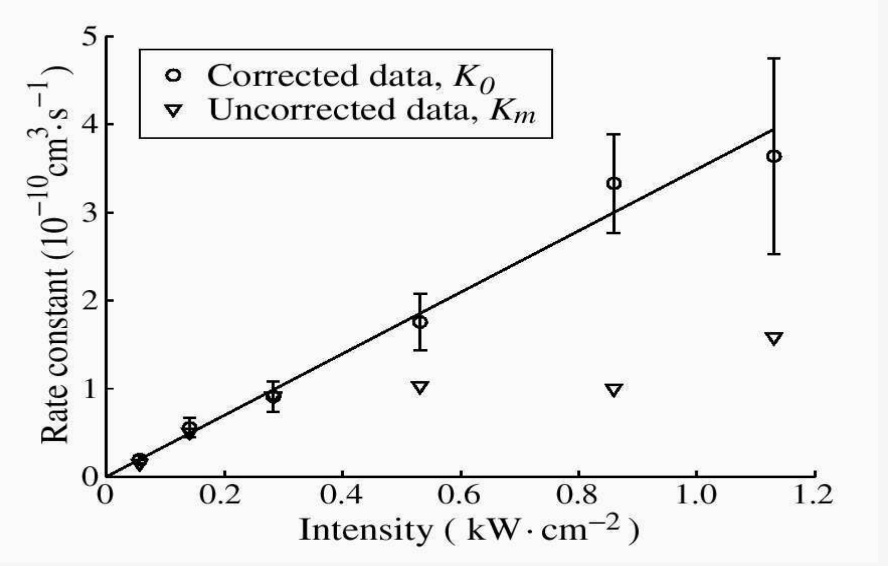
Single color PA of ^{23}**Na** (NIST).
[C. McKenzie, *et al.*, PRL **88**, 120403 (2002)]

GPE time evolution:

$$f = \frac{n(t,r)}{n(t_0,r)} = \frac{1}{1+K_0(I)n(t_0,r)t}$$







Related

Experimental and theoretical work

- **Experimental work** on rate limitation issues:
  C. Drag, *et al.*, J. Quant. Electron. **36**, 1378 (2000).
  C. McKenzie, *et al.*, PRL **88**, 120403 (2002).
  U. Schlöder, C. Silber, T. Deuschle, and C. Zimmermann, PRA **66**, 061403 (2002).
  I.D. Prodan, *et al.*, PRL **91**, 080402 (2003).
  M. Weidemüller, R. Wester, *priv. comm.* (2004).

- Various **discussion** of rate limitations:
  J.L. Bohn and P.S. Julienne, PRA **60**, 414 (1999).
  K. Góral, M. Gaida, and K. Rzazewski, PRL **86**, 1397 (2001).
  M. Holland, J. Park, and R. Walser, PRL **86**, 1915 (2001).
  J. Javanainen and M. Mackie, PRL **88**, 090403 (2002).

Microscopic Quantum Dynamics Approach

Cumulant Approach

- **Dynamic equations** for correlation functions:

$$i\hbar\frac{\partial}{\partial t}\langle\psi(\mathrm{x})\rangle_t = \langle[\psi(\mathrm{x}), H]\rangle_t$$

$$i\hbar\frac{\partial}{\partial t}\langle\psi(\mathrm{y})\psi(\mathrm{x})\rangle_t = \langle[\psi(\mathrm{y})\psi(\mathrm{x}), H]\rangle_t$$

$$\vdots$$

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- Resummation in terms of **cumulants (connected Green's functions)**:

$$\langle\psi(\mathrm{x})\rangle_t^c = \langle\psi(\mathrm{x})\rangle_t$$
$$\langle\psi(\mathrm{y})\psi(\mathrm{x})\rangle_t^c = \langle\psi(\mathrm{y})\psi(\mathrm{x})\rangle_t - \langle\psi(\mathrm{x})\rangle_t\,\langle\psi(\mathrm{y})\rangle_t$$
$$\langle\psi^\dagger(\mathrm{y})\psi(\mathrm{x})\rangle_t^c = \langle\psi^\dagger(\mathrm{y})\psi(\mathrm{x})\rangle_t - \langle\psi^\dagger(\mathrm{x})\rangle_t\,\langle\psi(\mathrm{y})\rangle_t$$
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- Systematically truncate system of dynamic equations for the cumulants.

$\Rightarrow$ Few-body dynamics enters through **T-matrices**.
$\Rightarrow$ Energy & number conservation.
$\Rightarrow$ Positivity of mode occupations.

Mean Field Dynamics beyond Gross-Pitaevskii

- Dynamic equation for mean fi eld $\Psi(\mathrm{x},t) = \langle\psi(\mathrm{x})\rangle_t^c$: $\qquad (\Phi = \langle\psi\psi\rangle_t^c)$

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathrm{x},t) = \left[-\frac{\hbar^2}{2m}\Delta + V_{\mathrm{trap}}(\mathrm{x})\right]\Psi(\mathrm{x},t)$$
$$+ \int d^3y\, V(\mathrm{x}-\mathrm{y},t)\Psi^*(\mathrm{y},t)\left[\Phi(\mathrm{x},\mathrm{y},t) + \Psi(\mathrm{x},t)\Psi(\mathrm{y},t)\right]$$

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- **Non-Markovian** Nonlinear Schrödinger Equation:

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- Coupling function :

$$g(t,\tau) = (2\pi\hbar)^3\langle 0|\underbrace{V(t)[\delta(t-\tau) + G_{2\mathrm{B}}(t,\tau)V(\tau)]}_{T\text{-matrix!}}|0\rangle$$

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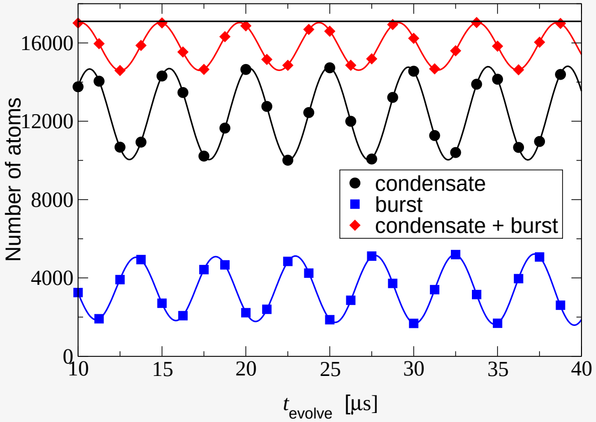
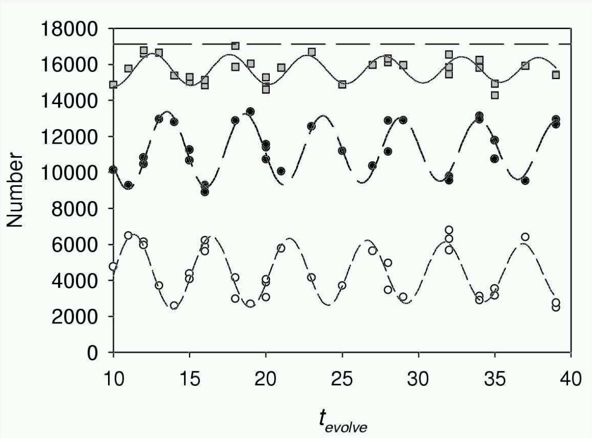
- Markovian limit:
  $\longrightarrow$ Gross-Pitaevskii dynamics:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathrm{x},t) = \left[-\frac{\hbar^2}{2m}\Delta + V_{\mathrm{trap}}(\mathrm{x})\right]\Psi(\mathrm{x},t) + \frac{4\pi\hbar^2 a}{m}|\Psi(\mathrm{x},t)|^2\Psi(\mathrm{x},t)$$

Previous applications of the
Microscopic Quantum Dynamics Approach

Atom-molecule oscillations (JILA)





T. Köhler, T.G., and K. Burnett, PRA **67**, 13601 (03);

T. Köhler, T.G., P.S. Julienne, and K. Burnett, PRL **91**, 230401 (03).

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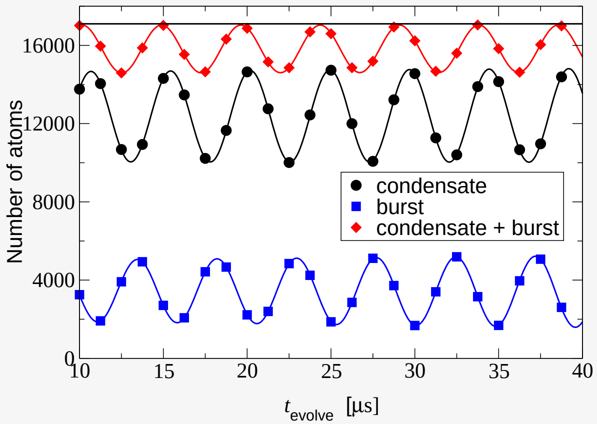

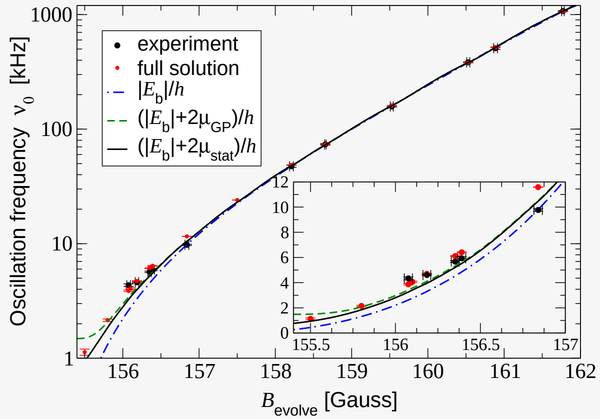

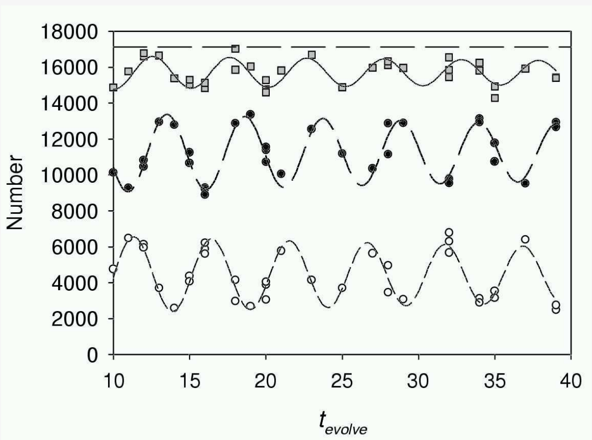

K. Góral, T. Köhler, and K. Burnett, cond-mat/0407627.

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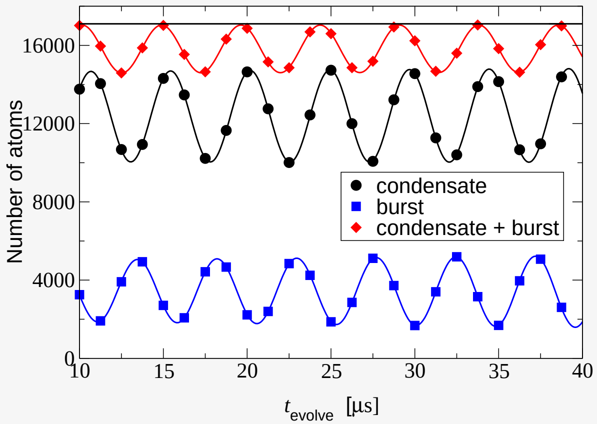

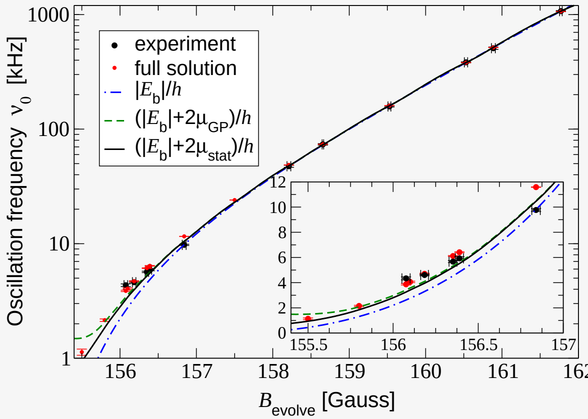

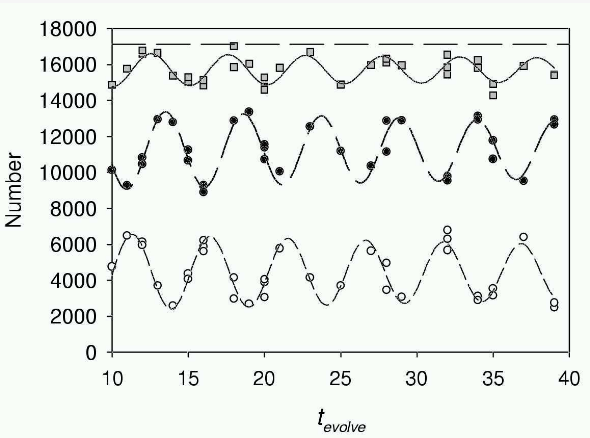

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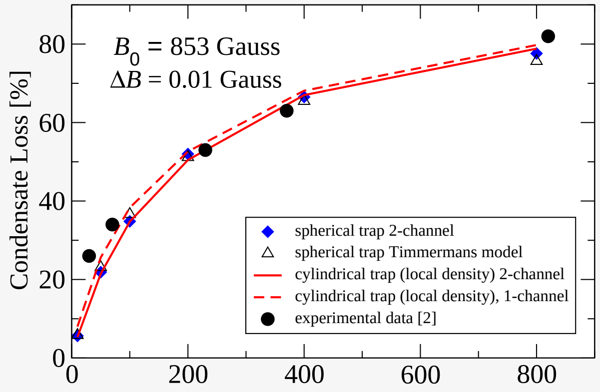
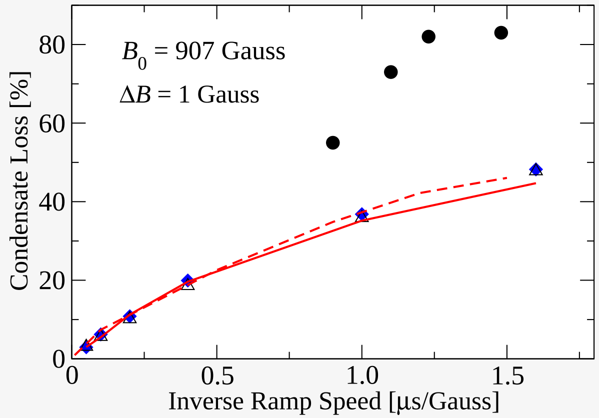
Feshbach ramps, e.g., @ MIT





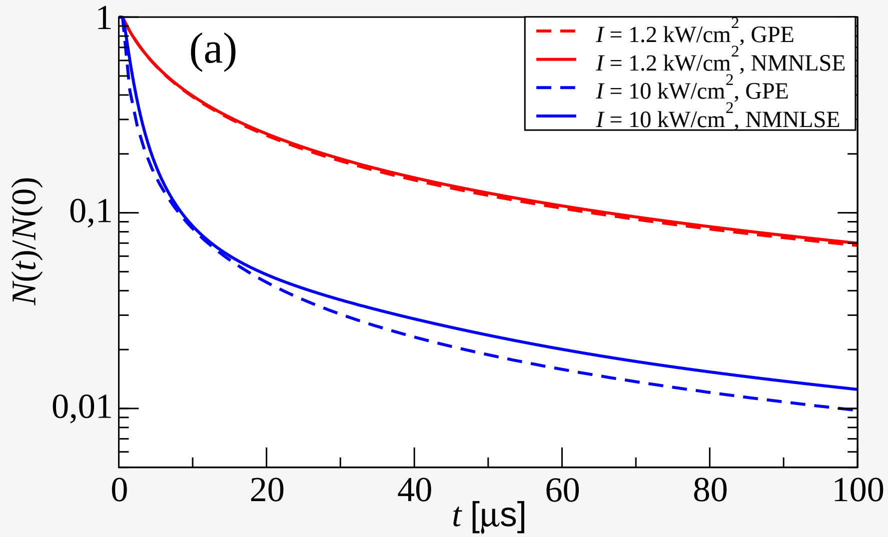
T. Köhler, K. Goral, and T.G., PRA **70**, 23613 (04).

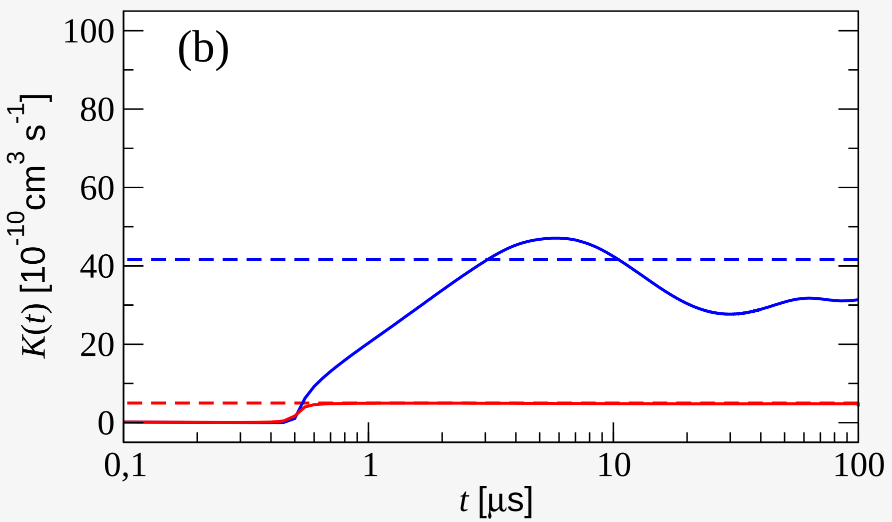
Photoassociation

Rate limits

Photoassociation in ^{23}Na condensate:

Is the loss rate limited?





Evolution of the **remaining fraction** of $N(0) = 4.0 \cdot 10^6$ condensate atoms in spherical harmonic ($\nu = 198$ Hz) trap, for laser intensity I.

GPE time evolution:

$$n(t, \mathbf{r}) = \frac{n(t_0, \mathbf{r})}{1 + K(I)\, n(t_0, \mathbf{r})\, t}$$

Temporally local decay 'rate':

$$K(t) = -\dot{N}(t) / \int d\mathbf{x}\, n(\mathbf{x}, t)^2$$

[T.G., PRA **70**, 021603(R) (2004); cond-mat/0406629 (PRA, in press)]

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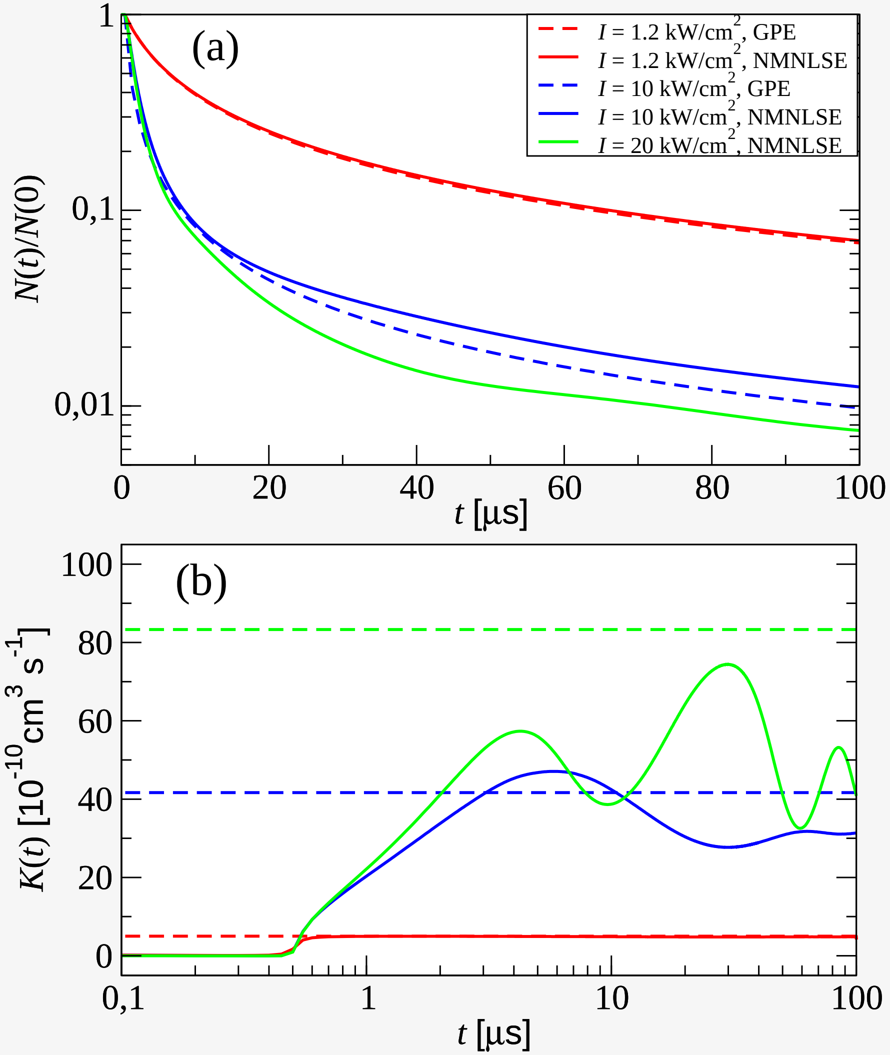

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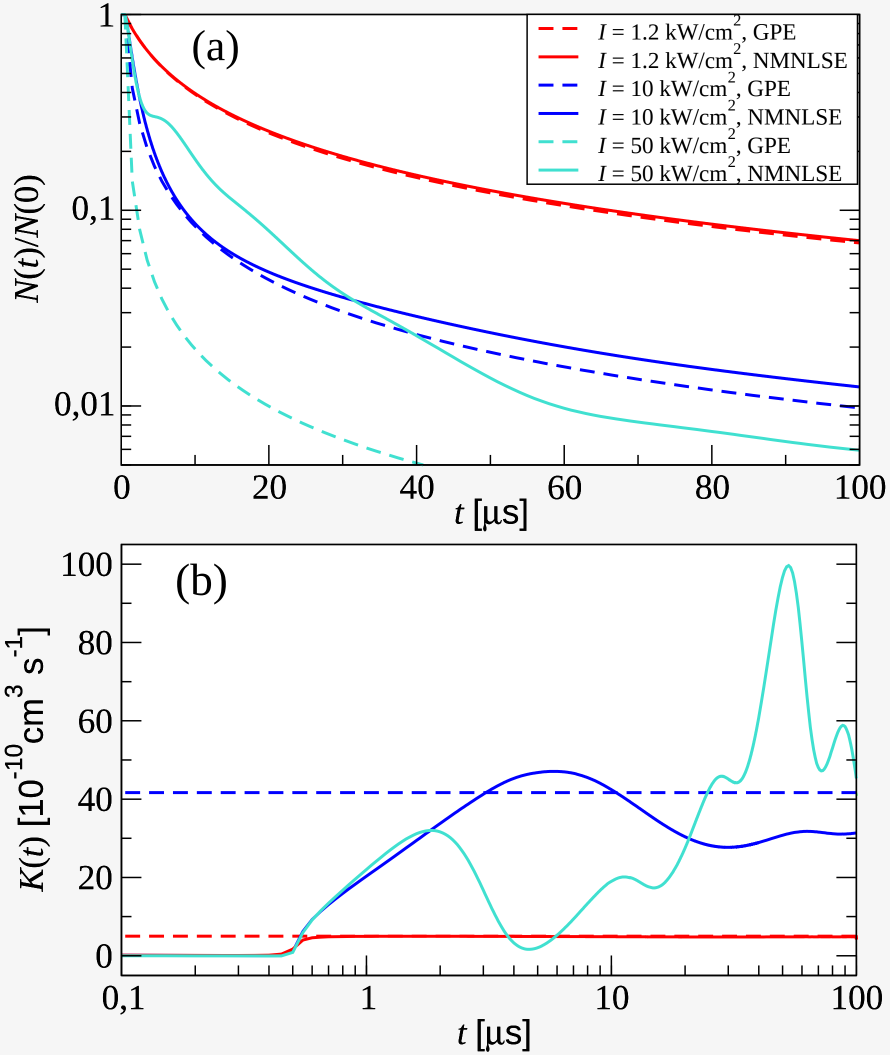

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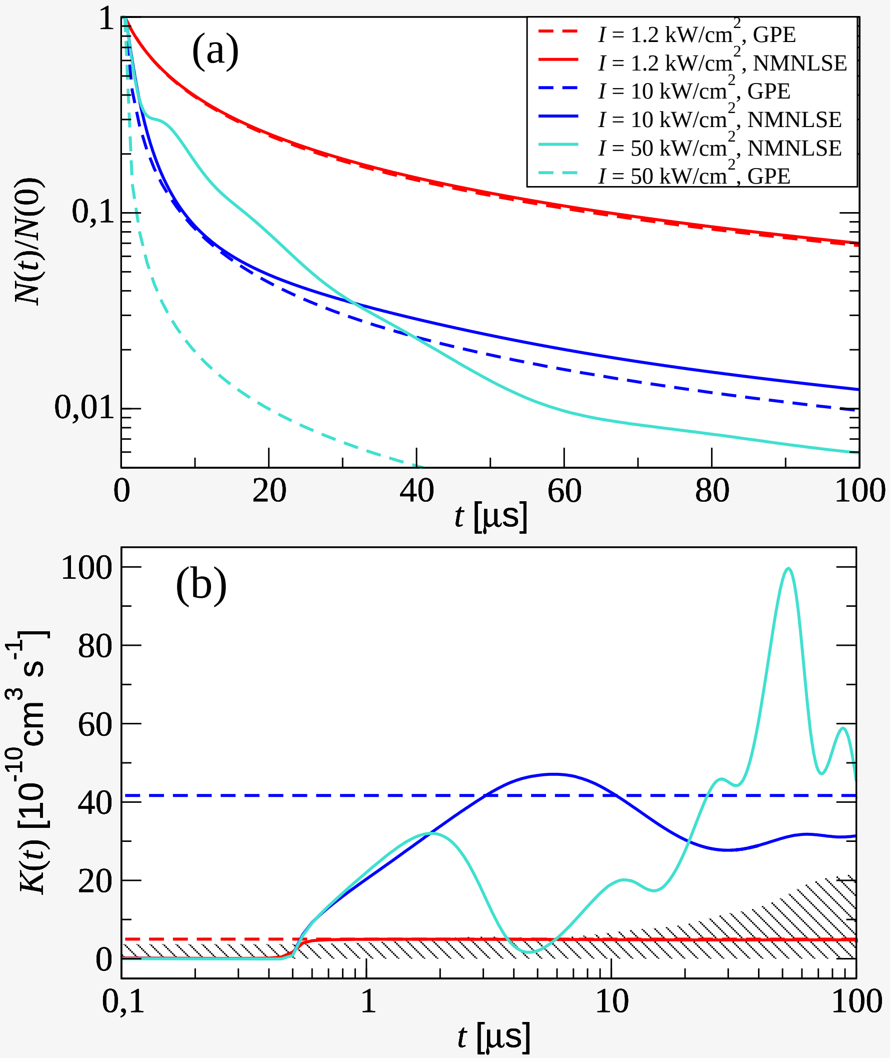

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Max. local decay rate according to

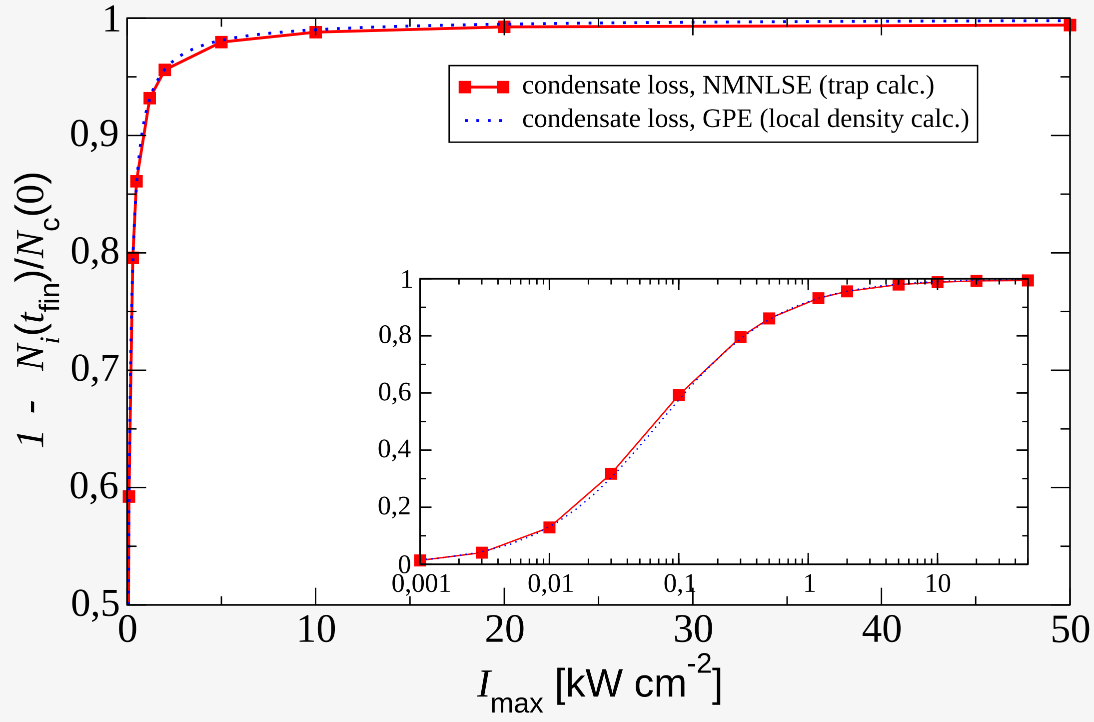
$K_\mathrm{J}(\mathbf{R}, t) = (\hbar/m)[n_\mathrm{c}(\mathbf{R}, t)]^{-1/3}$ (hatched)

[T.G., PRA **70**, 021603(R) (2004); cond-mat/0406629 (PRA, in press)]

Photoassociation in ^{23}Na condensate:

Is the molecule formation rate limited?

Fraction of condensate atoms lost after $t = 100\,\mu$s (red squares).



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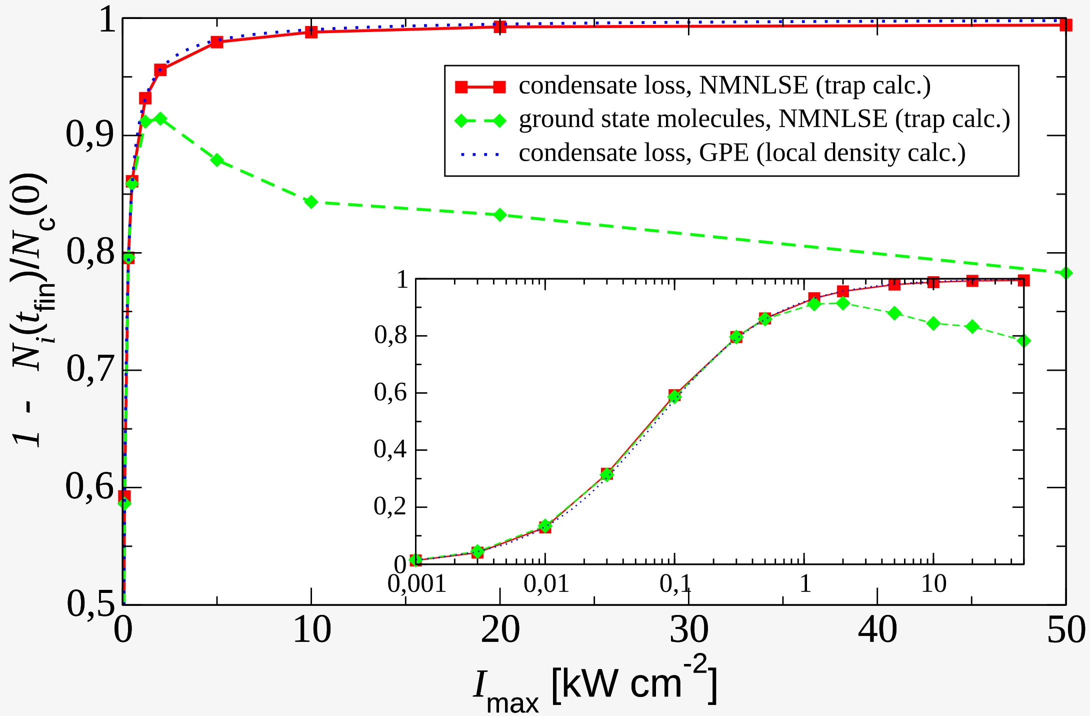
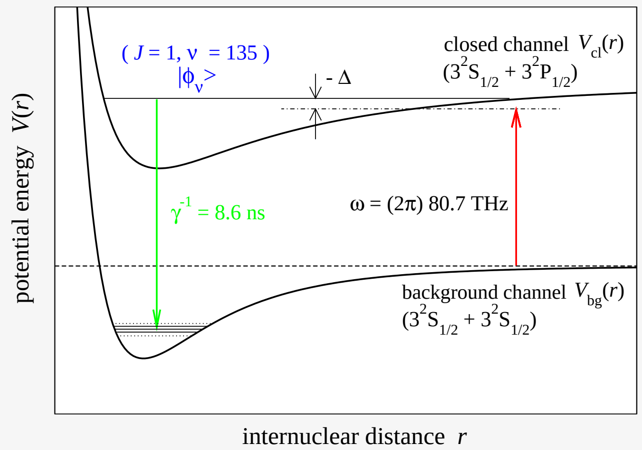


Compare this fraction to the fraction of the number of atoms lost via spontaneous decay (green diamonds):

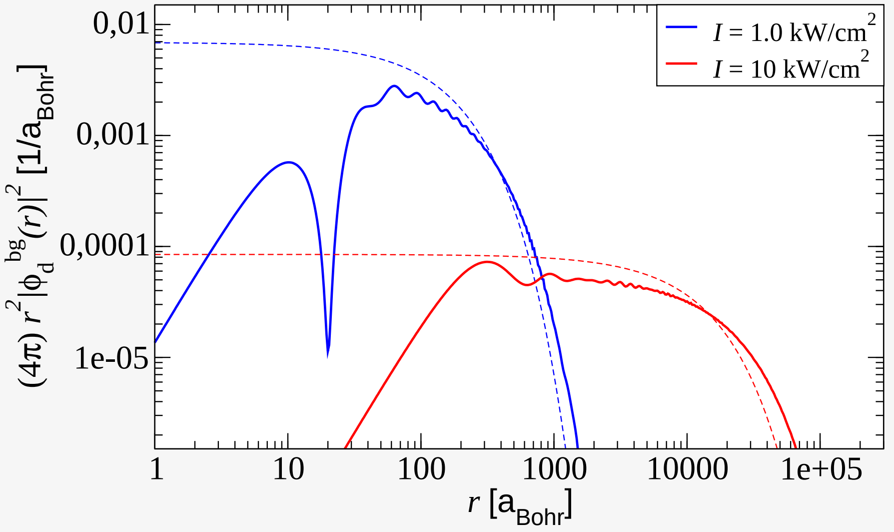
$$\hbar \dot{\mathbf{N}}_{\mathbf{tot}} = -\gamma \int d\mathbf{R}\, \left| \int d\mathbf{r}\, \tilde{\phi}^*_{\nu}(\mathbf{r}) \mathbf{\Phi}_{\mathbf{cl}}(\mathbf{R}, \mathbf{r}, t) \right|^2.$$



Two-body dressed states

Saturation of loss rates due to

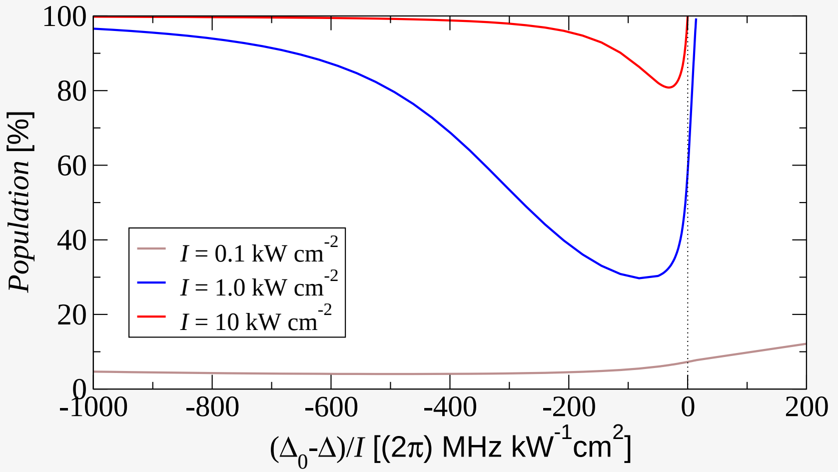
Long range nature of dressed states



Background channel component of radial **dressed state** density.

$$\Delta - \Delta_0(I) = 0.$$

$$\phi_d^{bg}(r) \simeq \frac{e^{-r/a(\Delta, I)}}{r} \quad (\text{---})$$



Population

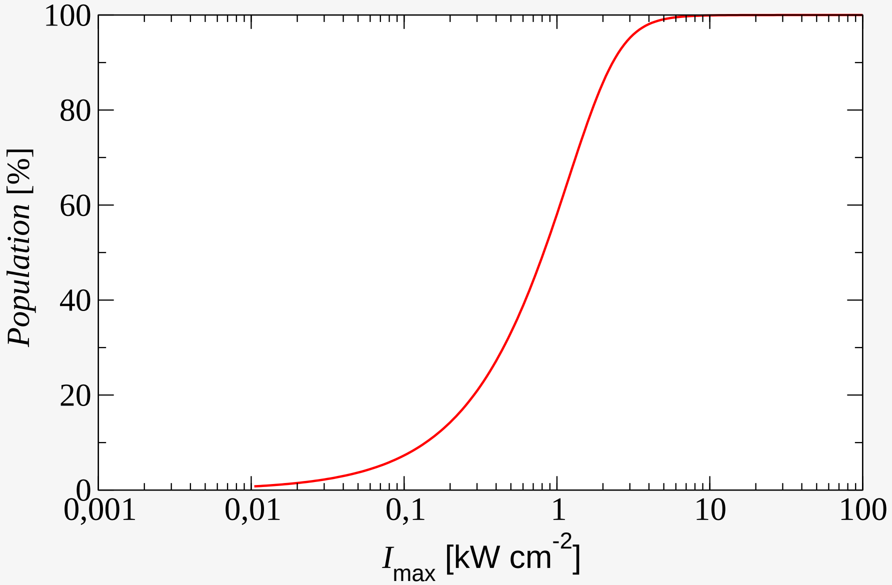
of background channel component:

$$(\mathcal{N}_d^2 - 1)/\mathcal{N}_d^2$$

[T.G., cond-mat/0401219]

Population of

Background channel dressed state component

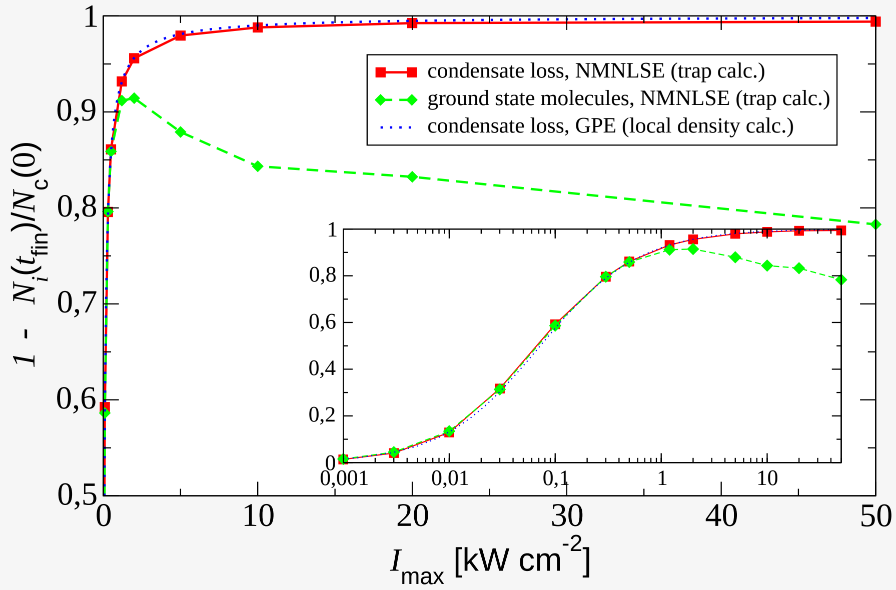


Population $(\mathcal{N}_d^2 - 1)/\mathcal{N}_d^2$ of **background channel component** vs. intensity I (on resonance, $\Delta = \Delta_0(I)$).

$$\begin{pmatrix} \phi_d^{bg} \\ \phi_d^{cl} \end{pmatrix} = \mathcal{N}_d^{-1} \begin{pmatrix} G_{bg}(E_d) W \phi_\nu \\ \phi_\nu \end{pmatrix}$$

Compare this to the fraction of the number of atoms lost via spontaneous decay (green diamonds):

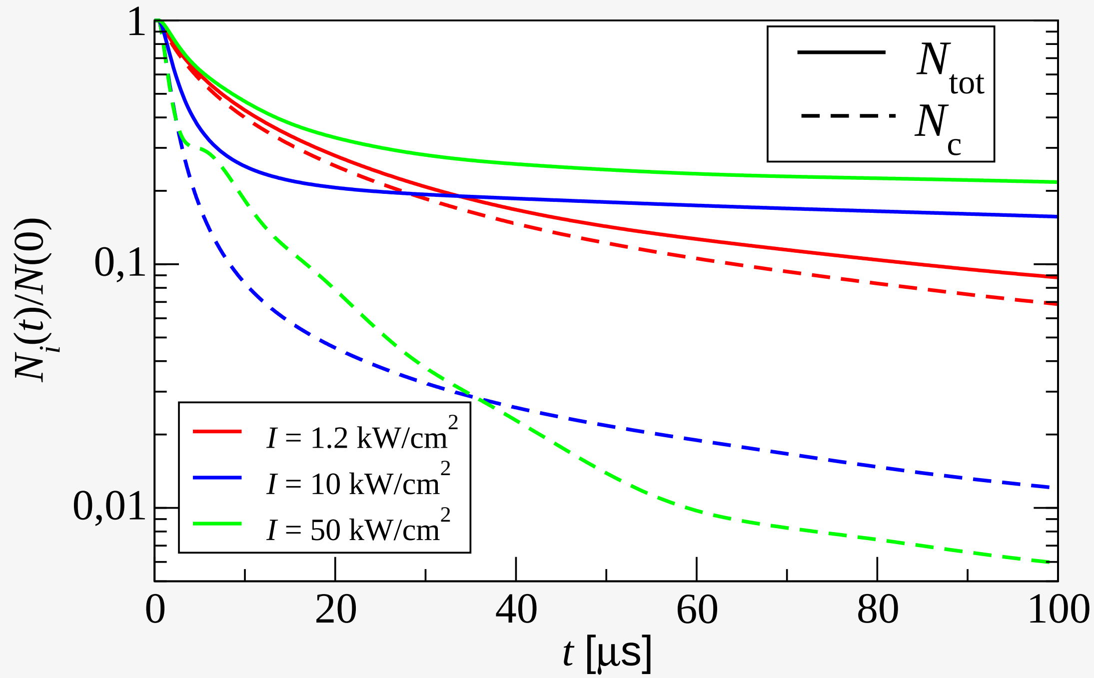
$$\hbar \dot{N}_{tot} = -\gamma \int d\mathbf{R} \, \left| \int d\mathbf{r} \, \tilde{\phi}_\nu^*(\mathbf{r}) \Phi_{cl}(\mathbf{R}, \mathbf{r}, t) \right|^2.$$



Photoassociation in ^{23}Na condensate:

Is the molecule formation rate limited?

Fractions of condensate (---) and total (—-) atom no. loss, as a function of time, for different intensities.



Outlook

- Photoassociation:
  – heteronuclear PA, STIRAP, ...
  – Alkaline earth elements: small decay width γ, large scattering lengths

  (⟶ metrology)
  – non-universal observables (many-body frequency shifts)

- Fermionic and mixed systems:
  – Molecular BEC
  – BCS pairing

- Improved truncation schemes on the basis of the 2PI effective action

$$\Gamma[\Psi, G] = S[\Psi] + \frac{i}{2}\mathrm{Tr}\ln(G^{-1} + G_0^{-1}(\Psi)G) + \Gamma_2[\Psi, G]$$

  [G. Aarts, *et al.*, PRD **66**, 045008 (2002)]

- Condensates in microtraps: far-from-equilibrium dynamics in (quasi) one-dimensional regime

- Long-time evolution (damping, drifting, thermalization)