Vortices Nucleation in Finite Bose-Einstein Condensates through edge states

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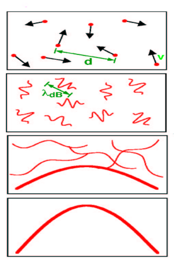
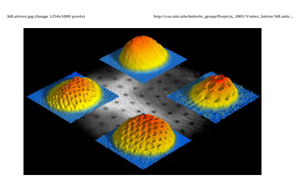
Background

Vortices in BEC



- BEC as a giant matter wave



- Vortices as holes in the density profile of the condensate (MIT,2001)

Highlights

- **Problem**: Explanation of Nucleation of Vortices in a finite domain via edge states [1, 2].
- **Experimental motivation**: Recent Experiments done at JILA, ENS, MIT [4, 5, 6] on vortices in BEC.
- **Relevant theoretical work**: Work done on the surface excitations in BEC by Stringari and collaborators, J. Anglin, U.A.Khawaza *et. al.* [8, 9, 12, 13] and others.
- **Technique** (Linear) Schrödinger equation of a rotatong trapped bosons in a disc has been solved with a set of non-local and chiral boundary conditions. These boundary conditions naturally split the Hilbert space in the bulk and the edge states [1, 3].
- **Main result** Critical Frequency of first Vortex nucleation. Change in the condensate size with faster rotation [1].

1 Theoretical Background

1.1 Superfluid under rotation

For a rigid-body $\boldsymbol{v} = \boldsymbol{\Omega} \times \boldsymbol{r} \Rightarrow \boldsymbol{\nabla} \times \boldsymbol{v} = 2\boldsymbol{\Omega}$

Superfluid (Mean-field) Order Parameter $\Psi_s = \sqrt{\rho(\boldsymbol{r},t)}e^{iS(\boldsymbol{r},t)}$

Superfluid velocity $\boldsymbol{v}_s(\boldsymbol{r},t) = \frac{\hbar}{m}\boldsymbol{\nabla}S(\boldsymbol{r},t) \Rightarrow \boldsymbol{\nabla} \times \boldsymbol{v}_s = 0$

In a cylinder $S(\vec{\boldsymbol{r}},t) = l\theta \Rightarrow \boldsymbol{v}_s = \frac{\hbar}{m}\frac{l}{r}\hat{\theta}$

$\oint \boldsymbol{v}_s.d\boldsymbol{r} = 2\pi l\frac{\hbar}{m}$

- This is a consequence of the fact that $\boldsymbol{\nabla} \times \boldsymbol{v}_s = 2\pi l\frac{\hbar}{m}\delta^2(\boldsymbol{r}_\perp)\hat{z}$
- The vortex state is characterized by $l \neq 0$.
- If the number of particles is N the vortex carries total angular momentum $L_z = Nl\hbar$.
- Thermodynamic critical frequency Ω_c is obtained by minimizing $F = E_{Lab} - \Omega L_z \Rightarrow \Omega_c = \frac{E_{Lab}}{Nl\hbar}$.

1.2 Effect of Finite Geometry on Ω_c

- Experimentally observed range for Ω_c (for the first vortex) $\rightarrow 0.1\omega_\perp$ to $0.7\omega_\perp$ and is higher than the thermodynamic critical frequency.
- Vortices are nucleated from the surface in a finite system. This effects the critical frequency of vortex nucleation as well as the mechanism of vortex nucleation.
- Initially higher angular momentum states are all localised at the boundary and there is a surafce potential barrier on the way to vortex nucleation.
- The role of these surface states and the barrier is qualitatively similar to that of the Meissner currents at the surface of a finite superconducting system. This is true even though the rotaion plays a kinematic role in the Gross-Pitaevskii theory rather than the dynamic role of a magnetic field in the Landau-Ginzburg theory
- Our approach to the problem of vortex nucleation identifies these surface (edge)states in a two-dimensional geometry by noting the chirality of the probability current in the edge and bulk region.

Theoretical Framework

2 Spectrum of Schrödinger Equation

2.1 Schrödinger equation in a rotating frame

- Time-independent Gross-Pitaevskii equation $\left(-\frac{\hbar^2\boldsymbol{\nabla}^2}{2m} - \mu + V_{ext} + g|\Psi_s(\boldsymbol{r})|^2\right)\Psi_s(\boldsymbol{r}) = 0$
- In the rotating frame is becomes $\left(-\frac{\hbar^2\boldsymbol{\nabla}^2}{2m} - \mu + V_{ext} + g|\Psi_s(\boldsymbol{r})|^2 - \Omega L_z\right)\Psi_s(\boldsymbol{r}) = 0$
- We solve the linear part of the above equation in two dimensions.

$$H = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\omega_\perp^2 r^2 - \Omega L_z$$

$$\text{define, } \mathbf{A_v} = \boldsymbol{\omega}_\perp \times \mathbf{r} \text{ with } \boldsymbol{\omega}_\perp = (0,0,\omega_\perp)$$

$$\Rightarrow H = \frac{1}{2m}(\mathbf{p} - m\mathbf{A_v})^2 + (\omega_\perp - \Omega)L_z$$

$$\text{Alternatively define, } \mathbf{A_\Omega} = \boldsymbol{\Omega} \times \mathbf{r} \text{ with } \boldsymbol{\Omega} = (0,0,\Omega)$$

$$\Rightarrow H = \left(\frac{1}{2m}(-i\hbar\nabla - m\mathbf{A_\Omega})^2 + \frac{1}{2}m(\omega_\perp^2 - \Omega^2)r^2\right)$$

2.2 Spectrum in the infinite plane

- $\Psi_{n,l}(r) = C_{n,l}r^{|l|}e^{il\theta}e^{-\frac{b_{\omega_\perp}r^2}{2}}\,{}_1F_1(a,|l|+1;b_{\omega_\perp}r^2)$
- $E_{n,l} = \hbar\omega_\perp(2n + |l| - \frac{\Omega}{\omega_\perp}l + 1), n \in \mathbf{N}, l \in \mathbf{Z}$

$\omega_\perp = \Omega$ corresponds to the Landau problem with $\omega_\perp = 2\omega_c$.
Also $b_{\omega_\perp} = \frac{m}{\hbar}\omega_\perp$, $b_\Omega = \frac{m}{\hbar}\Omega$, $\Phi = b_{\omega_\perp}R^2$, $\Phi_\Omega = b_\Omega R^2$.

2.3 Spectrum in a disc with radius R

Dirichlet Boundary Condition (DBC)

$${}_1F_1(a,|l|+1,\Phi) = 0$$

Chiral Boundary Condition (CBC)

The current density in a given eigenstate is

$$\mathbf{j} = \frac{\hbar}{2mi}(\Psi_{n,l}^*\boldsymbol{\nabla}\Psi_{n,l} - \Psi_{n,l}\boldsymbol{\nabla}\Psi_{n,l}^* - 2i\frac{m}{\hbar}\boldsymbol{A_\Omega}|\Psi_{n,l}|^2)$$

The nonvanishing azimuthal component is

$$j_\theta = \frac{\hbar}{m}(\frac{l}{r} - \frac{m}{\hbar}\Omega r)|\Psi_{n,l}|^2$$

- For $r < \sqrt{\frac{l}{b_\Omega}}$ it is positive and paramagnetic.
- For $r > \sqrt{\frac{l}{b_\Omega}}$ it is negative and diamagnetic.

The current flows with a different chirality in regions separated by a ring with radius $r_l = \sqrt{(\frac{l}{\Phi_\Omega})}$. We define the bulk and edge regions using this particular value of r_l as a reference for a given angular momentum.

The chiral boundary condition is defined as:
If $\lambda \geq 0$ which implies $0 < \Phi_\Omega \leq l$ (edge states)

$$\partial_r\psi_{n,l}|_R = 0 \tag{1}$$

If $\lambda < 0$ which implies $l < \Phi_\Omega$ (bulk states)

$$(\frac{\partial}{\partial r} + \frac{i\partial}{r\partial\theta} + b_\Omega r)\Psi_{n,l}|_{r=R} = 0 \tag{2}$$

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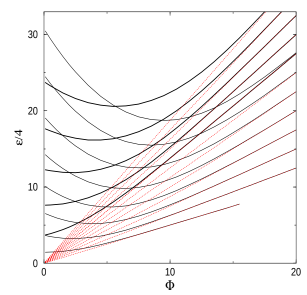
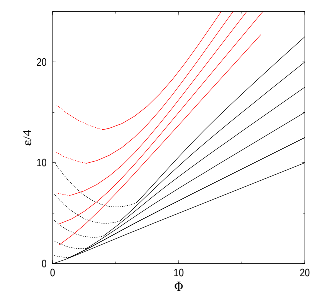
Results

3.1 Spectrum with DBC and CBC

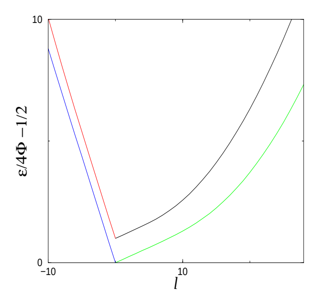


- Spectrum with DBC. Plots for $n = 0$ and $n = 1$ with first few l states. $\Omega = 0.75\omega_\perp$

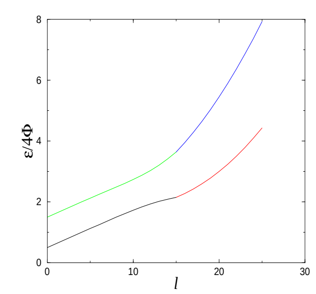


- Spectrum with CBC. Plots for $n = 0$ and $n = 1$ again for first few l states. $\Omega = 0.75\omega_\perp$

3.2 Bulk and Edge states with DBC and CBC

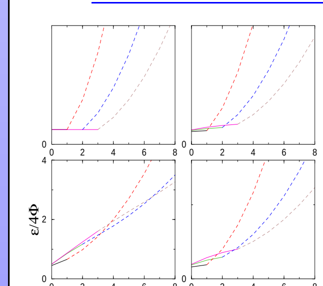


- Plots for $n = 0$ and $n = 1$ with first few l states $\Omega = 0.75\omega_\perp$

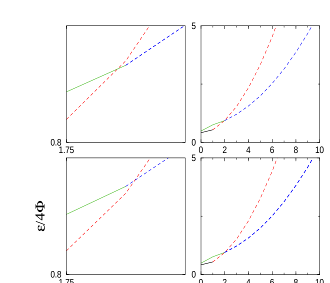


- Spectrum with CBC. Plots for $n = 0$ and $n = 1$ and $\Phi_\Omega = 15$, $\Omega = 0.75\omega_\perp$

3.3 Vortex Nucleation with CBC



- From bottom-left picture counterclockwise the bulk + edge spectrum corresponds to $\frac{\Omega}{\omega_\perp} = 0.25, 0.50, 0.75, 1$ are shown respectively .Each fig. corresponds to $\Phi_\Omega = 1, 2, 3$



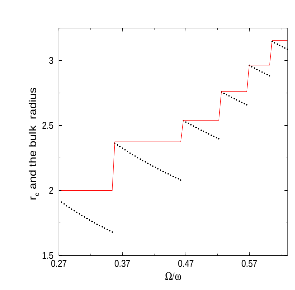
- The top and bottom set of figures correspond to $\frac{\Omega}{\omega_\perp} = 0.36, 0.35$. On the left hand side the region near point of intersection of two spectrum corresponds to $\Phi_\Omega = 1, 2$ is zoomed in.

3.4 successive vortex Nucleation, Size variation

Vortex	$\frac{\Omega}{\omega}$
1st	0.35 - 0.36
2nd	0.46-0.47
3rd	0.52-0.53
4th	0.569-0.57
5th	$0.603 - 0.604$
6th	$0.629 - 0.630$

No. of states transferred to the bulk	$\frac{\Omega}{\omega}$
2 (1, 2)	0.44 - 0.45
3 (1, 2, 3)	0.48-0.49
4 (1, 2, 3, 4)	0.494-0.495
5 (1, 2, 3, 4, 5)	0.498-0.499

- *Nucleation frequencies of successive ($l = 1$) vortices. (Above)*
- *Nucleation frequencies of multiple vortices. In parentheses we mention the angular momentum states which are transferred from the edge to the bulk at these values of Ω.(Below)*



- Change in the size of the condensate with increasing rotational frequency.

Discussion

4 Summary

- The problem of vortex nucleation is studied within the framework of independent bosons satisfying the **linear Schrödinger equation** with a set of non-local and chiral boundary conditions.
- These boundary conditions split the one particle Hilbert space into a direct sum of two orthogonal, *infinite dimensional* spaces with positive and negative chirality on the boundary. They correspond to bulk and edge states respectively. The chirality is determined from the direction of the azimuthal velocity on the boundary.
- The vortex nucleation is then interpreted as a transfer of angular momentum from the edge to the bulk.
- Physically our procedure resembles the **edge to bulk** quasiparticle transport in Quantum Hall effect suggested by Laughlin (1981) and Halperin (1982)[11]. The role of the edge states is similar to those of the Meissner states in a finite superconducting systems.
- The calculated critical frequency of first vortex nucleation lies in between $0.35\omega_\perp$ and $0.36\omega_\perp$.
- We have also studied the vortex nucleation at frequencies higher than the critical frequency of the first vortex nucleation.
- The size variation of the bulk region demonstrated is qualitatively similar to shape deformation and associated angular momentum transfer under a rotational perturbation suggested by Stringari and others [9, 10] as a possible mechanism of vortex nucleation. However here it is demonstrated for a circular geometry whereas in experiment the system is generally elliptical
- Unlike the infinite system a finite Bose-Einstein system has a surface energy and the corresponding surface tension. With the variation of condensate size this surface tension will also vary and may have observable consequences.

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