N fermionic atoms trapped in two spin states ↑, ↓ (N↑=N↓) by a spherical harmonic potential $\sum_i \frac{1}{2} m\omega^2 \vec{r}_i^{\,2}$

Zero-range atom-atom interaction:

$$\frac{4\pi\hbar^2 a}{m}\sum_{i<j}\delta^3\left(\vec{r}_i-\vec{r}_j\right)$$

a: s-wave scattering length (dilute gas)

$$g \equiv \frac{4\pi\hbar^2 a}{m}$$

g: coupling constant

Trap units: $\hbar = m = \omega = K_B = 1$

Hartree-Fock-Bogoliubov or Bogoliubov-de Gennes equations at finite temperature:

$$\left[H_0 + W(\vec{R})\right]u_\alpha\left(\vec{R}\right) + \Delta\left(\vec{R}\right)v_\alpha\left(\vec{R}\right) = E_\alpha u_\alpha\left(\vec{R}\right)$$

$$\Delta\left(\vec{R}\right)u_\alpha\left(\vec{R}\right) - \left[H_0 + W\left(\vec{R}\right)\right]v_\alpha\left(\vec{R}\right) = E_\alpha v_\alpha\left(R\right)$$

$$H_0 = T + \frac{1}{2}r^2 - \mu$$

T: kinetic term

μ: chemical potential

W: Hartree-Fock mean-field; Δ: pairing field

u, v: two-component quasiparticle wave functions

E: quasiparticle energies

Zero-range interaction:

$$\left\langle \Psi_\uparrow\left(\vec{R}+\frac{\vec{r}}{2}\right)\Psi_\downarrow\left(\vec{R}-\frac{\vec{r}}{2}\right)\right\rangle$$ **diverges as** $$\Delta/(4\pi r)$$ **when r->0**

PROBLEM IN CALCULATING THE PAIRING FIELD Δ

Regularization procedure:

The Green's function G^0_μ associated to the Hamiltonian H_0 diverges as $1/(2\pi r)$ **when r->0.**

Pseudo-potential prescription [1]:

$$\Delta(\vec{R}) = -g \lim_{r\to 0}\frac{\partial}{\partial r}\left\{ r\left[\left(\langle \Psi_\uparrow \Psi_\downarrow\rangle - \frac{\Delta(\vec{R})}{2}G^0_\mu(\vec{R},\vec{r})\right) + \frac{\Delta(\vec{R})}{2}G^0_\mu(\vec{R},\vec{r})\right]\right\}$$

One has to calculate the regular part of G^0_μ [2]. To simplify: Thomas-Fermi approximation [3, 4].

$$G^0_\mu(\vec{R},\vec{r}) \approx \sum_{\substack{nlm \\ \varepsilon^0_{nl}\le\varepsilon_C}} \frac{\Phi^0_{nlm}\left(\vec{R}+\frac{\vec{r}}{2}\right)\Phi^{0*}_{nlm}\left(\vec{R}-\frac{\vec{r}}{2}\right)}{\varepsilon^0_{nl}-\mu} + \int_{k_C(R)}^{+\infty}\frac{d^3k}{(2\pi)^3}\frac{e^{i\vec{k}\cdot\vec{r}}}{\frac{k^2}{2}+\frac{R^2}{2}-\mu}$$

with: $$k_C(R) = \sqrt{2\varepsilon_C - R^2}$$ **;** ε_C **cutoff**

Then:

$$G^{0reg}_\mu(\vec{r}) \approx \sum_{\substack{nl \\ \varepsilon^0_{nl}\le\varepsilon_C}} \frac{(2l+1)R^2_{nl}(r)}{4\pi(\varepsilon^0_{nl}-\mu)} + \frac{k^0_F(r)}{2\pi^2}\times\ln\frac{k_C(r)+k^0_F(r)}{k_C(r)-k^0_F(r)} - \frac{k_C(r)}{\pi^2}$$

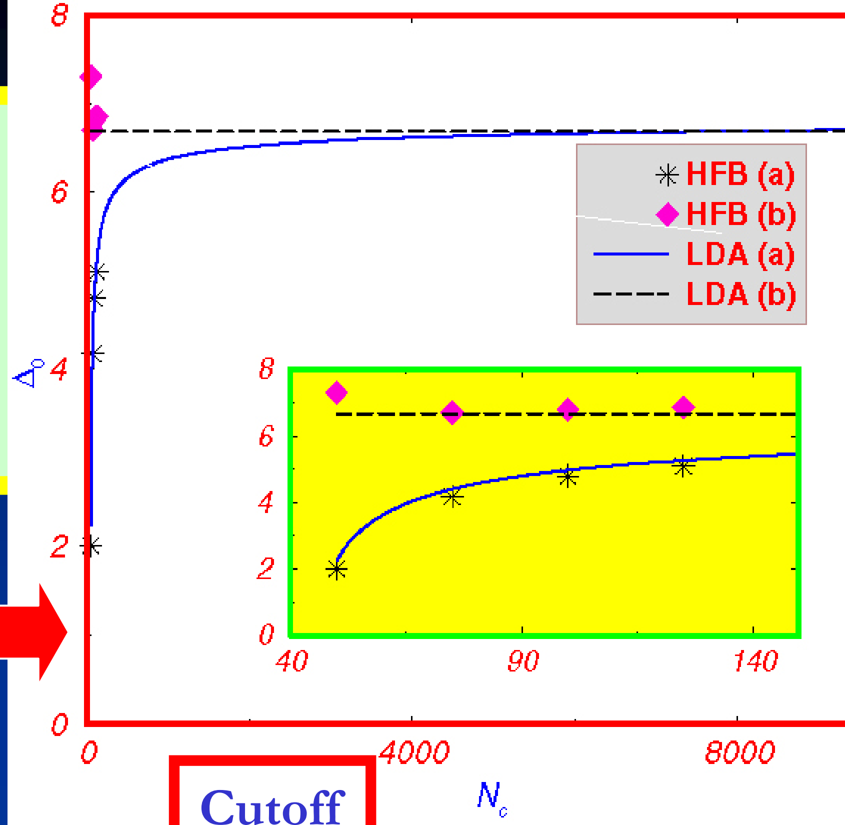
[1] K. Huang, Statistical Mechanics (Wiley, New York, 1987); [2] G. Brunn, et al., Eur. Phys. J.D. 7, 433 (1999); [3] A. Bulgac and Y. Yu, Phys. Rev. Lett. 88, 042504 (2002); [4] Marcella Grasso and Michael Urban, Phys. Rev. A 68, 033610 (2003)

Two possible choices:

(a) $$k_F^0(r) = \sqrt{2\mu - r^2}$$

(b) $$k_F^0(r) = \sqrt{2\mu - r^2 - 2W(r)}$$

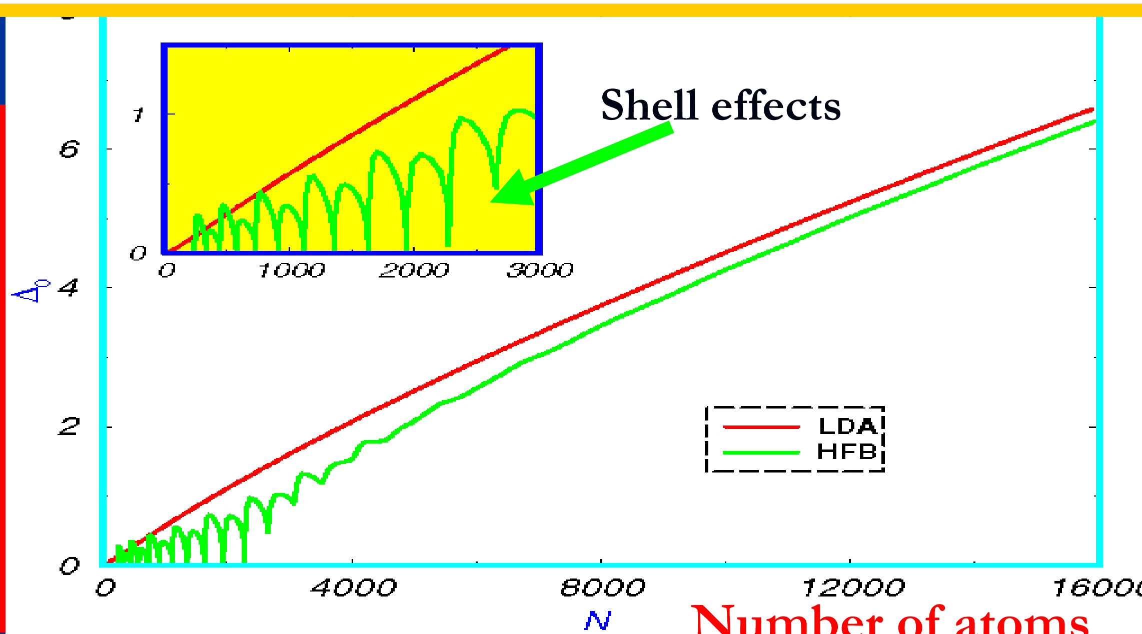
Pairing field at the center of the trap (ħω)



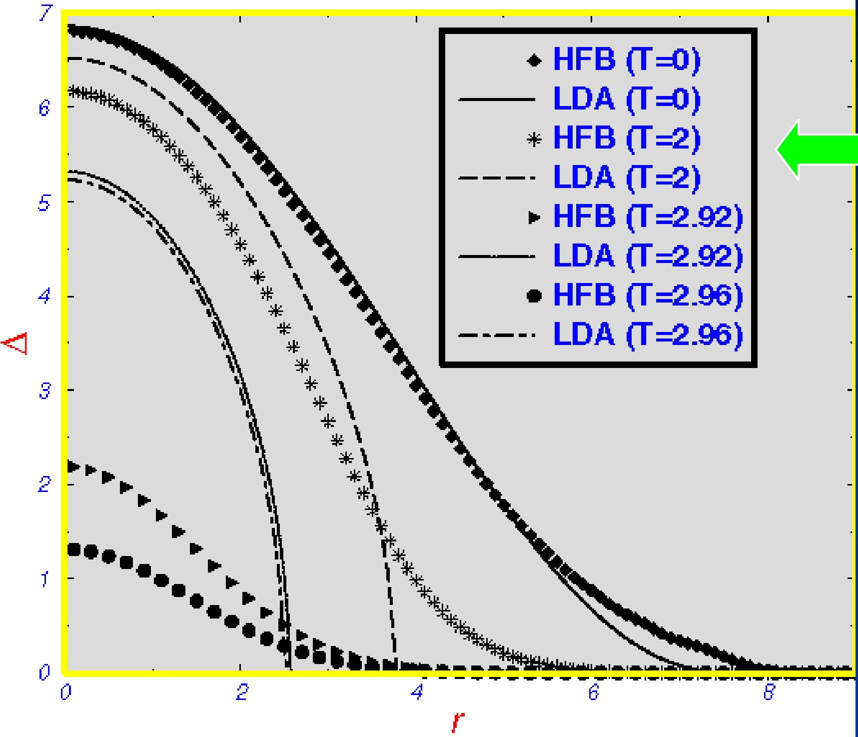
The regularization method (b) converges much faster

HFB AND LDA AT T=0: good agreement for systems with a large value of N

Pairing field at the center of the trap (ħω)



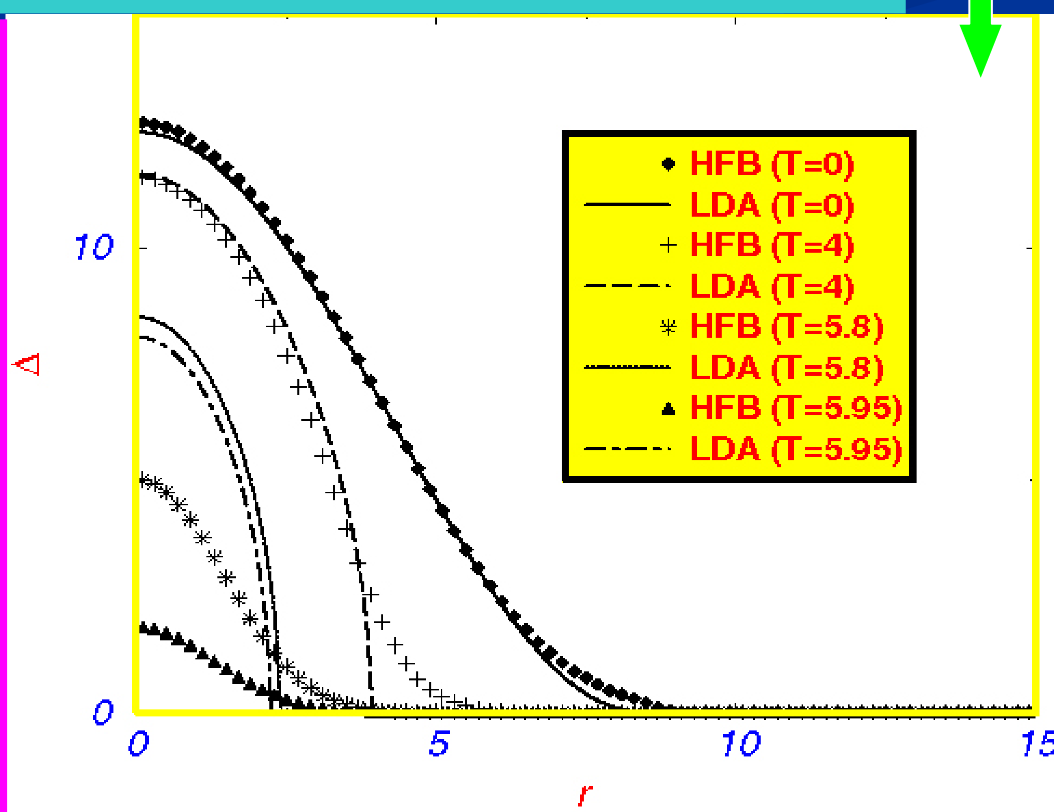
HFB and LDA at nonzero temperature



$N \approx 1.7 \times 10^4$

$N \approx 4 \times 10^4$

Δ: pairing field (ħω); r: radius $((\hbar/(m\omega))^{1/2})$



LDA overestimates the central value of Δ, cuts too drastically the tail of the radial profile of Δ at large distances and overestimates the critical temperature with respect to HFB