2-Body Descriptions of Bose and Fermi Gases

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Support: NSF

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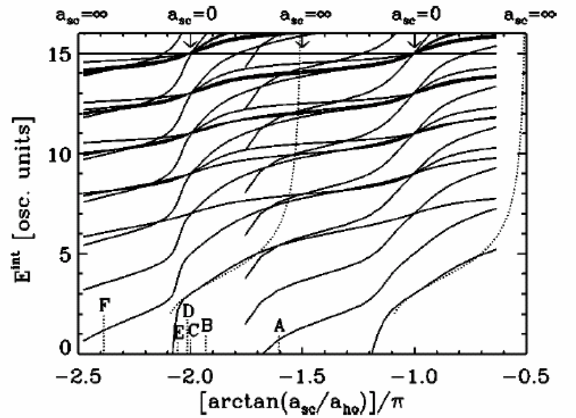
Topics:

1. What is a Feshbach resonance, really? (And what is not.)
2. Atom-molecule condensate oscillations in a Bose gas
3. Feshbach resonance cooling of two trapped atoms
4. Pair formation in a degenerate Fermi gas with attraction
5. Sub-Doppler laser cooling of a fermionic alkaline earth gas

This talk is an overview of the behavior of the two-body scattering and bound state properties that arise in the vicinity of a low-energy Feshbach resonance. It is aimed at clarifying what aspects of the two-body physics must be incorporated in order to correctly describe many-body phenomena in Bose-Einstein condensates or degenerate Fermi gases.

Three-body physics I will NOT talk about today:



PHYSICAL REVIEW A **66**, 013601 (2002)

Three particles in an external trap: Nature of the complete $J=0$ spectrum

D. Blume[1] and Chris H. Greene[2]

VOLUME 90, NUMBER 5 PHYSICAL REVIEW LETTERS week ending 7 FEBRUARY 2003

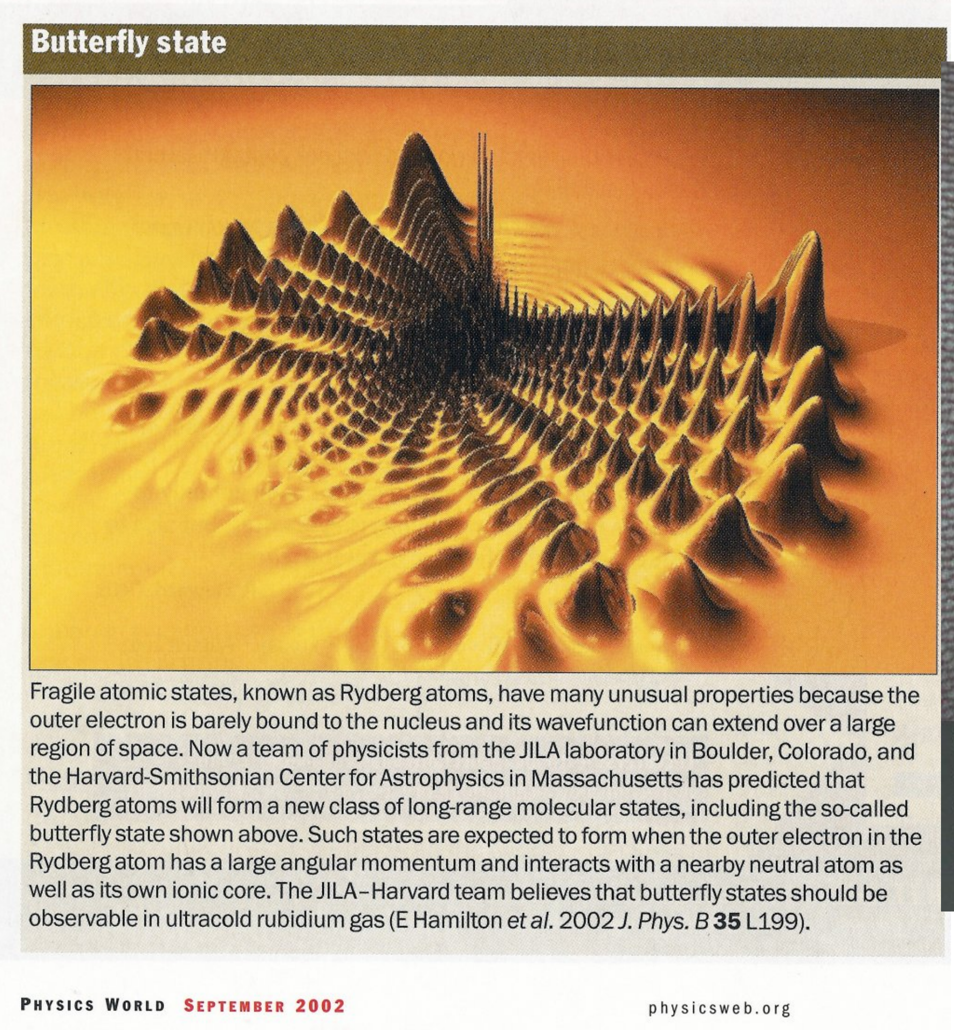
Recombination of Three Ultracold Fermionic Atoms

H. Suno,[1] B. D. Esry,[1] and Chris H. Greene[2]

A new class of ultra-long-range Rydberg molecules predicted

Shape-resonance-induced long-range molecular Rydberg states, E Hamilton and C Greene, J Phys. B Letter, May 2002. (Also Greene et al. PRL, Sept. 2000)

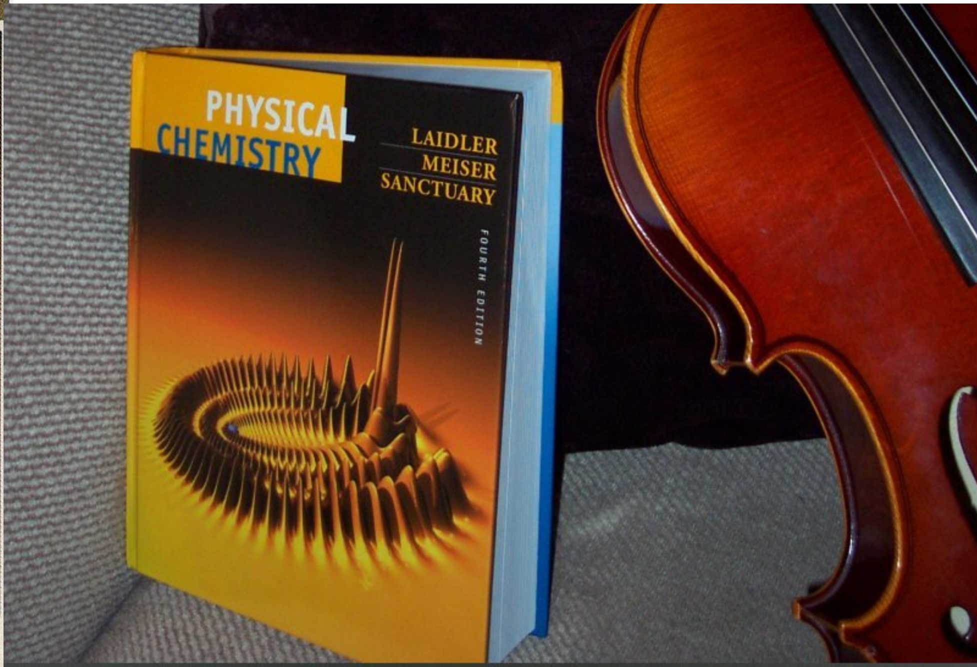
“Butterfly” and “Trilobite” Rydberg molecules (with one Rydberg alkali atom and one ground state atom)



Butterfly state

Fragile atomic states, known as Rydberg atoms, have many unusual properties because the outer electron is barely bound to the nucleus and its wavefunction can extend over a large region of space. Now a team of physicists from the JILA laboratory in Boulder, Colorado, and the Harvard-Smithsonian Center for Astrophysics in Massachusetts has predicted that Rydberg atoms will form a new class of long-range molecular states, including the so-called butterfly state shown above. Such states are expected to form when the outer electron in the Rydberg atom has a large angular momentum and interacts with a nearby neutral atom as well as its own ionic core. The JILA–Harvard team believes that butterfly states should be observable in ultracold rubidium gas (E Hamilton *et al.* 2002 *J. Phys. B* **35** L199).

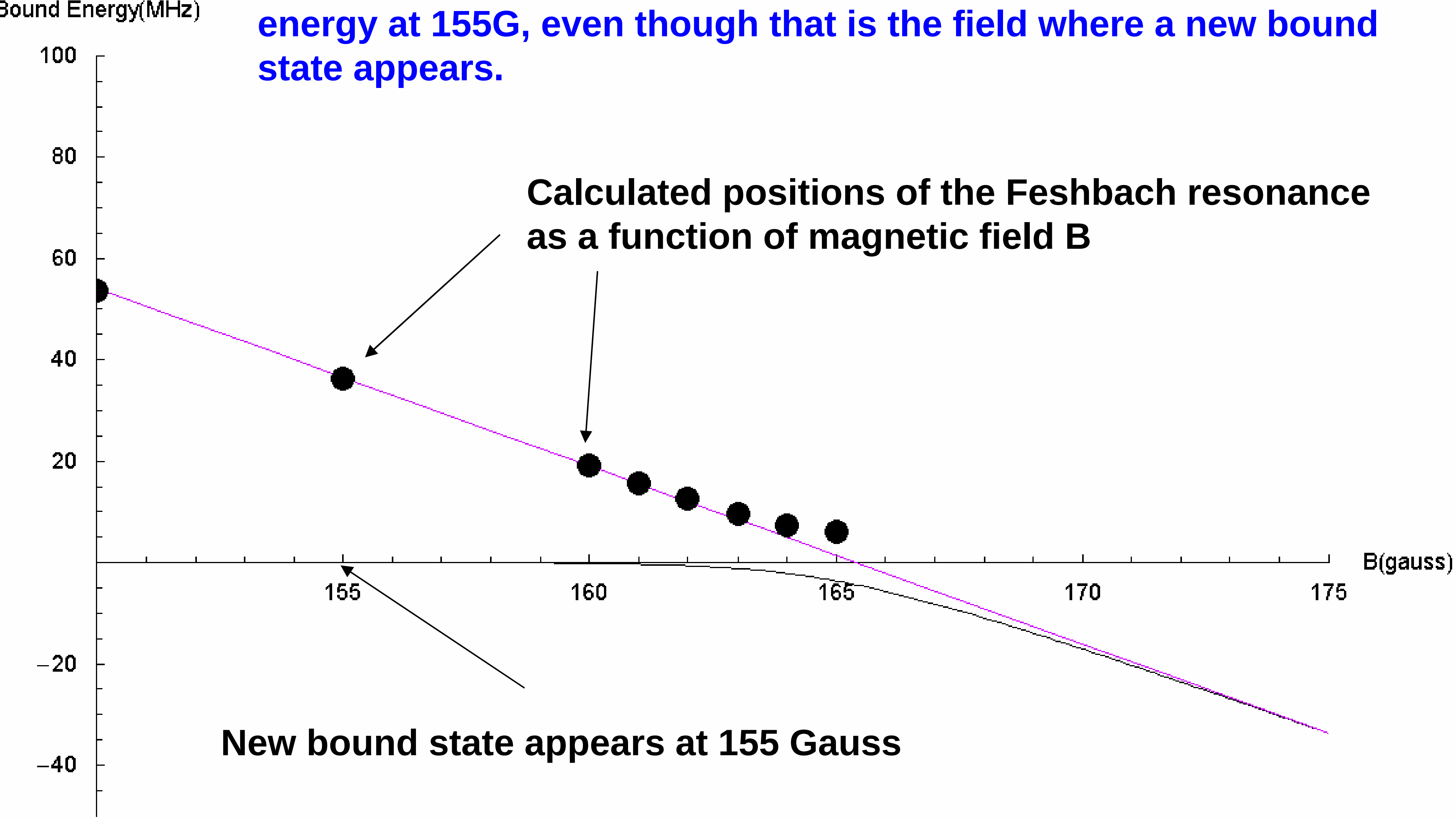
Physics World **September 2002** physicsweb.org



On the cover is a trilobite-like long-range Rydberg rubidium dimer showing a cylindrical coordinate surface plot of the electronic probability density for the lowest Born-Oppenheimer state. The equilibrium internuclear distance is $R = 1232$ a. u. for the $^3\Sigma$ perturbed hydrogenic state, with $n = 30$. The Rb(5s) atom is beneath the towers to the right and the Rb$^+$ ion is represented as a small blue sphere to the left. From the work of C. H. Greene, A. S. Dickinson, and H. R. Sadeghpour, *Phys. Rev. Lett.*, 85, 2458-2461 (2000).

Motivation: the ^{85}Rb – ^{85}Rb Feshbach resonance near B=155 Gauss

Note that the “resonance”, as defined by where the bound state portion of the wavefunction is a maximum, does not lie at zero energy at 155G, even though that is the field where a new bound state appears.



Calculated positions of the Feshbach resonance as a function of magnetic field B

New bound state appears at 155 Gauss

Historical Overview

1. O. K. Rice, JCP 1, 375 (1933) – basic treatment of how a bound state autoionizes into a degenerate continuum
2. U. Fano, Nuovo Cimento 12, 156 (1935) – shows that quantum interference has opposite signs above and below the resonance, leading to asymmetric line profiles analogous to anomalous dispersion.
3. H. Beutler, Z. Physik 93, 177 (1935) – experimental observation of highly asymmetric line profiles in rare gas photoionization spectra
4. G. Breit and E. Wigner, Phys. Rev. 49, 519 (1936) – Basic formula developed for symmetric resonance profile when only the “bound part” of the reaction dominates – i.e., showing no asymmetry in this case.
5. Other nuclear theorists treat interference of “direct path” and “resonance path” for scattering, giving Beutler-Fano-type asymmetric lineshapes, e.g. Blatt & Weisskopf, Theoretical Nuclear Physics, 1952.
6. H. Feshbach, Ann. Phys. 5, 357 (1958) and 19, 287 (1962) – developed general projection operator formalism that cleanly separates “bound” and “continuum” subspaces and systematically treats their interaction.
7. U. Fano, Phys. Rev. 124, 1866 (1961) – more elegant reformulation of his 1935 theory of asymmetric line profiles from discrete-continuum interactions
8. P. Anderson, Phys. Rev. 124, 41 (1961) – model of localized impurity state in a continuous band.

Feshbach resonances in neutron-sulfur scattering, from Blatt&Weisskopf, 1952

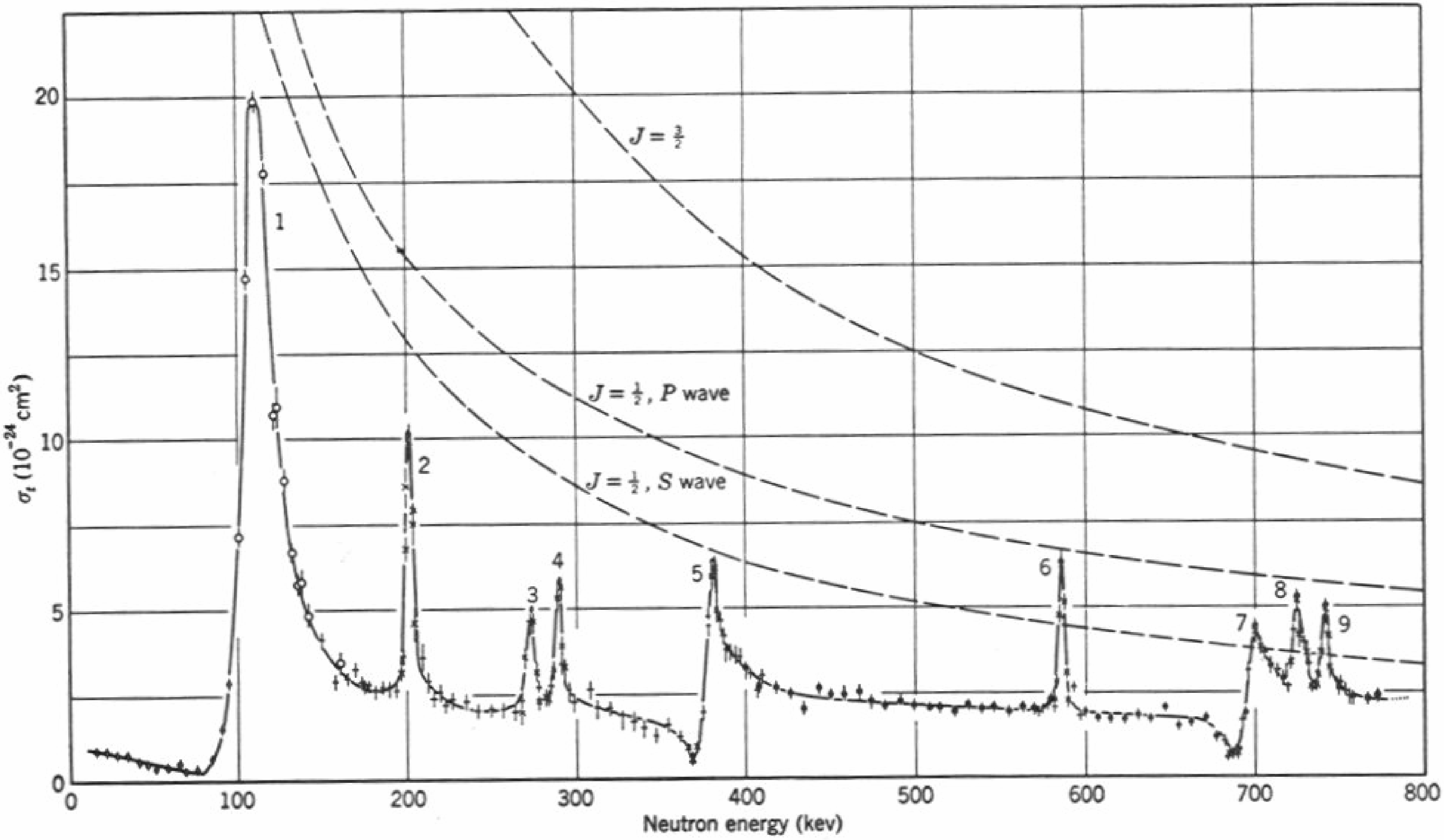


FIG. 2.2. Total neutron cross section for sulfur; experimental data taken from Adair (49) and Peterson (50).

468 IX. Nuclear Reactions; Comparison with Experiments

For a review showing many Feshbach resonances seen in photoionization, see Aymar, Greene, and Luc-Koenig, Rev. Mod. Phys. 1996.

Wigner and Eisenbud's "R-matrix" Theory (1947 Phys. Rev.) made it very simple to characterize near-threshold resonances and threshold nonanalyticities. Let any independent solution of the 2-body Schrodinger equation regular at the origin be written as a channel expansion:

$$\boxed{\Psi(r,\Omega) = \sum_i \Phi_i(\Omega) F_i(r)}$$

In this representation the full TISE $H\Psi(r,\Omega) = E\Psi(r,\Omega)$ becomes

$$\boxed{-\frac{\hbar^2}{2m}\vec{F}''(r) + (\underline{V}(r) - E\underline{I})\vec{F}(r) = 0.}$$

If we call r_0 a radius beyond which all interactions can be neglected, the R-matrix is defined as

$$\boxed{R(E) = \underline{F}(r_0)[\underline{F}'(r_0)]^{-1}.}$$

The key result from Wigner and Eisenbud: $R(E)$ must be meromorphic, with its poles on the real energy axis. Implications are that, for short-range forces, $R(E)$ is approximately energy-independent and can be taken to be constant throughout the ultracold. Consequently, this predicts the threshold behavior of T-matrixes to be:

$$\boxed{Tij = S_{ij} - \delta_{ij} \propto k_i^{l_i+\frac{1}{2}} k_j^{l_j+\frac{1}{2}}}$$

The characteristic energy range relative to threshold over which this type of approximation is quantitatively accurate is $|\Delta E| \lesssim \frac{\hbar^2}{2mr_0^2}$ with r_0 the range of the potential (van der Waals radius)

Suppose we go ahead and solve for the S-wave scattering phaseshift in terms of the (assumed constant) 1-channel R-matrix, $R(E) \to R \simeq$ constant. Then this one constant determines S-wave **scattering properties**, at positive energy $E = \frac{\hbar^2 k^2}{2m}$:

Since $F(r) = N\sin(kr + \delta)$, at $r > r_0$, we demand that logarithmic derivatives match:

i.e.

$$\boxed{-\frac{F(r_0)}{F'(r_0)} = R = \frac{\sin(kr_0 + \delta)}{-k\cos(kr_0 + \delta)} = -\frac{\tan(kr_0 + \delta)}{k}}$$

whereby the scattering phaseshift and elastic scattering cross section are

$$\delta = -kr_0 - \tan^{-1}(kR) \simeq \tan\delta \to -ka, \text{ at } kr_0 << 1 \text{ and } ka << 1.$$

$$\sigma^{elastic}_{\text{identical}} = \frac{8\pi}{k^2}\sin^2\delta \to 8\pi\frac{a^2}{1 + k^2a^2} \text{ and the S-matrix is } S \to \frac{1 - ika}{1 + ika}.$$

Here the zero-energy scattering length is $a = r_0 + R$.

This same analysis now also determines the bound state properties, since we can find the energy at which the long range solution has no exponential growth:

$$\boxed{F(r) = N' e^{-\kappa r}, \text{ where } \kappa = +\sqrt{\frac{-2mE}{\hbar^2}},}$$

that is, we demand that

$$\boxed{-\frac{F(r_0)}{F'(r_0)} = \frac{1}{\kappa} = R \simeq a, \text{ provided } |a| >> r_0, \text{ or } E \simeq -\frac{\hbar^2}{2ma^2}}$$

Behavior of scattering length versus well depth for a potential D $\mathrm{sech}^2(r)$, Poschl-Teller potential.

This is a single-channel problem, with **no Feshbach resonances**.

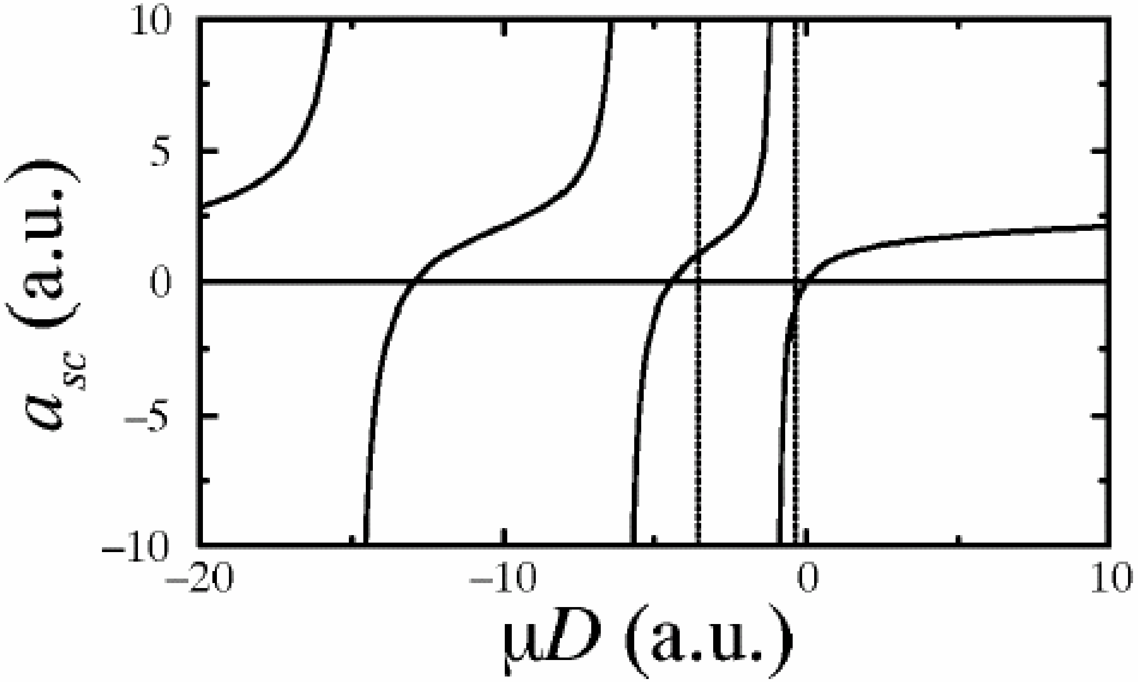


Figure 1. The two-body scattering length as a function of reduced mass-scaled potential depth, μD, for the potential $v(r) = D\ \mathrm{sech}^2 r$, with all quantities in au. The dotted lines enclose the parameter range considered in our numerical studies. A stable condensate is predicted to occur in conventional formulations for $\mu D > 0$.

Note: The above development is all for a single channel possessing no Feshbach resonances.

In practice, what usually ALLOWS the scattering length to be TUNABLE is the nearby proximity of a Feshbach resonance whose position is B-field dependent, which means that **a=a(B)**

So, the question remains, WHAT IS A FESHBACH RESONANCE?

ANSWER: A Feshbach resonance is the presence of a temporary trapping of the system at short range, in a different configuration. In particular the trapping of the system should be (at least) somewhat longer than the time it takes a particle to come in and go out in a nonresonant, ordinary single-channel collision.

Mathematically, a Feshbach resonance is a peak in the time delay,

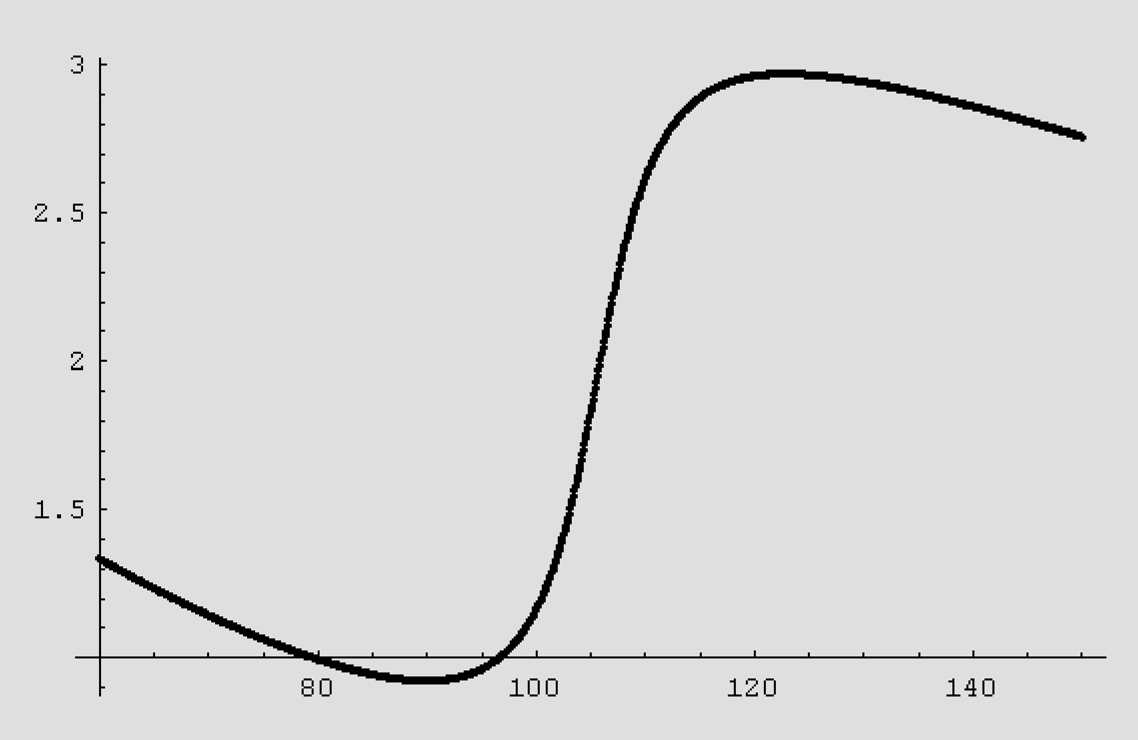
$$\boxed{Q = 2\hbar \frac{d\delta}{dE}, \text{ provided } Q > 2\,\mathrm{Re} \int^{r_0} \frac{dr}{v(r)} = 2\int^{r_0} \frac{dr}{\sqrt{2(E - V(r))/m}}}$$

An alternative, and somewhat more general definition of a Feshbach resonance is that it is a POLE of the SCATTERING MATRIX occuring on the physical sheet (i.e. with no exponentially growing components at $r \to \infty$), and at positive energy, Re(E)>0. Our one-channel derivation has no Feshbach resonance

$$\boxed{S \simeq \frac{1 - ika(B)}{1 + ika(B)}.}$$

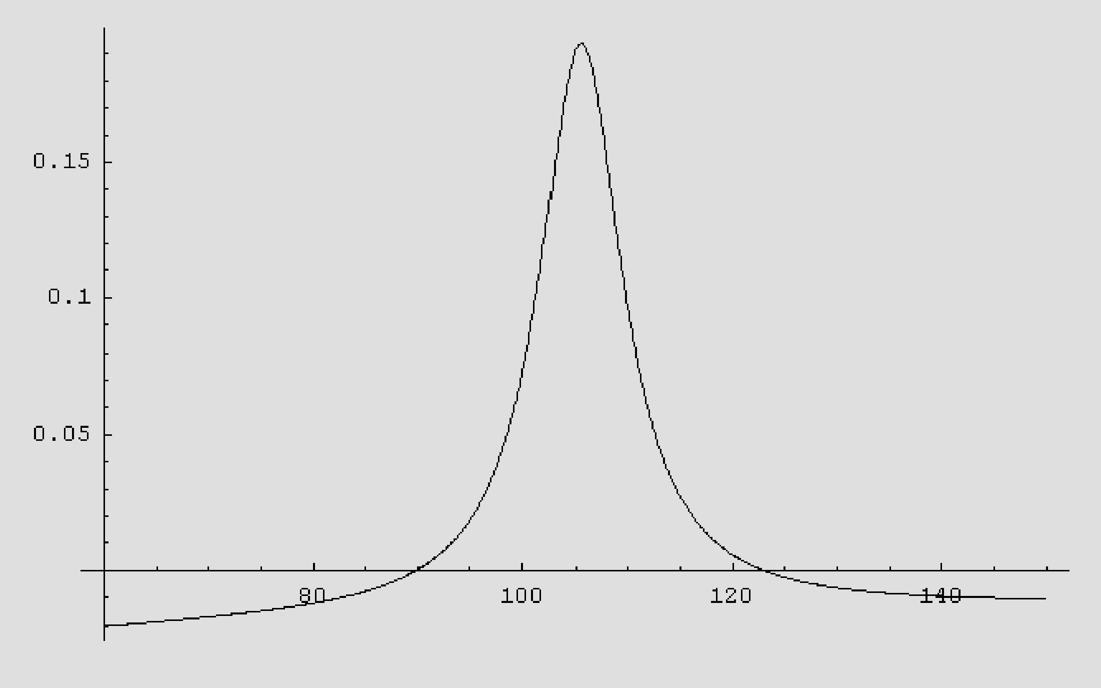
The only poles of this S-matrix occur when the denominator vanishes, i.e. at $k = \frac{i}{a(B)}$. When $a(B) > 0,$ this gives a TRUE MOLECULAR BOUND STATE whose wavefunction decays exponentially.

When $a(B) < 0,$ this gives a VIRTUAL STATE, which corresponds to pure exponential GROWTH rather than decay. (This is not a physical state, in any sense.)

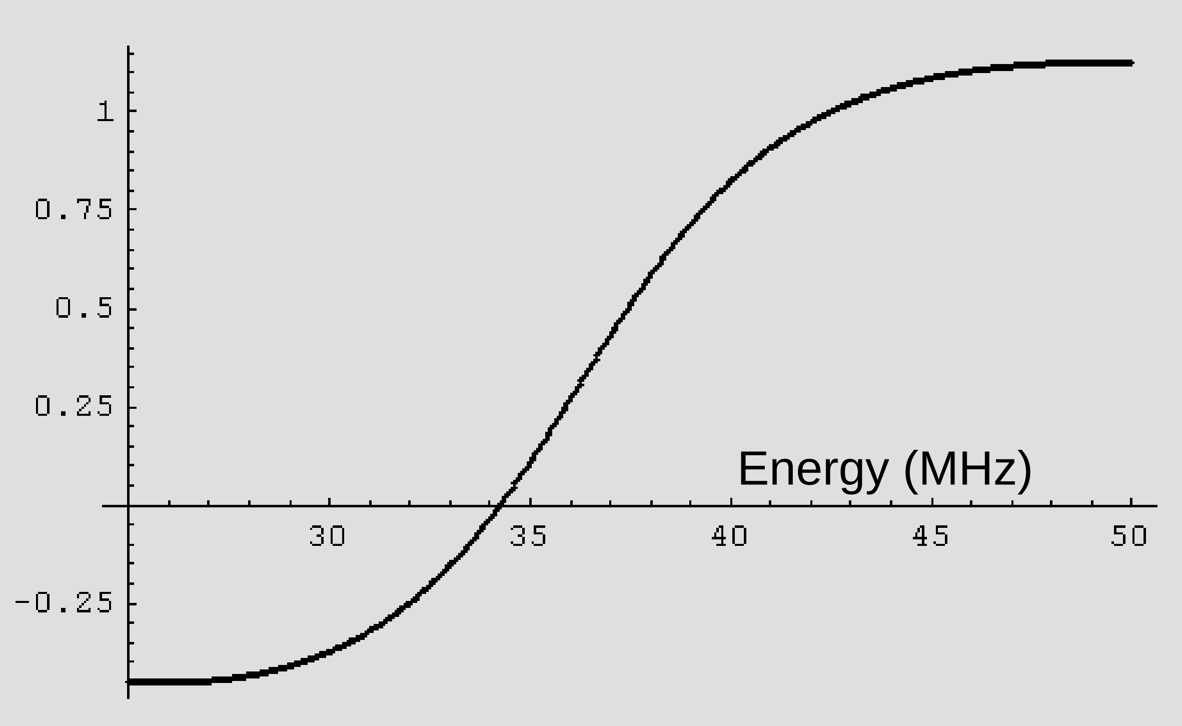


Let's look at the ^{85}Rb-^{85}Rb Feshbach resonance in more detail.

← The S-wave scattering phaseshift for ^{85}Rb-^{85}Rb at B=135 G, as a function of energy in MHz.

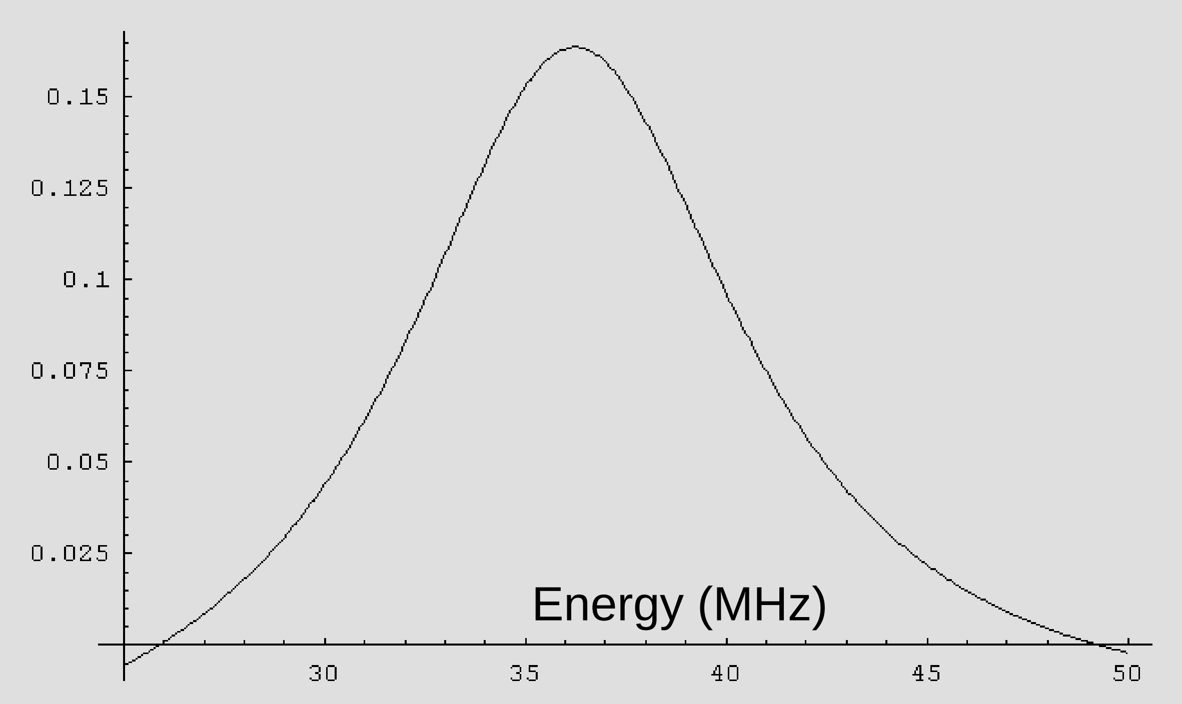


The energy derivative of the S-wave phaseshift at B=135 G versus the energy in MHz. Scattering theory defines the "resonance position" as the energy at which this curve is a local maximum.



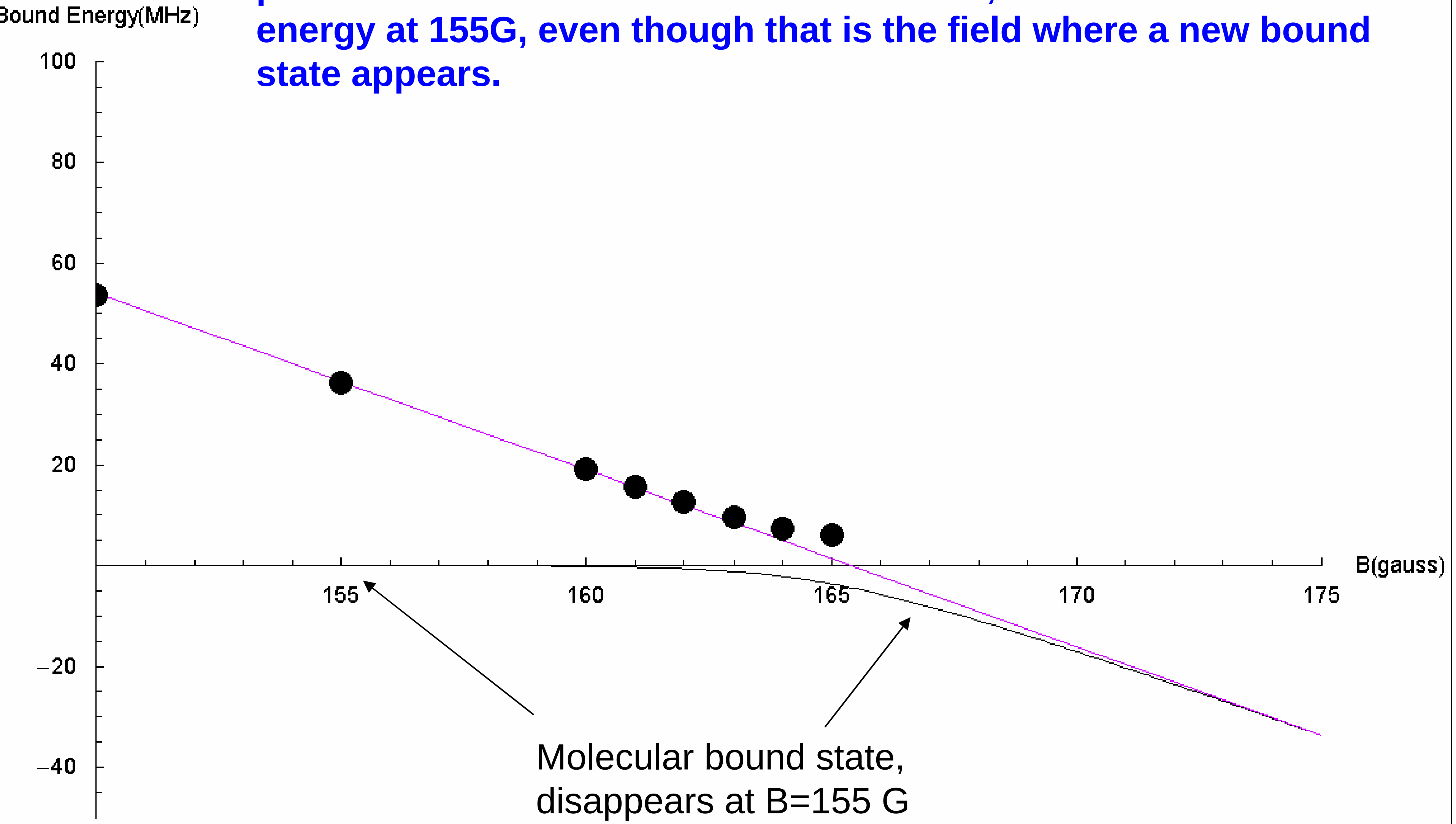
← The S-wave scattering phaseshift for 85Rb-85Rb at B=155 G, as a function of energy in MHz.

The energy derivative of the S-wave phaseshift at B=155 G versus the energy in MHz. Scattering theory defines the "resonance position" as the energy at which this curve is a local maximum.



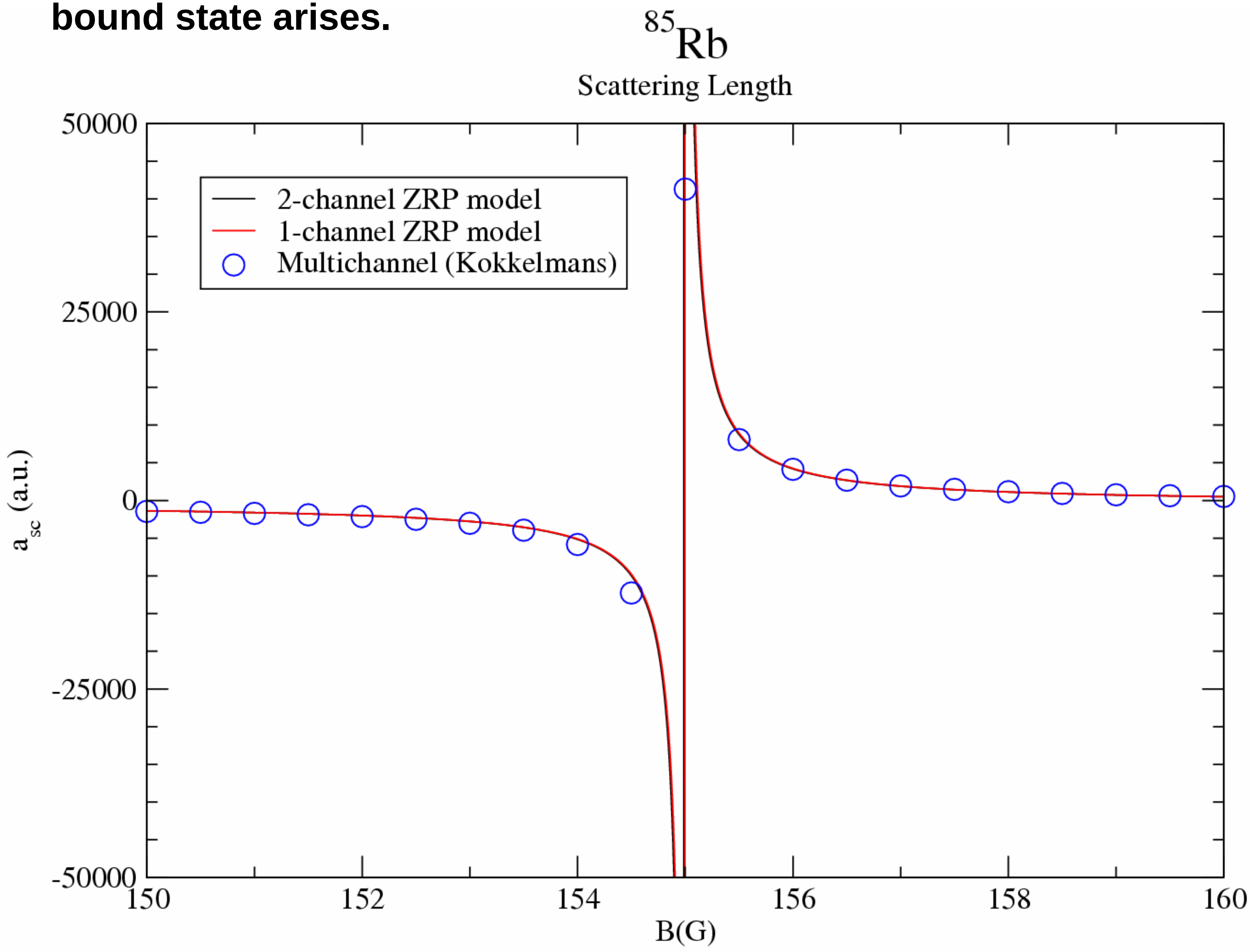
A study of the ^{85}Rb – ^{85}Rb Feshbach resonance near B=155 Gauss

Note that the “resonance”, as defined by where the bound state portion of the wavefunction is a maximum, does not lie at zero energy at 155G, even though that is the field where a new bound state appears.



Molecular bound state, disappears at B=155 G

Comparison of 3 different ways of describing the 85Rb-85Rb scattering length, near the magnetic field (B=155G) where a new bound state arises.



The main result of this multichannel effective range theory is that for sufficiently low energies, close to the point B_0, the scattering phaseshift is accurately given throughout the nearby complex E,B planes by the expression:

$$\boxed{-\frac{\tan\delta}{k} \equiv a(E,B) = a_{bg}\left(1-\frac{\Delta}{B-B_0+\gamma E}\right)}$$

The main way this differs from the expression usually adopted in ultracold collisions and BEC studies is the presence of an energy dependent resonance slope in the resonance denominator. The slope γ is typically of order 0.2 to 0.5 Gauss/MHz.

The energy range over which this expression holds is again typically about

$$\boxed{|\Delta E| \lesssim \frac{\hbar^2}{2mr_0^2} \simeq \frac{\hbar^2}{2m}\left(\frac{\hbar^2}{2mC_6}\right)^{1/2}}$$

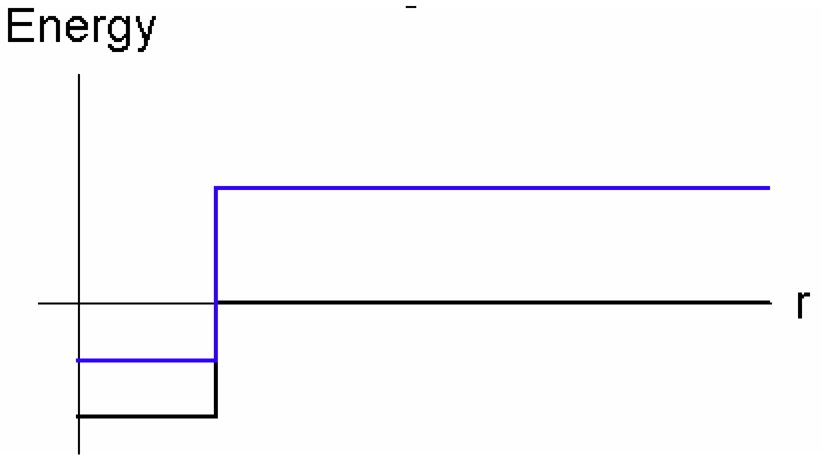
Let's examine a two-channel model for which there IS a Feshbach resonance, and see what is the interconnection between the scattering properties, the molecular bound state properties, and the resonance.

$$H = -\frac{\hbar^2}{2m}\underline{1}\frac{d^2}{dr^2} + \underline{V}(r) + \underline{E}^{th}.$$

$$\underline{V}(r) = \begin{pmatrix} -V_1 & V_{12} \\ V_{12} & -V_2 \end{pmatrix} \theta(r_0 - r). \qquad \underline{E}^{th} = \begin{pmatrix} 0 & 0 \\ 0 & E_2^{th} \end{pmatrix}$$

$$\underline{W} = \left(-\frac{2m}{\hbar^2}\right)\left(\underline{V} + \underline{E}^{th} - \varepsilon\underline{1}\right) = \left(\frac{2m}{\hbar^2}\right)\begin{pmatrix} \varepsilon + V_1 & -V_{12} \\ -V_{12} & \varepsilon - E_2^{th} + V_2 \end{pmatrix}$$

Note: Can solve this model problem exactly by diagonalizing the above constant matrix, and then matching the trigonometric solutions at r<r0 to a scattering solution at r>r0 in channel 1 and a decaying solution in channel 2.



The solution at r>r_0 thus has the form:

$$\vec{\psi}^{phys}(r) = \begin{pmatrix} e^{ikr}S - e^{-ikr} \\ Ne^{-qr} \end{pmatrix}$$

And we can solve exactly for the S-matrix:

$$\boxed{S = e^{-2ikr_0} \frac{R_{12}^2 - R_{11}R_{22} - qR_{11} - ik(q+R_{22})}{R_{12}^2 - R_{11}R_{22} - qR_{11} + ik(q+R_{22})}.}$$

Because the closed channel threshold is assumed to be so far away in energy, we can extract the ultracold scattering behavior by using a linear expansion of the closed channel wavenumber in energy and in magnetic field,

$$q(\varepsilon, B) \simeq q_0 + \zeta k^2 + \gamma(B - B_0)$$

The poles of this S-matrix in the complex k-plane or energy-plane are now readily determined as the roots of a cubic equation, for any chosen B-field (written in terms of B'=B-B$_o$. Note that B'=0 is the point where the new bound state appears or disappears:

$$ikX + (Y - ik)\left(ZB' + k^2\right) = 0.$$

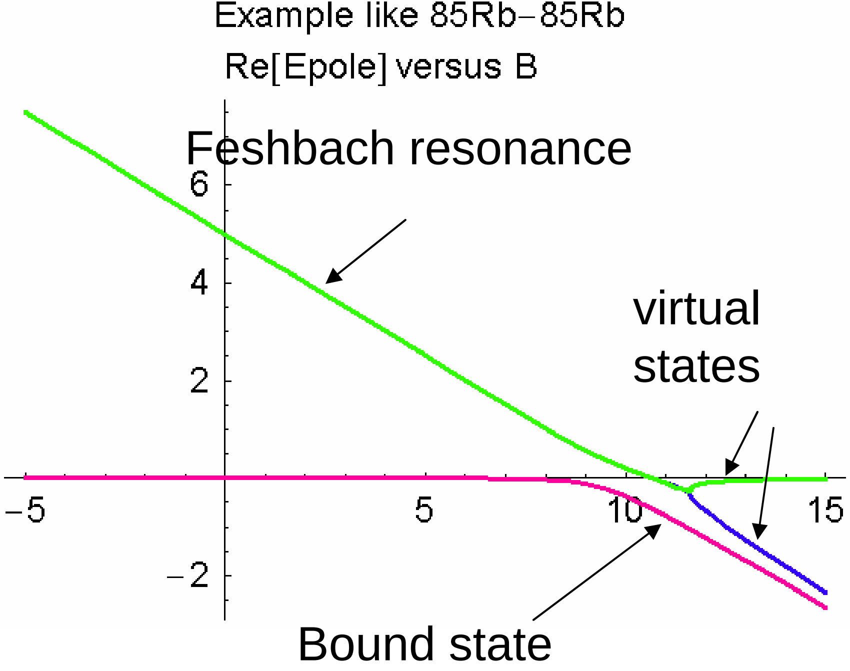
There are just 3 different real parameters that control the behavior of these S-matrix poles in the complex plane which determine the interplay of molecular bound states, resonances, and virtual states:

the parameter $X \equiv (R_{12}^2/\zeta R_{11})$ is a measure of the coupling strength

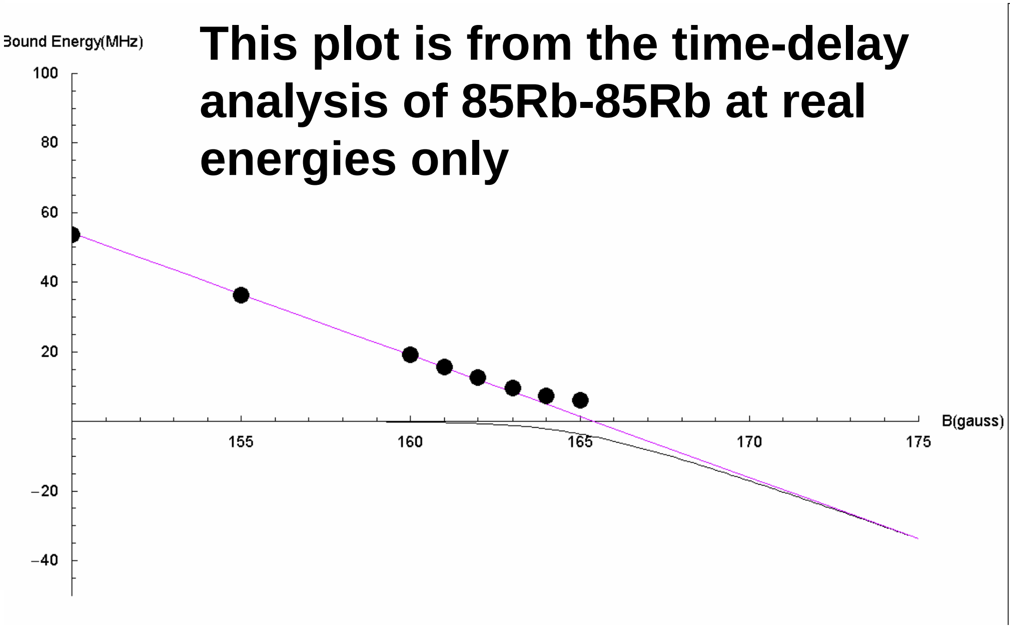
$$Y \equiv R_{11} \simeq -1/A_{bg}$$

can be approximately associated with the background scattering length

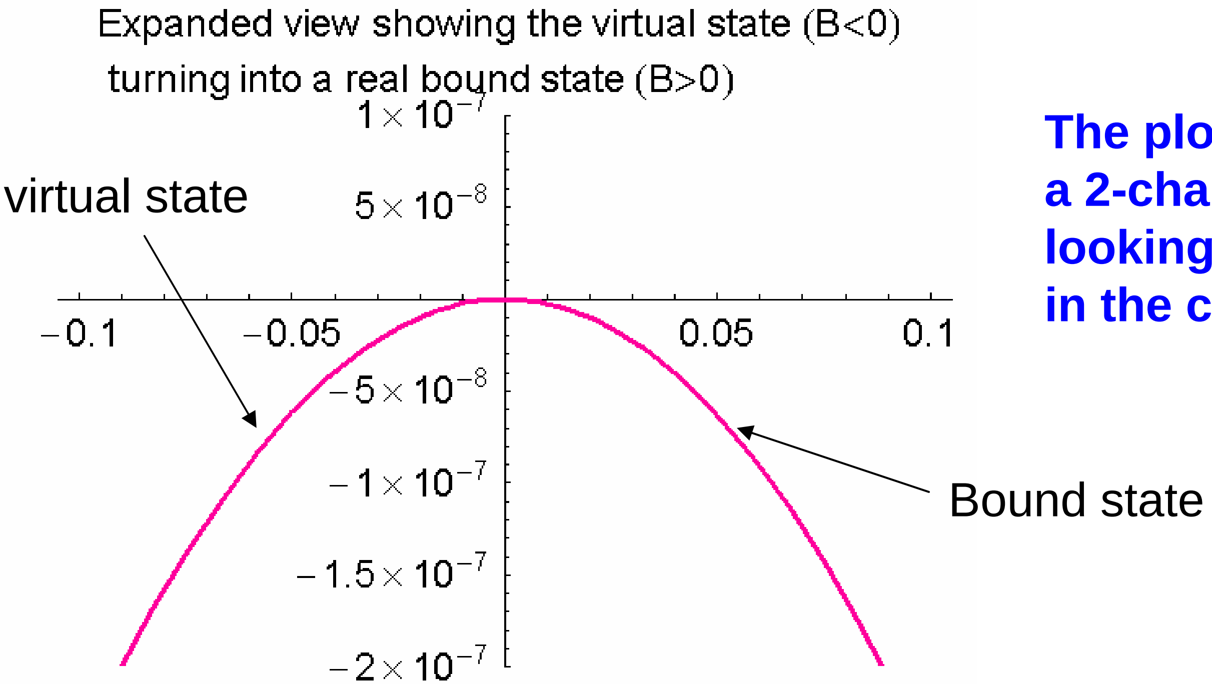
$Z \equiv (\hbar^2\gamma/2m\zeta)$ is the slope of the Feshbach resonance,


Example like 85Rb−85Rb

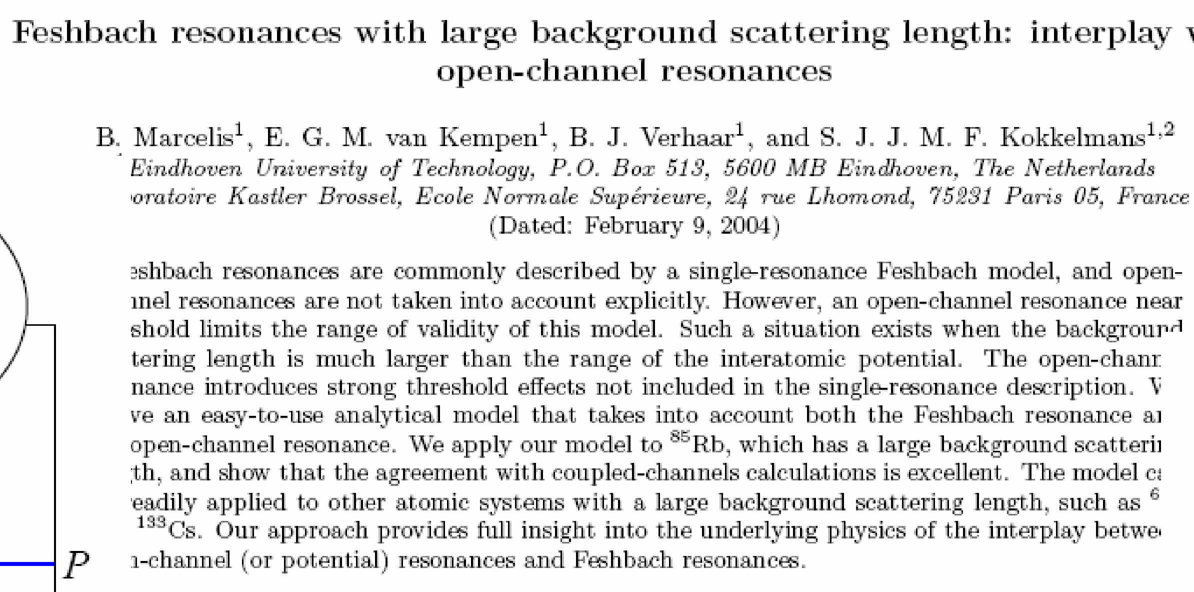
Re[Epole] versus B



This plot is from the time-delay analysis of 85Rb-85Rb at real energies only


Expanded view showing the virtual state (B<0) turning into a real bound state (B>0)

The plots on the left are from a 2-channel treatment, looking at the S-matrix poles in the complex energy plane.



Feshbach resonances with large background scattering length: interplay with open-channel resonances

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(Dated: February 9, 2004)

eshbach resonances are commonly described by a single-resonance Feshbach model, and open-nel resonances are not taken into account explicitly. However, an open-channel resonance near shold limits the range of validity of this model. Such a situation exists when the backgroun tering length is much larger than the range of the interatomic potential. The open-chann nance introduces strong threshold effects not included in the single-resonance description. W ve an easy-to-use analytical model that takes into account both the Feshbach resonance a open-channel resonance. We apply our model to ^{85}Rb, which has a large background scatteri th, and show that the agreement with coupled-channels calculations is excellent. The model c readily applied to other atomic systems with a large background scattering length, such as ^{6} ^{133}Cs. Our approach provides full insight into the underlying physics of the interplay betwe n-channel (or potential) resonances and Feshbach resonances.

arXiv:cond-mat/0402278 v1 10 Feb 2004

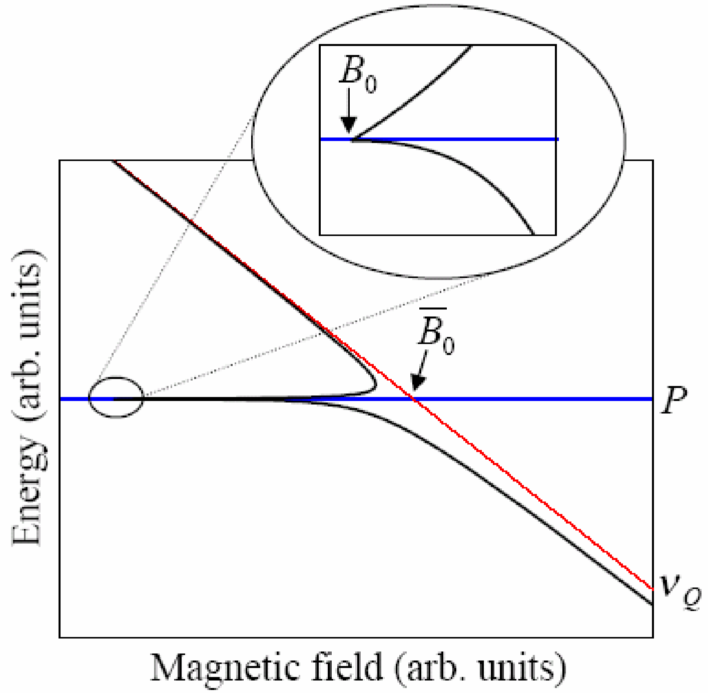


FIG. 5: Effect of virtual state on Feshbach resonance in ^{85}Rb in the $|2,-2\rangle$ hyperfine channel. Shown are the energy of the unperturbed Q-channel bound state (dotted line), and of the (quasi-)bound state (solid line) which is 'dressed' by the coupling to the P-channel. The (quasi-)bound state crosses the collision threshold at B_0, the unperturbed Q-channel bound state crosses the collision threshold at $\overline{B}_0$. Note that the energies are give relative to the P-channel collision threshold (horizontal solid line).

While the predictions of this Marcelis et al. paper initially appeared to be inconsistent with our conclusions, we have seen that their results can be reproduced by changing the way we search for resonance poles. Instead of looking for complex E-poles at real B, we must look for complex B-poles of S, at real E.

However, we do not agree that an "extra" resonance must be added to the open channel when the background scattering length is large and negative.

Or, search for maxima in d(delta) /dB rather than d(delta) /dE

Dispelling some myths about low energy Feshbach resonances:

Myth #1. The “Feshbach resonance” occurs when the scattering length goes to Infinity.

Reality: Even 1-channel models having NO RESONANCES still possess points where a(B)→infinity, whenever the last bound state disappears

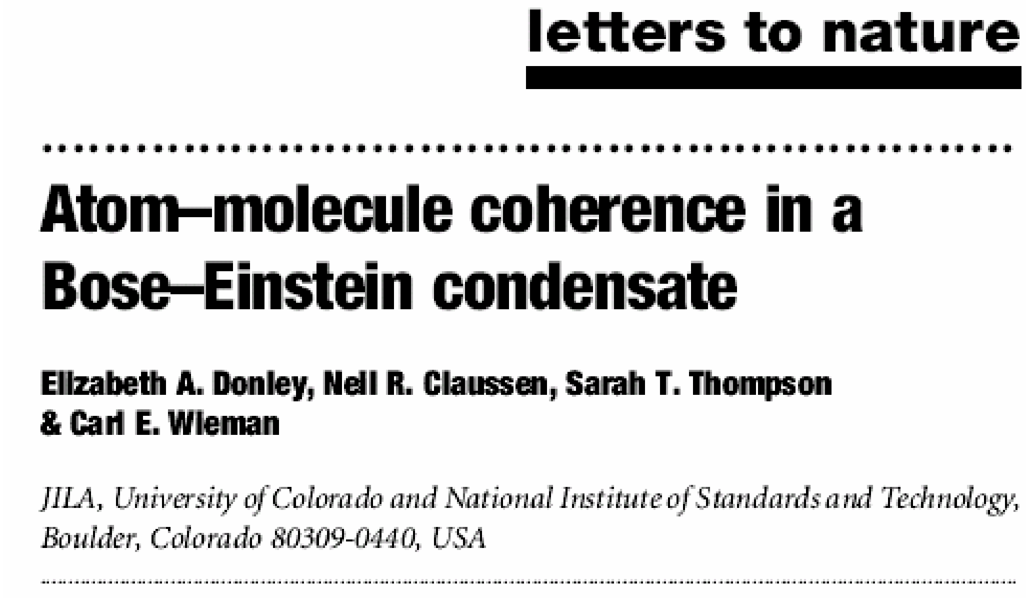
Myth #2. In a problem that involves a true Feshbach resonance in a closed channel, when a NEW bound state appears for the molecule, its wavefunction is mostly in the higher closed channels.

Reality: The two-channel effective range description, confirmed by detailed, realistic coupled channel calculations, shows that the LOWEST CHANNEL component is the dominant one as Asc→+infinity. (The higher channel component probabilities fall off in proportion to 1/Asc.)

→An implication: This makes it difficult to extract “molecular populations” from diabatic field theory descriptions, owing to their inherent nonlinearity.

Myth #3. The Feshbach resonance lifetime 1/Gamma diverges as 1/sqrt(E_{res}) when the Re(E_{res}) →0.

Reality: The decay width remains finite right down to the point where the true resonance energy crosses E=0. Close to the point where the resonance energy crosses zero, it does have a cusp and typically bifurcates into two virtual states. The bifurcation point is where the width of the resonance actually vanishes.



letters to nature

Atom–molecule coherence in a Bose–Einstein condensate

Elizabeth A. Donley, Neil R. Claussen, Sarah T. Thompson & Carl E. Wieman

JILA, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA

2002 JILA Experiment with N=17,000 Rb atoms subjected to B-field ramps that change *a*

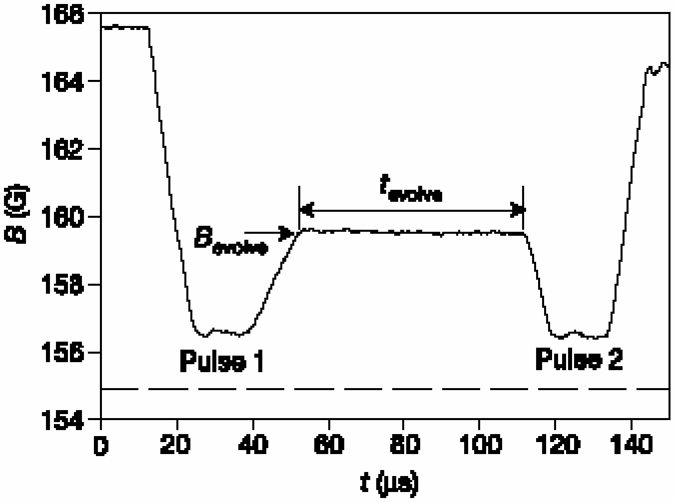


Figure 2 Magnetic field pulse shape. Fields shown for pulses 1 and 2 correspond to scattering lengths of ~2,500a_0, and the free precession field B_{evolve} corresponds to a scattering length of ~570a_0. The dashed line indicates the position of the Feshbach resonance. In the text, we refer to the free precession time as t_{evolve}. The average rise/fall time for all of the pulses that we used was 14 μs.

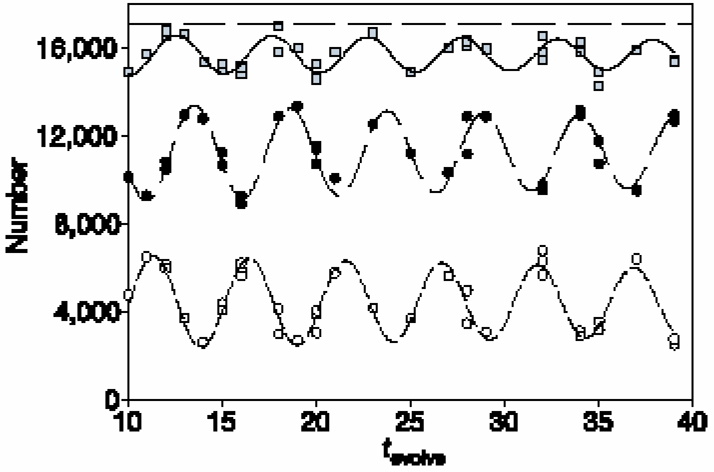


Figure 6 Number versus t_{evolve} for $n_0 = 1.1 \times 10^{13}\,\mathrm{cm}^{-3}$. From bottom to top, the data are plots of N_{burst} (open circles), N_{remnant} (filled circles), and the total number of observed atoms (grey squares). Each data set was fitted to a damped sine wave resulting in the displayed fits. $N_{\mathrm{init}} = 17{,}100$ is indicated by the flat dashed line. $B_{\mathrm{evolve}} = 159.84(2)$ G and the remnant data fitted to an oscillation frequency of 196(1) kHz and $\tau_{\mathrm{decay}} =$ 91(33) μs. Not all of the data used to determine τ_{decay} are shown. To produce condensates with lower density for these measurements, the initial magnetic field before the fast-pulse sequence was 162.2(1) G and the amplitudes for pulses 1 and 2 were reduced to ~7 G.

VOLUME 91, NUMBER 8 PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2003

Conversion of an Atomic Fermi Gas to a Long-Lived Molecular Bose Gas

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(Received 21 July 2003; published 22 August 2003)

We have converted an ultracold Fermi gas of ^{6}Li atoms into an ultracold gas of $^6\mathrm{Li}_2$ molecules by adiabatic passage through a Feshbach resonance. Approximately 1.5×10^5 molecules in the least-bound, $v = 38$, vibrational level of the $X^1\Sigma_g^+$ singlet state are produced with an efficiency of 50%. The molecules remain confined in an optical trap for times of up to 1 s before we dissociate them by a reverse adiabatic sweep.

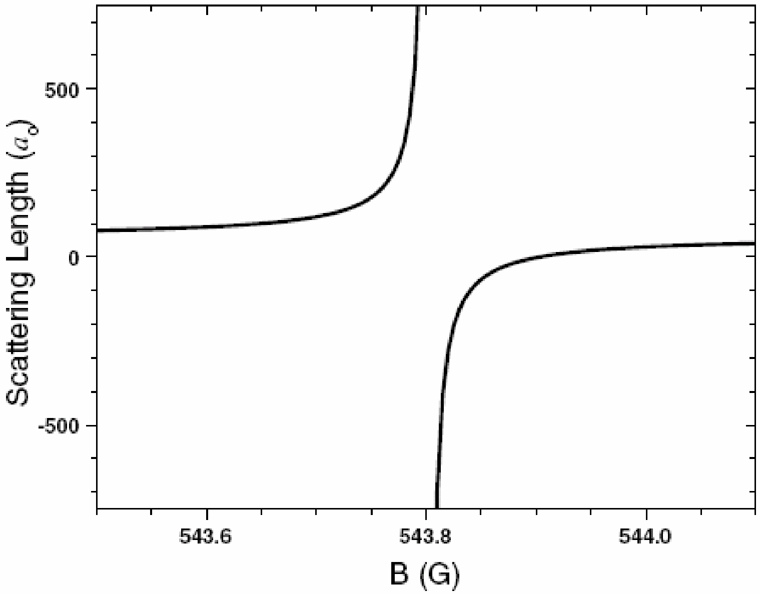


FIG. 1. Coupled-channels calculation of the narrow Feshbach resonance between the two lowest Zeeman sublevels of ^{6}Li. The scattering length is shown in units of the Bohr radius. The predicted location of the resonance is at a slightly higher field than observed (Figs. 2 and 4).

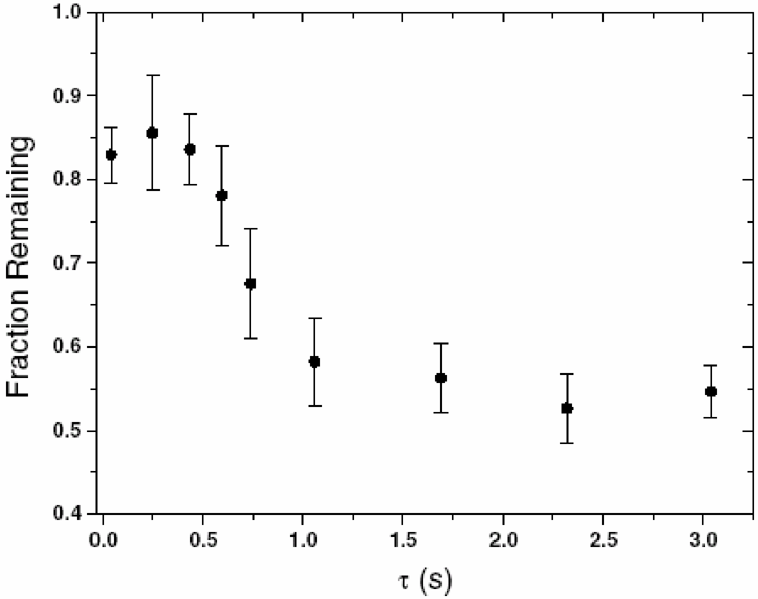
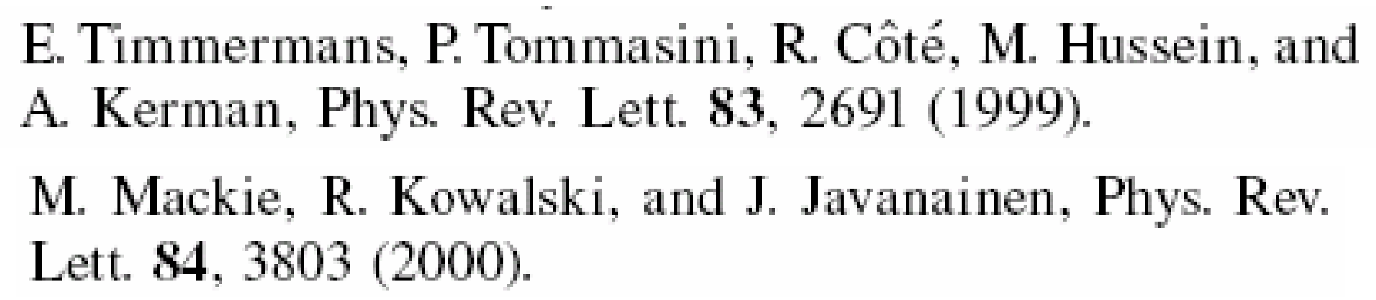


FIG. 5. Measurement of the molecular lifetime. The field is ramped downward through the Feshbach resonance and back to the starting field. The time τ is defined as the interval between traversing the field B_o on the downward sweep and again on the upward sweep. The inverse sweep rate is 3.5 ms/G and the starting field is 549 G. The field is ramped down to a final field,

Some relevant theory papers:



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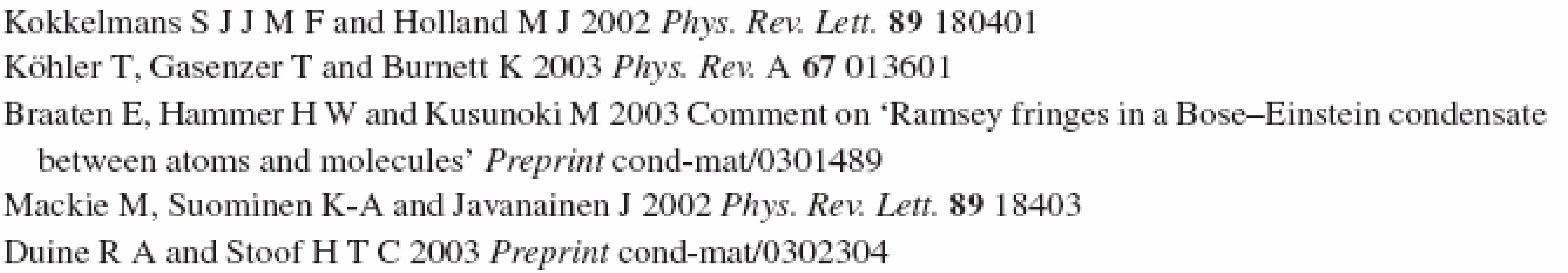

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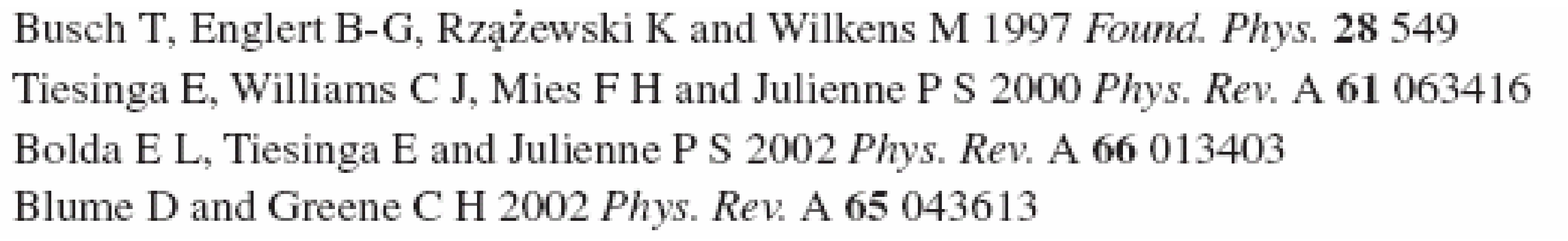

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Blume D and Greene C H 2002 *Phys. Rev.* A **65** 043613

VOLUME 89, NUMBER 18 PHYSICAL REVIEW LETTERS 28 OCTOBER 2002

Ramsey Fringes in a Bose-Einstein Condensate between Atoms and Molecules

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(Received 24 April 2002; published 11 October 2002)

In a recent experiment, a Feshbach scattering resonance was exploited to observe Ramsey fringes in a ^{85}Rb Bose-Einstein condensate. The oscillation frequency corresponded to the binding energy of the molecular state. We show that the observations are remarkably consistent with predictions of a resonance field theory in which the fringes arise from oscillations between atoms and molecules.

$$i\hbar \frac{d\phi_a}{dt} = V\mathcal{N}(0)\phi_a + [VG_A(0) + g\phi_m]\phi_a^*, \tag{2}$$

$$i\hbar \frac{d\phi_m}{dt} = \frac{g}{2}\mathcal{P}(0) + \nu\phi_m, \tag{3}$$

$$\hbar \frac{dG_N(r)}{dt} = 2\mathrm{Im}[V\mathcal{P}(0)G_A^*(r) + g\phi_m G_A^*(r)], \tag{4}$$

$$\begin{aligned} i\hbar \frac{dG_A(r)}{dt} = {} & -\frac{\hbar^2\nabla^2}{2\mu}G_A(r) + 4V[|\phi_a|^2 + G_N(0)]G_A(r) \\ & + [V\mathcal{P}(0) + g\phi_m][2G_N(r) + \delta(r)], \end{aligned} \tag{5}$$

with μ the reduced mass, $\mathcal{N}(r) = |\phi_a|^2 + 2G_N(r)$, and $\mathcal{P}(r) = \phi_a^2 + G_A(r)$. This is the complete closed set of equations to be dynamically solved. The binary collision physics encapsulated in the HFB equations is extracted by setting the density-dependent shifts to zero [8].

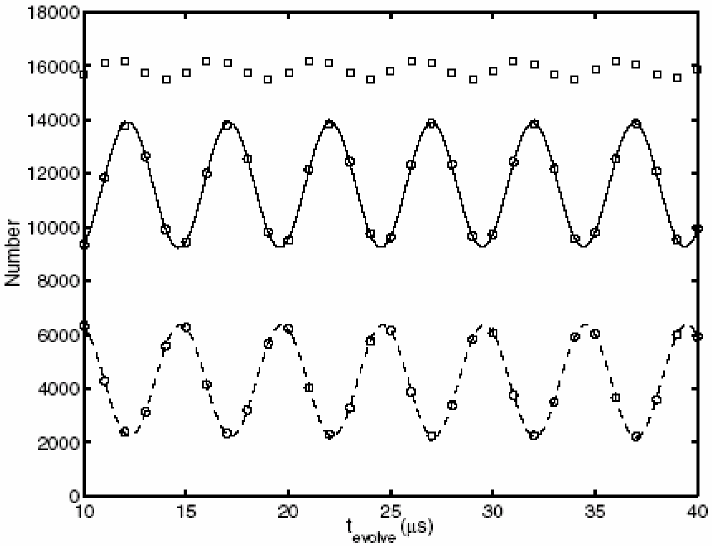


FIG. 5. Oscillations between the atomic condensate (solid line) and the normal field $G_N(0)$ (dashed line). These two numbers add up to the total number of recovered atoms (squares), which excludes the molecular component. The calculation is performed for a mean density of $\langle n\rangle = 3.9 \times 10^{12}$ cm^{-3}.

See, e.g.:

Tiesinga et al., PRA 61, 063416 (2000);

Blume, Greene, PRA 65, 043613 (2002);

Bolda, Tiesinga, Julienne, PRA 66, 013403 (2002).

$$\frac{2\Gamma\left(-\frac{\epsilon}{2}+\frac{3}{4}\right)}{\Gamma\left(-\frac{\epsilon}{2}+\frac{1}{4}\right)}=\frac{1}{a_{\rm sc}/a_{\rm ho}}$$

🡐Determines exact 2-atom energy levels in the trap, while the following formula determines the projection of an eigenstate at one value of a(B) onto an eigenstate at a different a(B’):

$$O_{\nu_2,\nu_1}=N_{\nu_1}N_{\nu_2}\Big[2\cos(\pi\nu_1)\Gamma(\frac{3}{2}+\nu_1)\Gamma(1+\nu_2)\sin(\pi\nu_2) \\ -2\cos(\pi\nu_2)\Gamma(1+\nu_1)\Gamma(\frac{3}{2}+\nu_2)\sin(\pi\nu_1)\Big]/[4\pi(\nu_2-\nu_1)$$

and

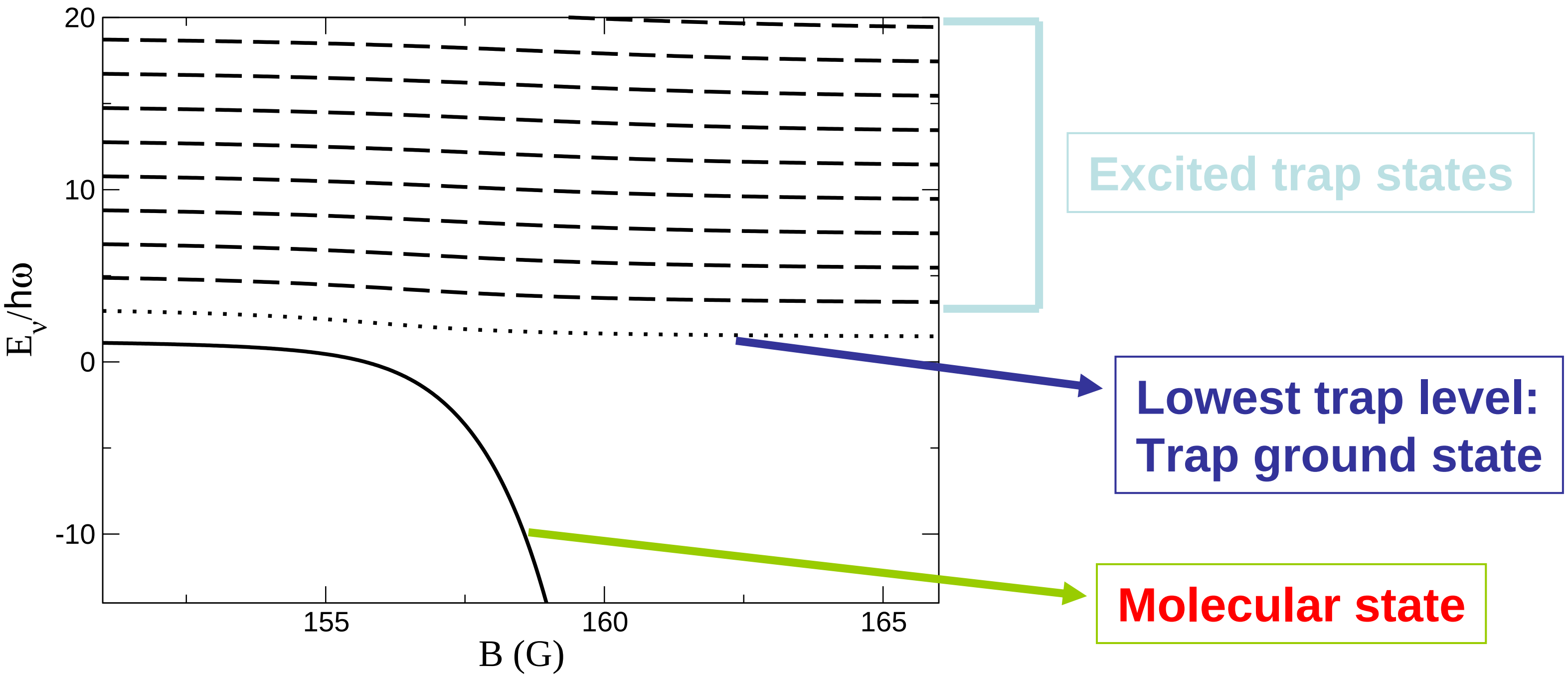
$$N_\nu=\left\{\Gamma(1+\nu)\Gamma(\frac{3}{2}+\nu)\left[2\pi+\left(\psi(\nu)-\psi(\frac{1}{2}+\nu)\right)\sin(2\pi\nu)\right]\right\}^{-1/2},$$

A Two-Atom Picture of Coherent Atom-Molecule Quantum Beats

Bogdan Borca†, D. Blume‡, Chris H. Greene†

New Journal of Physics 5 (2003) 111.1–111.14 (http://www.njp.org/

Two-Particle Energy Levels: $\varpi/2\pi$ = 3.5kHz



Excited trap states

Lowest trap level: Trap ground state

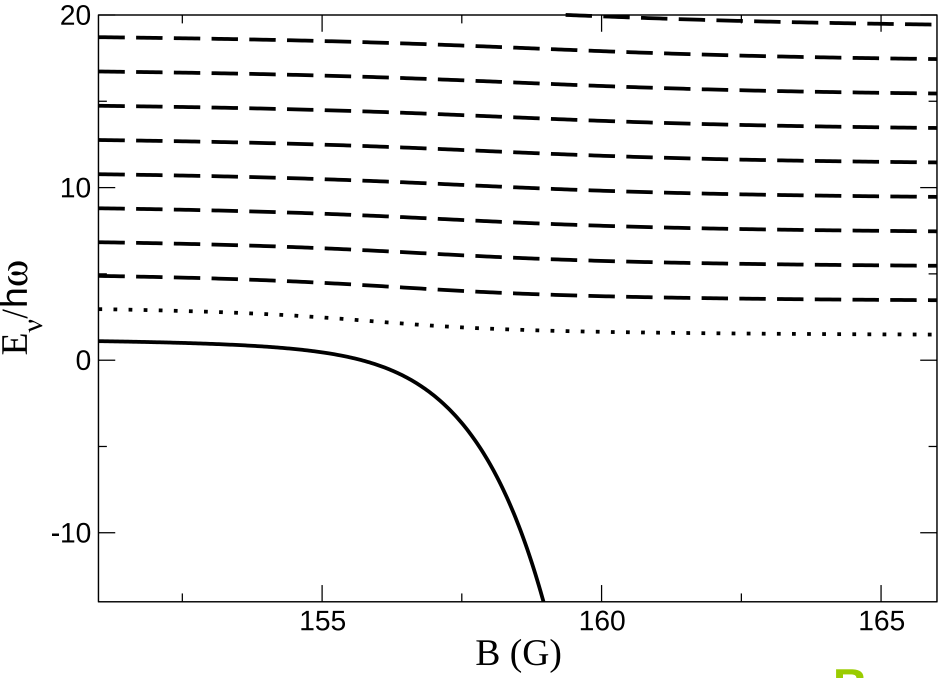
Molecular state

Perform time-dependent calculation:

At time t=0: Initial state is chosen to be lowest trap level.

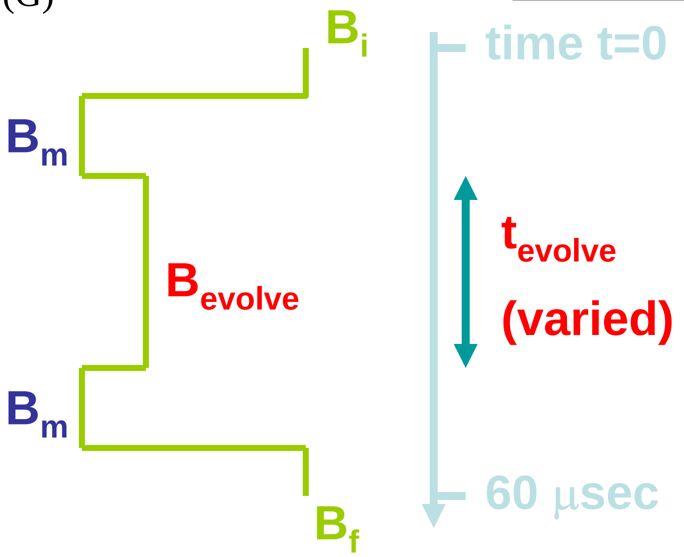
Apply a sequence of magnetic field pulses [two-body Hamiltonian becomes time-dependent through scattering length term a(B(t))].

eld Pulses



Just like the field pulses in JILA experiment
Donley et al., Nature 417, 529 (2002)
– but we're only treating two atoms!
Kokkelmans,Holland, PRL 89, 180401 (2002); Kohler, Gasenzer, Burnett, PRA 67, 013601 (2003); and others have treated this via field theory methods.

Magnetic field sequence:



Time propagation for instantaneously changed field pulses (sudden approximation) gives us occupation probability of energy states as a function of time.

Derivation of the quantum beats between atomic and molecular modes as a two-state system

After a sudden change of the magnetic field from B_0 to B_{evolve}, the wave-function of the pair is projected suddenly onto the eigenstates of the two-body Schrodinger equation at the new field:

$$\begin{aligned} \left|\psi^{2\ atoms}(B_0)\right\rangle \quad \rightarrow \quad & \left|\psi^{2\ atoms}(B_{\mathrm{evolve}})\right\rangle \left\langle\psi^{2\ atoms}(B_{\mathrm{evolve}})\right|\left.\psi^{2\ atoms}(B_0)\right\rangle \\ & + \left|\psi^{molecule}(B_{\mathrm{evolve}})\right\rangle \left\langle\psi^{molecule}(B_{\mathrm{evolve}})\right|\left.\psi^{2\ atoms}(B_0)\right\rangle \end{aligned}$$

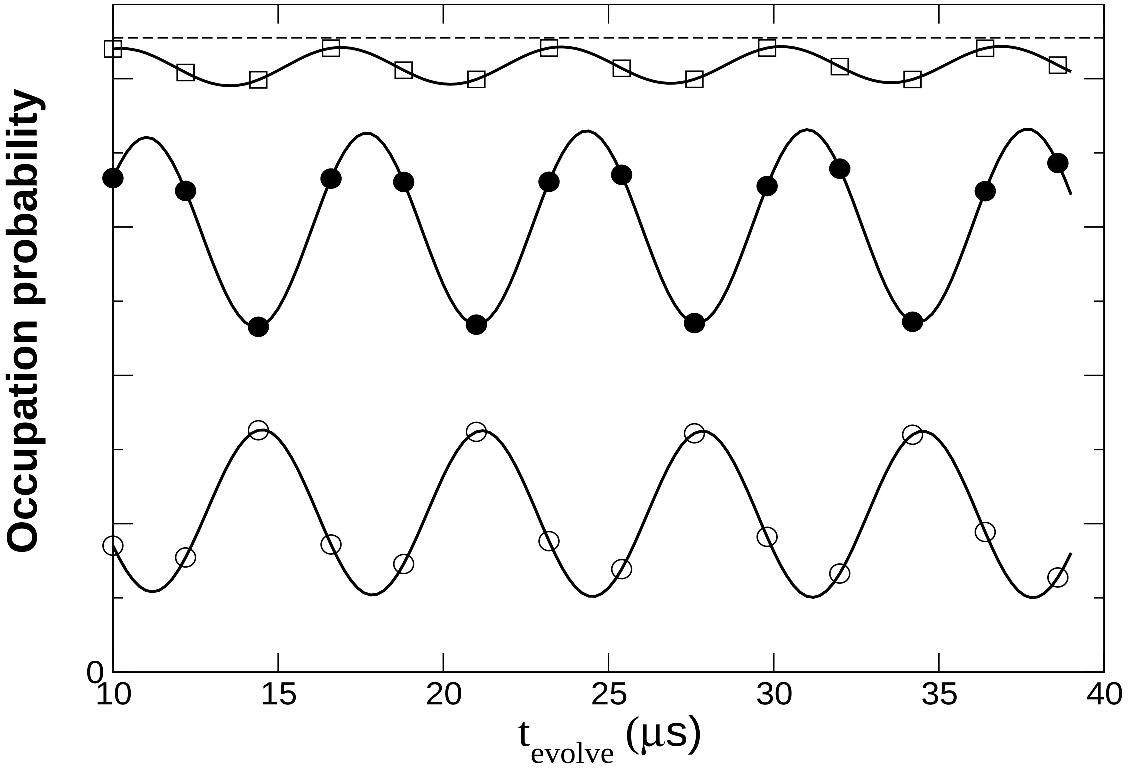
This coherent superposition now evolves in time according to:

$$\left|\Psi(t)\right\rangle = \left|\psi^{2\ atoms}(B_{\mathrm{evolve}})\right\rangle A_1 + \left|\psi^{molecule}(B_{\mathrm{evolve}})\right\rangle A_2 e^{-iE_b t/\hbar}$$

When the final magnetic field pulse reprojects this state vector onto the final two-body eigenstates, one obtains probabilities for the system to be found, as either two atoms or else a molecule, e.g.

$$P_{molecule}(t) = |A_1 A_2|^2 \left(1 - \cos\frac{E_b t}{\hbar}\right)$$

End-of-the-Pulse Occupation Probability of States: Quantum Beats



Population of molecular level

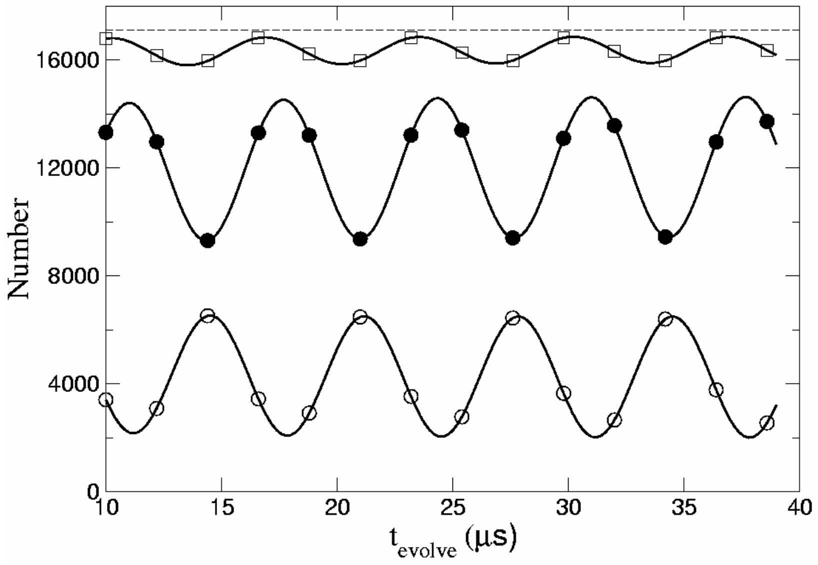
Population of lowest trap level (BEC)

Population of excited trap levels

Coherent atom - molecule quantum beats: **Interference of quantum paths that go through the "intermediate" molecular state with those that go through the "intermediate" ground trap state (excited trap states).**

Numbers of Molecules, Condensate Atoms, and Excited Atoms following the sequence of magnetic field ramps

Present 2-atom theory



2002 JILA Experiment in Nature magazine with 17,000 Rb atoms

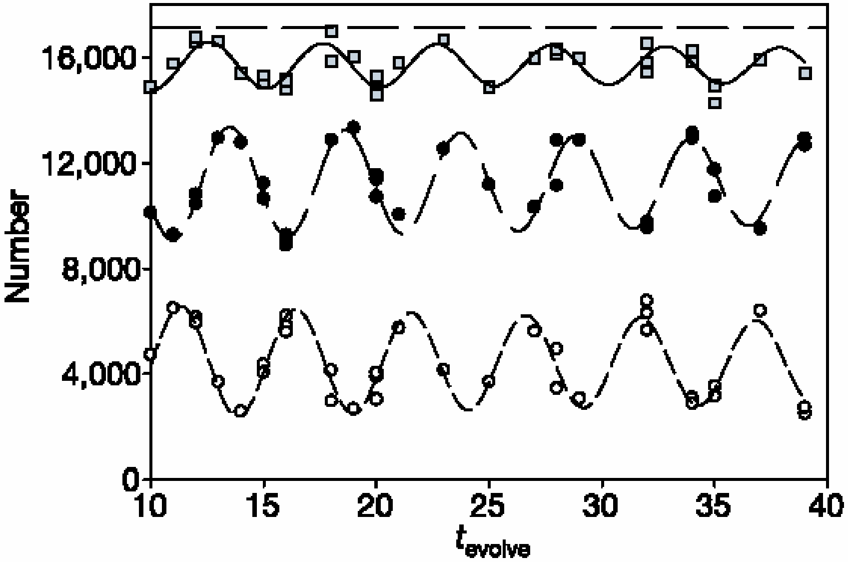
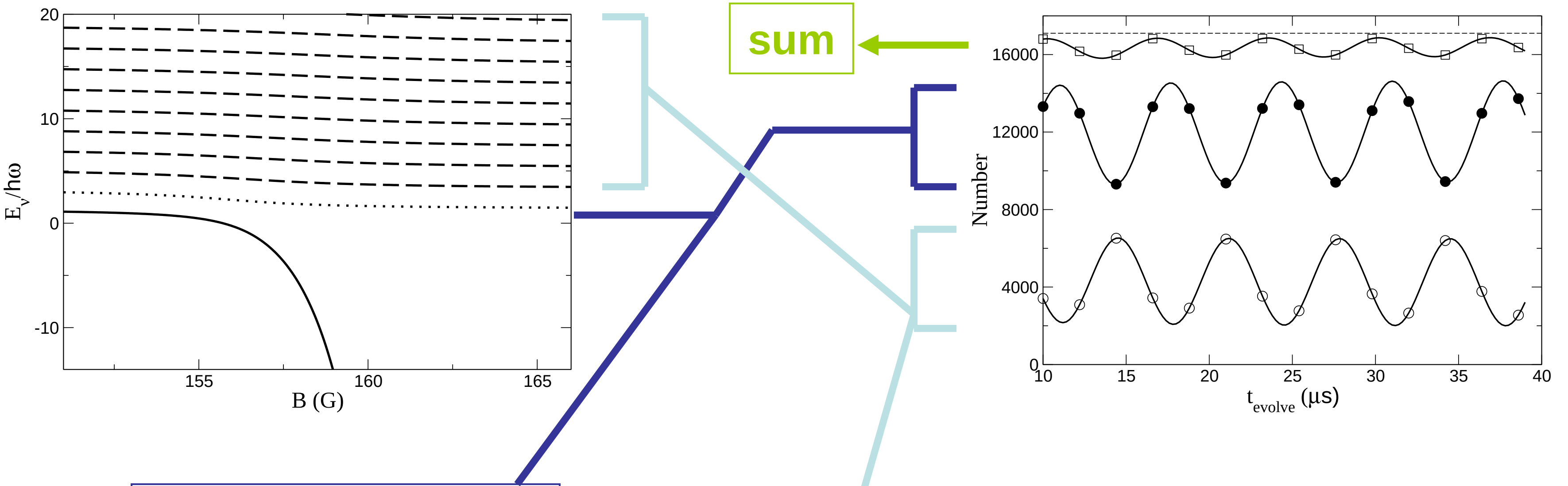


Figure 6 Number versus t_{evolve} for $n_0 = 1.1 \times 10^{13}\,\text{cm}^{-3}$. From bottom to top, the data are plots of N_{burst} (open circles), N_{remnant} (filled circles), and the total number of observed atoms (grey squares). Each data set was fitted to a damped sine wave resulting in the displayed fits. $N_{\text{init}} = 17{,}100$ is indicated by the flat dashed line. $B_{\text{evolve}} = 159.84(2)$ G and the remnant data fitted to an oscillation frequency of 196(1) kHz and $\tau_{\text{decay}} = 91(33)\,\mu\text{s}$. Not all of the data used to determine τ_{decay} are shown. To produce condensates with lower density for these measurements, the initial magnetic field before the fast-pulse sequence was 162.2(1) G and the amplitudes for pulses 1 and 2 were reduced to ~7 G.

Mapping: From a Two-Body System to the Many-Body System



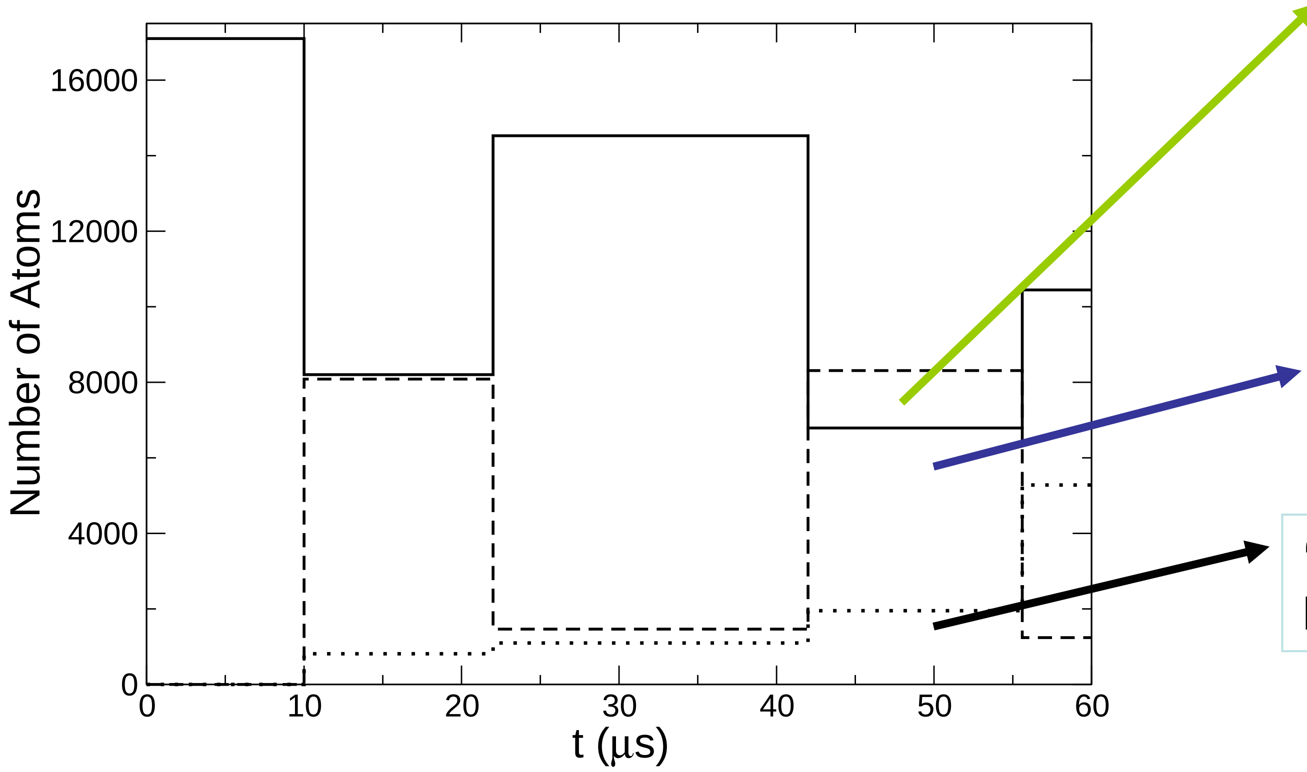
Above resonance: Condensate atoms

Non-condensed atoms: “Hot jet of atoms” in JILA experiment

Trapping frequency of two-atom simulation is chosen such that peak density in two-atom system equals that of the experiment (to first order, diluteness parameter $N(a/a_{ho})^3$):

Experiment: ν = 10.9Hz, N = 17100 **Model: ν = 3.5kHz, N = 2**

Population as a Function of Time for one Double Pulse Sequence



Molecules

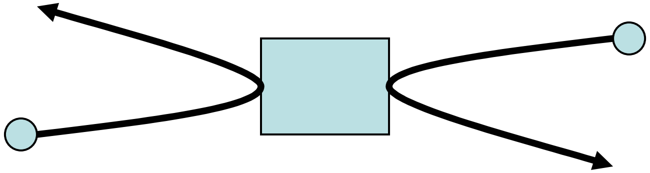
Condensate population: Ground state atoms

"Hot atoms": Excited atoms

We can watch the molecule population throughout the magnetic field sequence: Our molecules are truly molecules!

Summary of the two-body picture

- Two-atom study does a good job of reproducing experimental ^{85}Rb BEC results near a Feshbach resonance qualitatively, and even semi-quantitatively.
- Nice, simple complement to mean-field type studies.
- Seems to suggest that near a Feshbach resonance, much of the physics is two-body in nature.
- Borca, Blume, Greene, cond-mat/0304341, or New Journal of Physics 5, 111 (2003).



•Deficiencies: The main one is that the oscillator levels are more widely spaced, causing energy estimations to deviate from experiment.

Pair formation in a degenerate Fermi gas

Collaborator: Javier von Stecher

Questions

In the range of large negative scattering lengths, are the pairs “really molecules” in some sense?

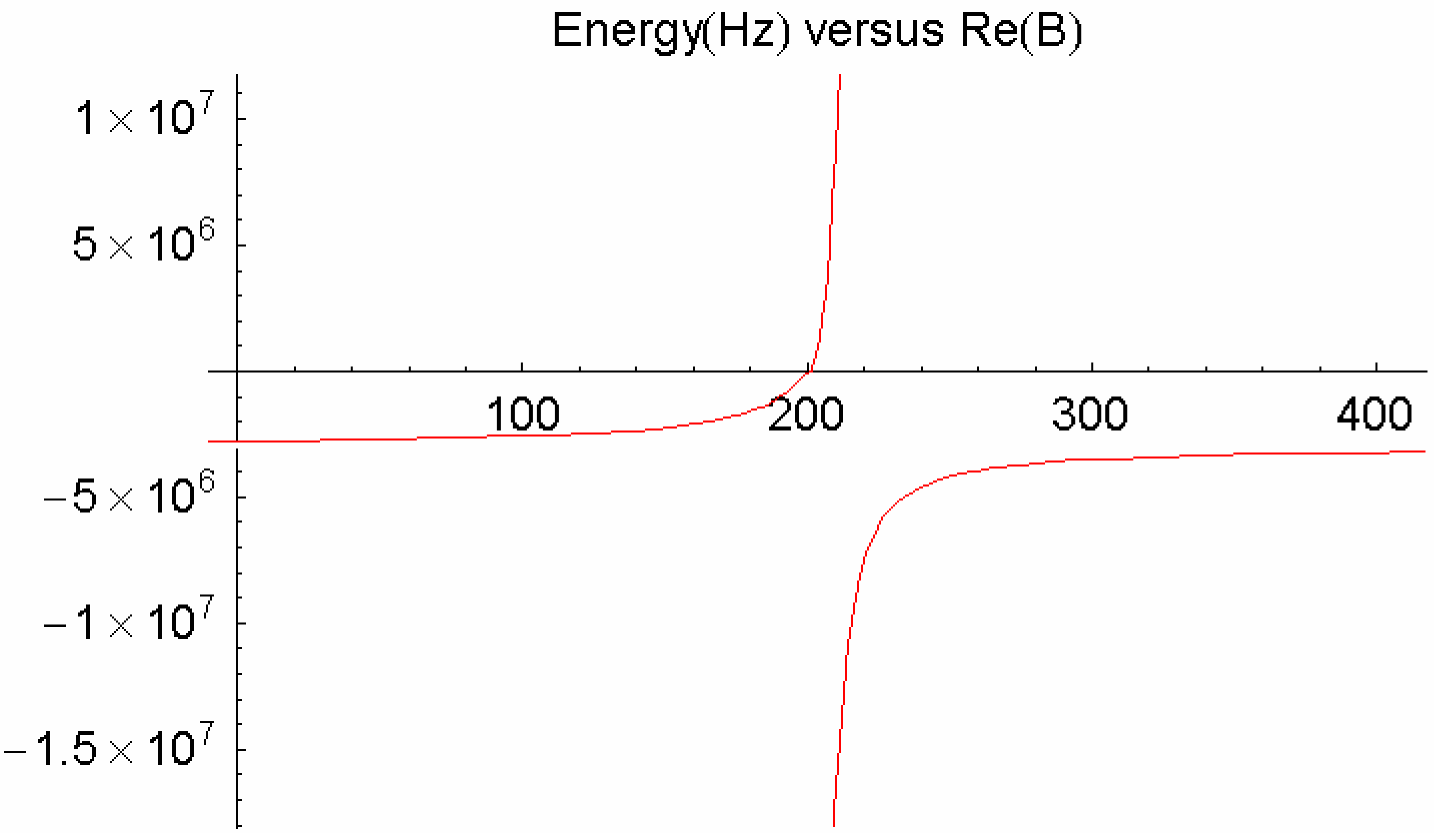
Do they have a well-defined binding energy and size?

Can one write an effective Schroedinger equation for two fermionic atoms, and include their interactions nonperturbatively, in the BEC-BCS crossover region?

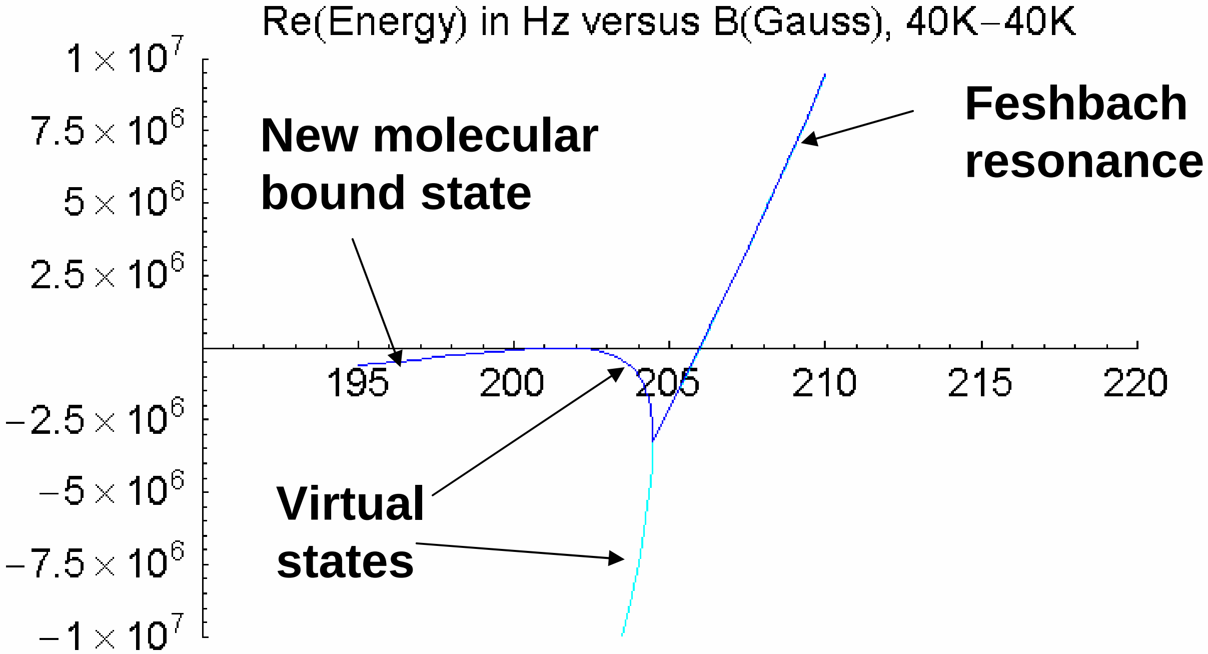
Our method of attack

1. Single particle (Thomas-Fermi) description of a degenerate Fermi gas with two spin components.
2. 2 fermions interacting with each other and with the mean field produced by the DFG.
3. Use of pseudopotentials to enforce Pauli blocking
4. Pair formation in 40K-40K near the scattering length pole.

The ^{40}K-^{40}K Fesbach resonance and its avoided crossing with a deeply-bound molecular state – Viewed from a large scale, the “resonance” turns into a bound level



The case of fermionic potassium, studied in recent experiments by D. Jin, M. Greiner, C. Regal

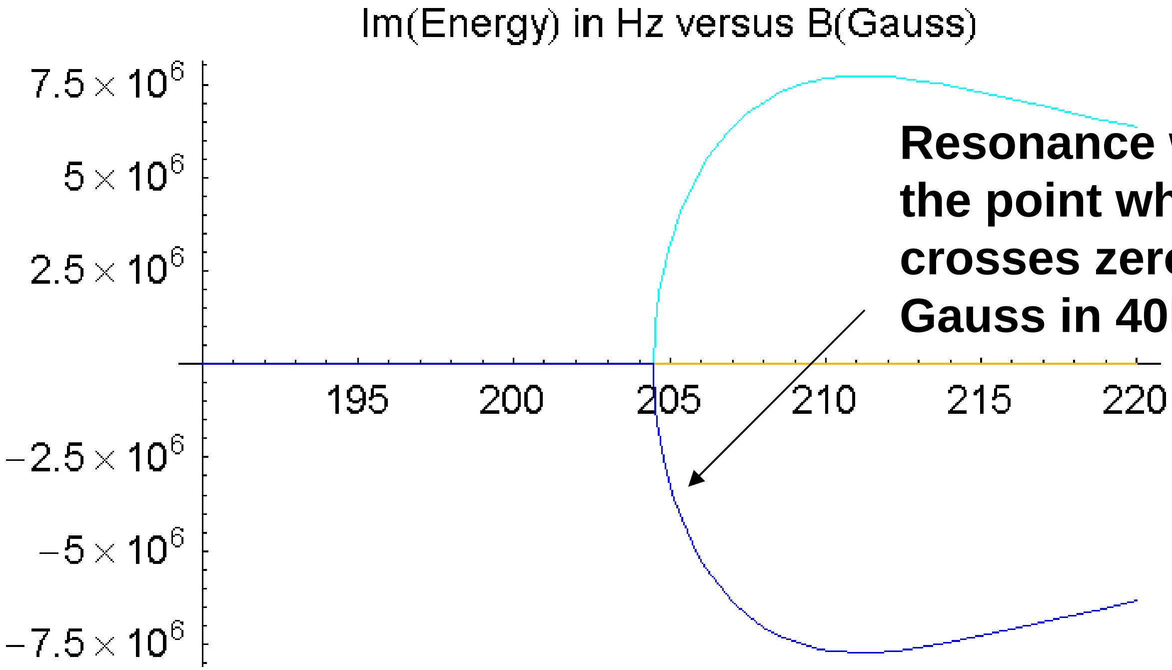


Re(Energy) in Hz versus B(Gauss), 40K–40K

New molecular bound state

Feshbach resonance

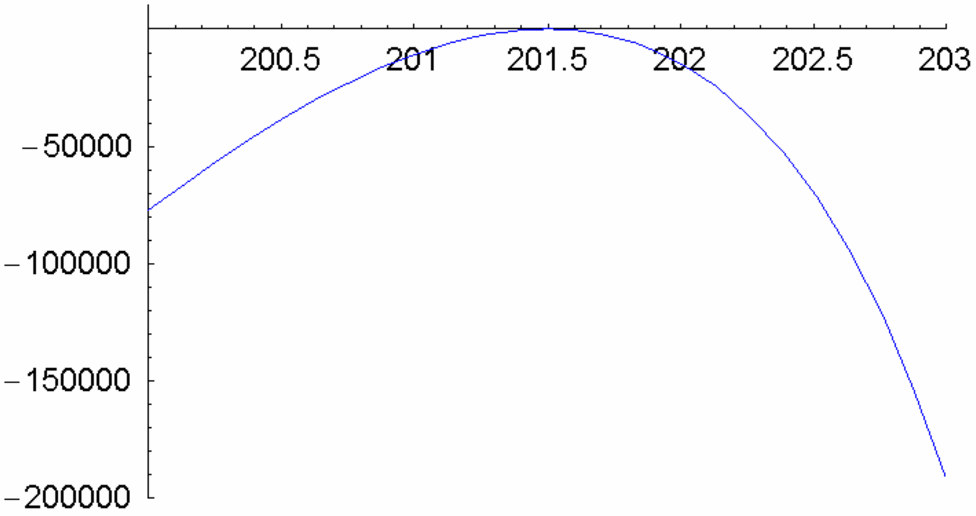
Virtual states



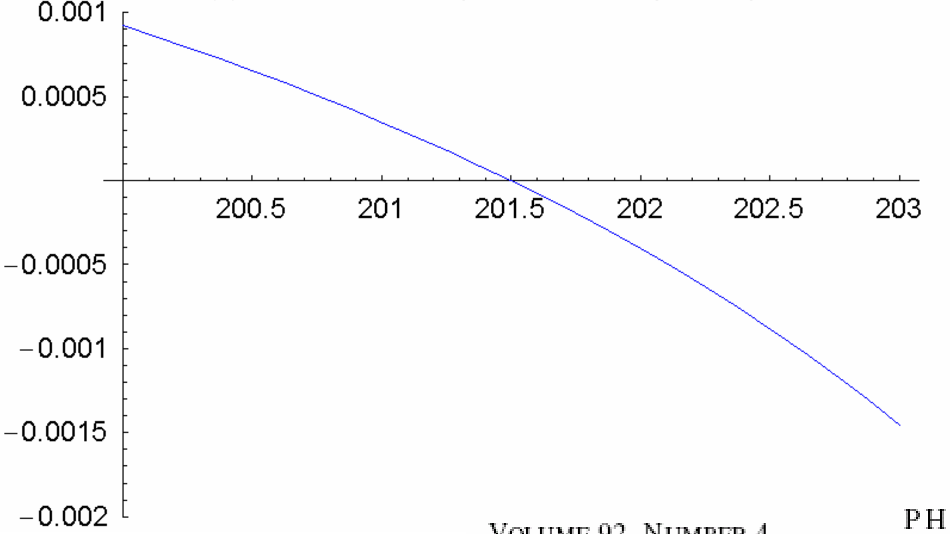
Im(Energy) in Hz versus B(Gauss)

Resonance width is still nonzero at the point where the resonance crosses zero energy, at around 206 Gauss in 40K.

Re(Energy) in Hz versus B(Gauss), 40K−40K



40K−40K, the change from bound state to virtual state
Im(k) for the S−matrix pole in the complex k−plane



Note: All the "action" in the new Jin group experiment is within about 0.5 Gauss of the creation point for the new molecular bound state

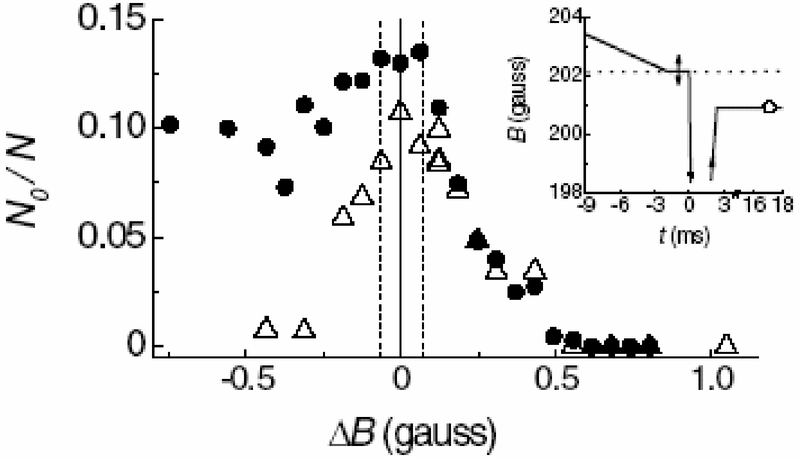


FIG. 2. Measured condensate fraction as a function of detuning from the Feshbach resonance $\Delta B = B_{\text{hold}} - B_0$. Data here were taken for $t_{\text{hold}} = 2$ ms (●) and $t_{\text{hold}} = 30$ ms (△) with an initial cloud at $T/T_F = 0.08$ and $T_F = 0.35\ \mu$K. The area between the dashed lines around $\Delta B = 0$ reflects the uncertainty in the Feshbach resonance position based on the 10%–90% width of the feature in Fig. 1. Condensation of fermionic atom pairs is seen near and on either side of the Feshbach resonance.

VOLUME 92, NUMBER 4 PHYSICAL REVIEW LETTERS week ending 30 JANUARY 2004

Observation of Resonance Condensation of Fermionic Atom Pairs

C. A. Regal, M. Greiner, and D. S. Jin*

← Increasing Attraction between the two fermions

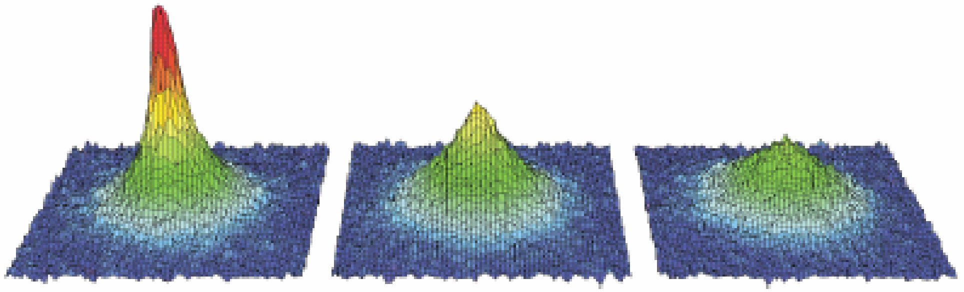
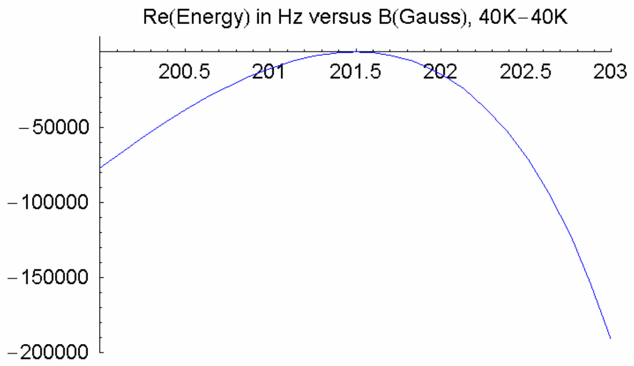


FIG. 3 (color online). Time of flight images showing the fermionic condensate. The images, taken after the projection of the fermionic system onto a molecular gas, are shown for $\Delta B = 0.12$, 0.25, and 0.55 G (left to right) on the BCS side of the resonance. The original atom cloud starts at $T/T_F = 0.07$,



PHYSICAL REVIEW A VOLUME 54, NUMBER 1 JULY 1996

Adiabatic hyperspherical study of the helium trimer

B. D. Esry, C. D. Lin,* and Chris H. Greene

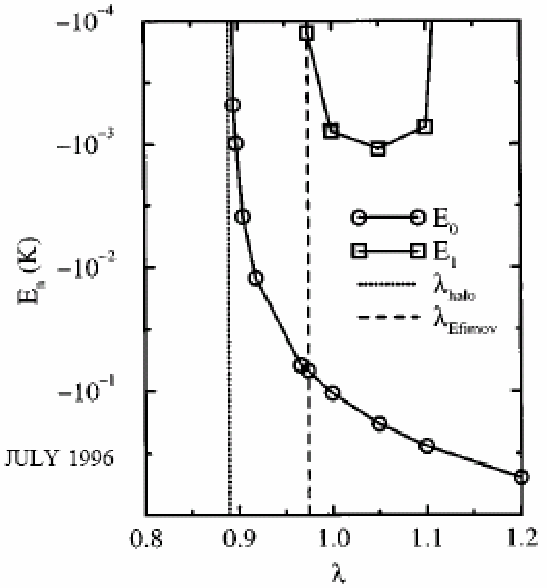
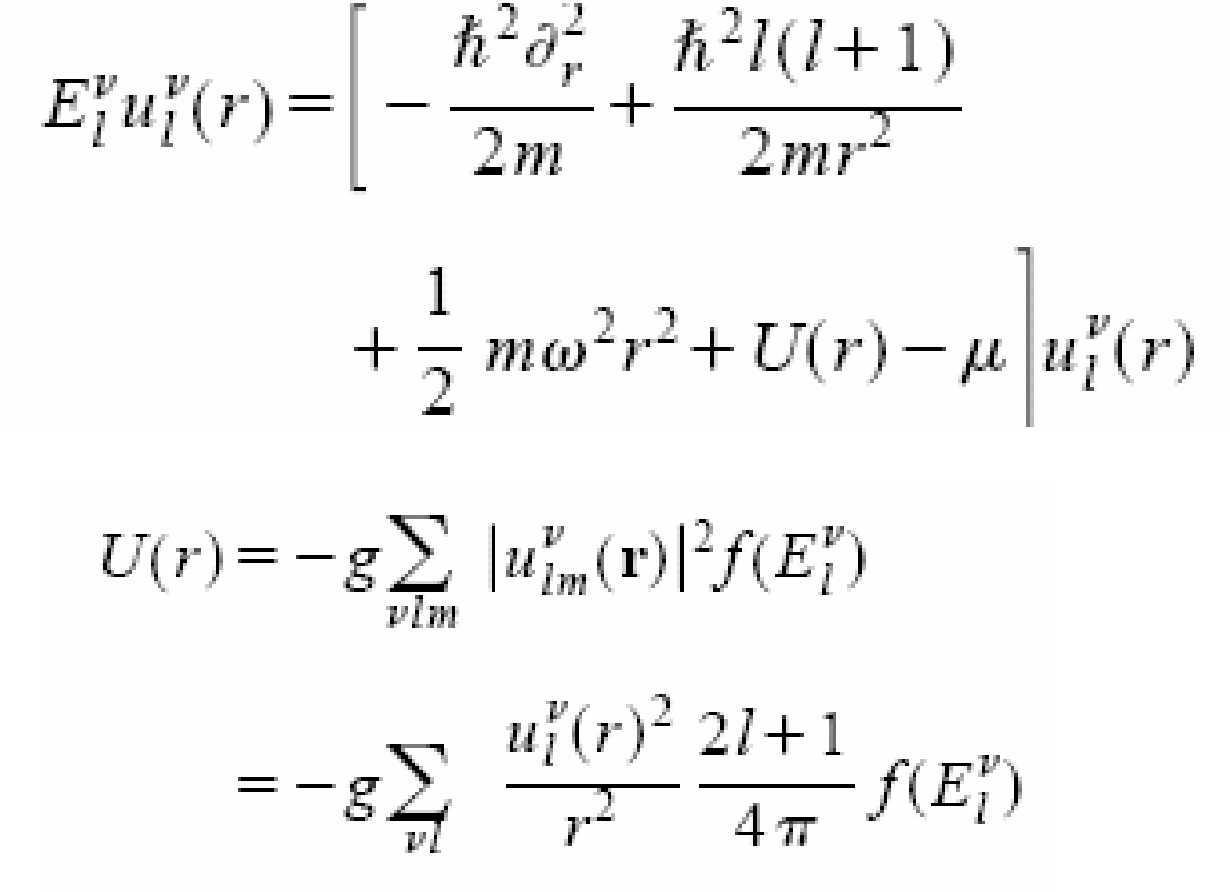


FIG. 3. The trimer binding energies as a function of the interaction strength, λ, for the physical value $\lambda = 1$. Shown are $\lambda = \lambda_{\text{halo}} = 0.89$ (dotted line), $\lambda = \lambda_{\text{Efimov}} = 0.9741$ (dashed line), the ground state (circles), and the first excited state (squares).

Straightforward description of the DFG at the single-particle level

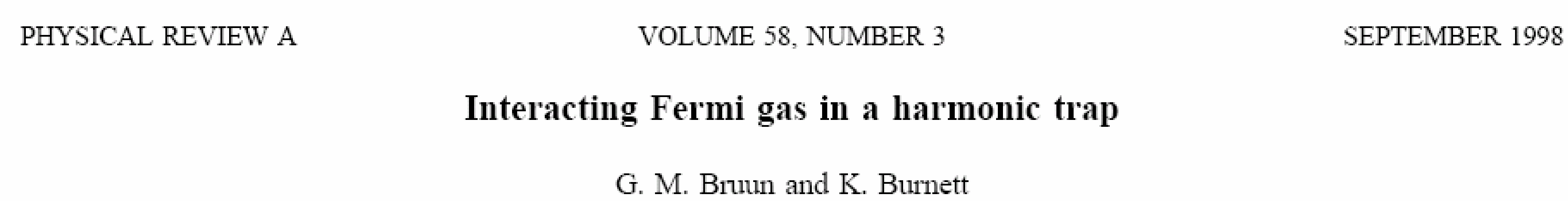


$$E_l^{\nu} u_l^{\nu}(r) = \left[-\frac{\hbar^2 \partial_r^2}{2m} + \frac{\hbar^2 l(l+1)}{2mr^2} + \frac{1}{2} m\omega^2 r^2 + U(r) - \mu \right] u_l^{\nu}(r)$$

$$U(r) = -g \sum_{\nu l m} |u_{lm}^{\nu}(\mathbf{r})|^2 f(E_l^{\nu})$$

$$= -g \sum_{\nu l} \frac{u_l^{\nu}(r)^2}{r^2} \frac{2l+1}{4\pi} f(E_l^{\nu})$$

Note that one should probably switch over, for fields very close to the pole, to a DFG description based more on the “unitarity limit”, of the type discussed by Pandharipande and others.

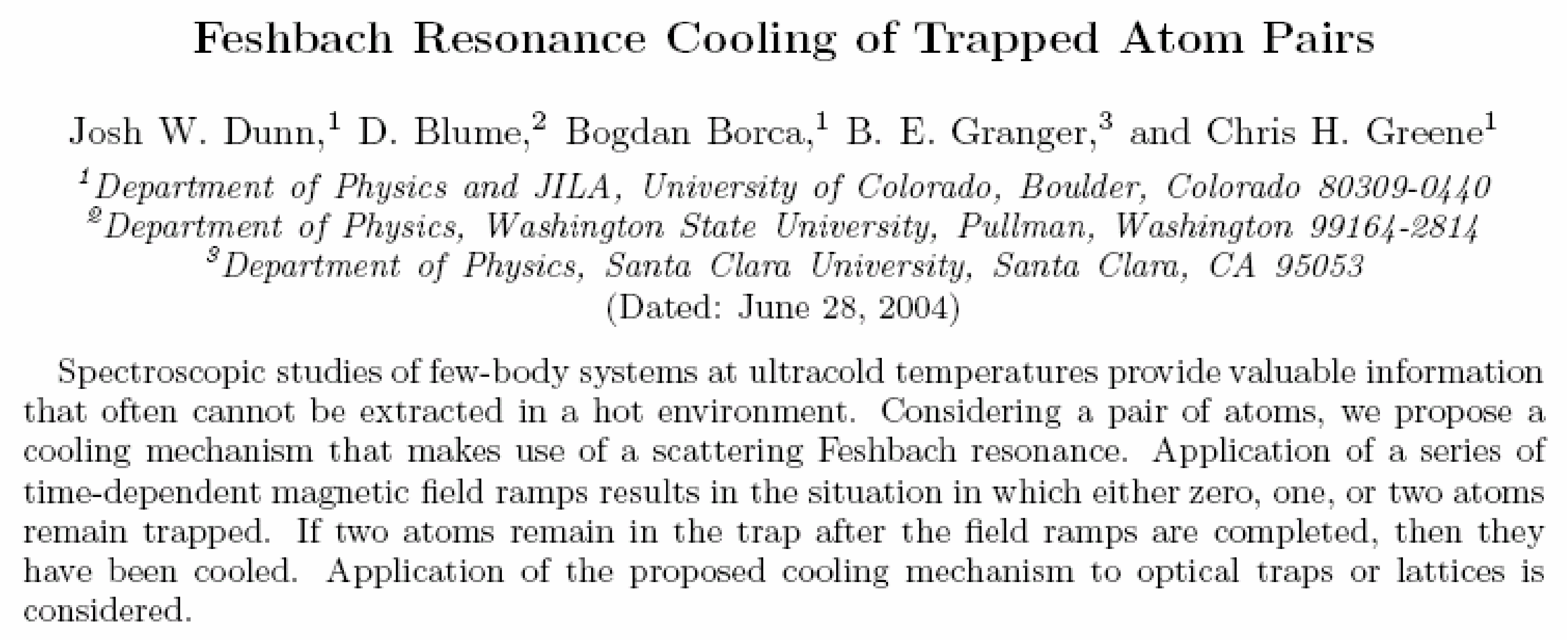


PHYSICAL REVIEW A VOLUME 58, NUMBER 3 SEPTEMBER 1998

Interacting Fermi gas in a harmonic trap

G. M. Bruun and K. Burnett

A proposed "Schroedinger Refrigeration" scheme that can cool atom pairs in a thermal gas, e.g. in an optical lattice



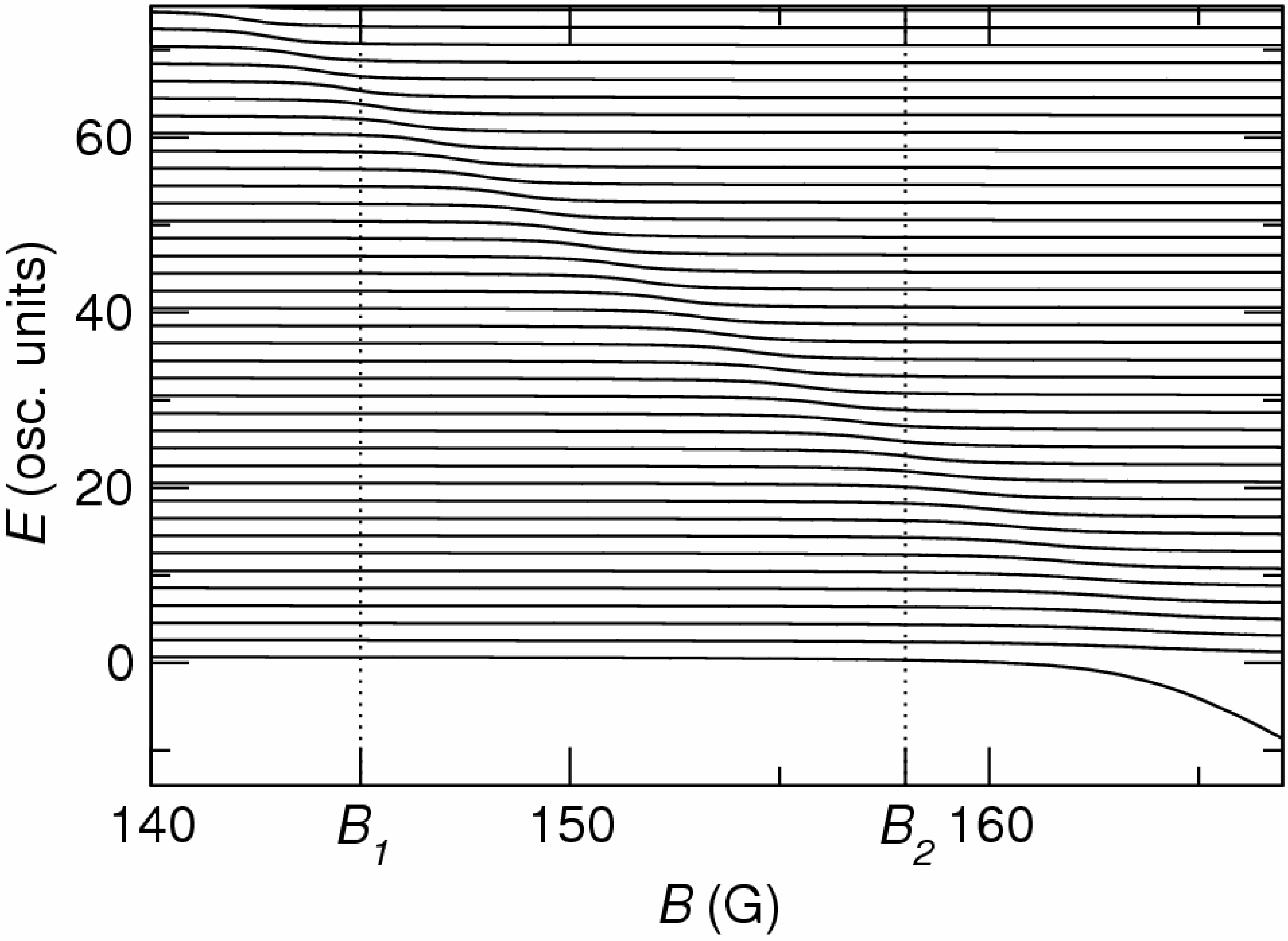
Feshbach Resonance Cooling of Trapped Atom Pairs

Josh W. Dunn,[1] D. Blume,[2] Bogdan Borca,[1] B. E. Granger,[3] and Chris H. Greene[1]
[1]*Department of Physics and JILA, University of Colorado, Boulder, Colorado 80309-0440*
[2]*Department of Physics, Washington State University, Pullman, Washington 99164-2814*
[3]*Department of Physics, Santa Clara University, Santa Clara, CA 95053*

(Dated: June 28, 2004)

Spectroscopic studies of few-body systems at ultracold temperatures provide valuable information that often cannot be extracted in a hot environment. Considering a pair of atoms, we propose a cooling mechanism that makes use of a scattering Feshbach resonance. Application of a series of time-dependent magnetic field ramps results in the situation in which either zero, one, or two atoms remain trapped. If two atoms remain in the trap after the field ramps are completed, then they have been cooled. Application of the proposed cooling mechanism to optical traps or lattices is considered.

Energy levels of two ^{85}Rb atoms, trapped in a spherical oscillator potential, in the relative degree of freedom, versus magnetic field.



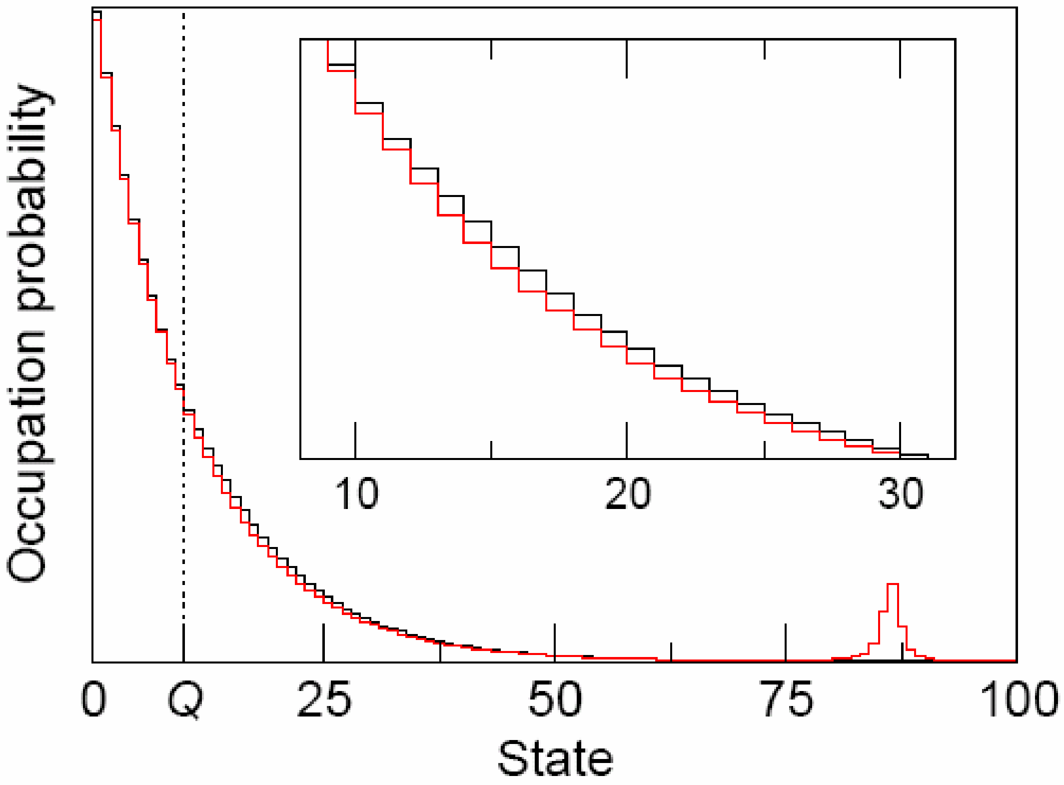


FIG. 2: (Color online) Illustration of the effect of a single Feshbach resonance cooling cycle for $T = 1\,\mathrm{mK}$ and $\nu = 1\,\mathrm{MHz}$. The black (red) line represents the population distribution before (after) application of one slow and one fast magnetic field ramp. The state Q (here $Q = 10$) is indicated. Population from $n = Q$ and nearby states is moved to higher states with $n \approx 85$. Population initially in a state with $n > Q$ is moved to the next-lowest state (see the inset close-up).

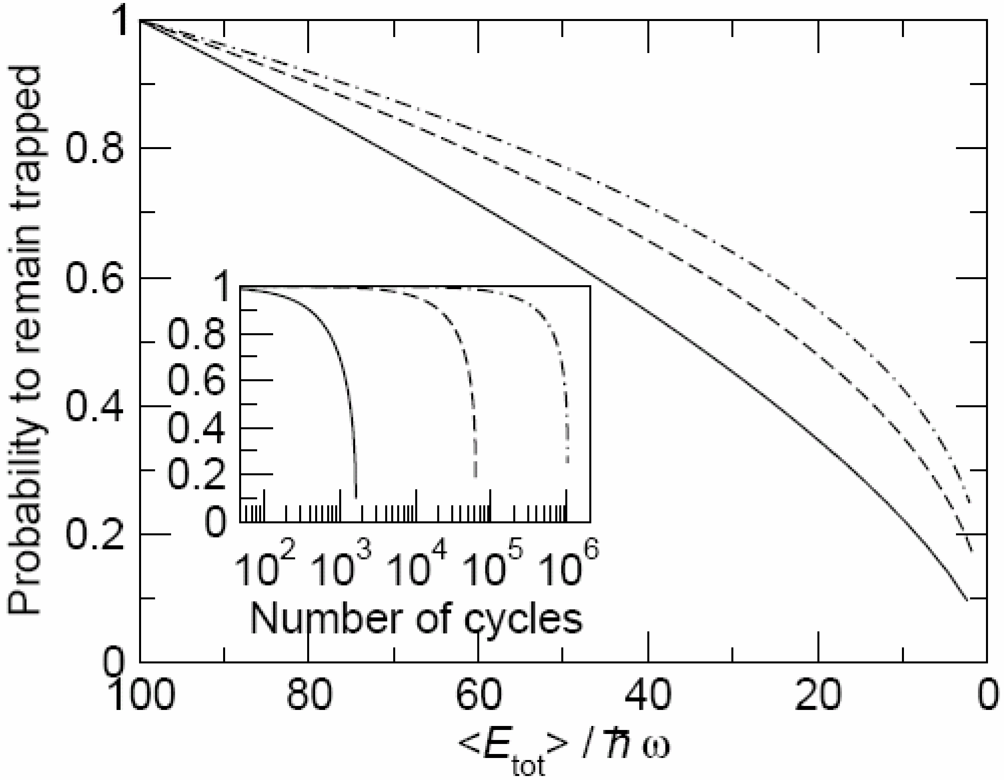


FIG. 3: Probability that a pair of atoms remains trapped vs. the average total kinetic energy of the two atoms in oscillator units (note that $k_B T = \langle E_{\rm tot} \rangle / 6$ for two harmonically trapped atoms). Three different cooling parameters are used: $2\hbar\omega Q = 5\tau$ (solid line), 9τ (dashed line), and 12τ (dot-dashed line). It is assumed that rethermalization occurs between cooling cycles (see text). Inset: probability to remain trapped vs. the number of cooling cycles for the same three cooling parameters.