Spinor Bose-Einstein condensation and Bose ferromagnetism

Qiang Gu†

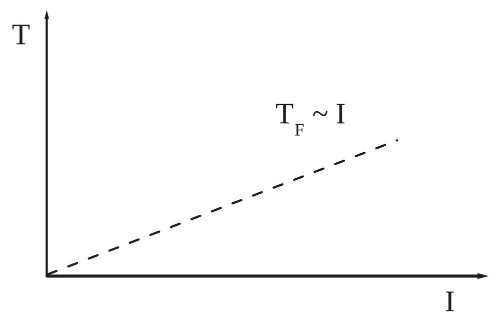
Institut für Laser-Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg

ABSTRACT We discuss ferromagnetism in spinor Bose gases with effective ferromagnetic (FM) couplings between particles. Some basic problems, including interplay between the FM transition and Bose-Einstein condensation, the low-lying collective excitations as Goldstone modes, and the structure of the ground state, are considered. We show on the mean-field level that the FM transition occurs above Bose-Einstein condensation. Under the spin conservation rule, the FM transition corresponds to the domain formation. Our results can be applied to the ultracold ^{87}Rb gas and magnetic dipolar gases.

Ferromagnetism: an overview [1]

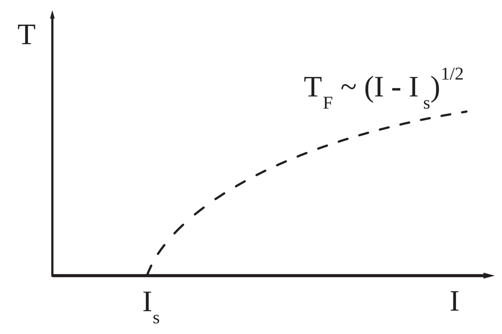
Weiss molecular-field theory: for classical particles

- The model
$$H = -I\sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$
  - Introducing the molecular-field: $M = \langle \mathbf{S}_i \rangle = \langle \mathbf{S}_j \rangle$
  - Decoupling the interaction:
$$\mathbf{S}_i \cdot \mathbf{S}_j \approx \langle \mathbf{S}_i \rangle \cdot \mathbf{S}_j + \mathbf{S}_i \cdot \langle \mathbf{S}_j \rangle - \langle \mathbf{S}_i \rangle \cdot \langle \mathbf{S}_j \rangle = 2MS_i^z - M^2$$
- The result
  - The molecular-field energy: (z-the coordination number)
$$E_m = \langle H \rangle = -zNIM^2 \leq 0$$
  - The FM transition point:
$$k_B T_F = \frac{1}{3} S(S+1) zI \sim I$$



Stoner mean-field theory: for fermions

- M increases the band energy due to the Fermi surface splitting:
$$E_b = \frac{M^2}{4\mu_B^2 N(\epsilon_F)} \qquad N(\epsilon_F)\text{—the density of state}$$
- M leads to a negative molecular-field energy density:
$$E_m = -\frac{1}{2} IM^2$$
- The Stoner criterion results,
$$I > I_s = \frac{1}{2\mu_B^2 N(\epsilon_F)}$$
- The transition temperature:
$$T_F^2 \sim I - I_s$$



Spin waves as Goldstone modes

- The spin-wave in Heisenberg ferromagnets:
$$\omega_k = Dk^2, \qquad D = ISa^2 \qquad a\text{—the lattice constant}$$
- The spin-wave in fermionic ferromagnets:
$$\omega_k = Dk^2$$
  with the spin-wave stiffness
$$D = \frac{1}{6MN}\left[\sum_k n_k \Delta E_k - \frac{2}{MI}\sum_k n_k (\nabla E_k)^2\right]$$
- Summary: $\omega_k \sim k^2$
  gapless at $k = 0$

Ferromagnetism in spinor Bose gases in the thermodynamic limit [2,3]

Phase diagram: a microscopic model [2]

- The model Hamiltonian:
$$H = -t\sum_{\langle ij\rangle\sigma} \psi_{i\sigma}^\dagger \psi_{j\sigma} - I\sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$
- Suppose $\langle \mathbf{S} \rangle = (0, 0, \langle S^z \rangle) = (0, 0, M)$, then we arrive at the mean-field Hamiltonian:
$$H = \sum_{\mathbf{k}\sigma} (\epsilon_{\mathbf{k}} - \sigma H_m) n_{\mathbf{k}\sigma}$$
  with the kinetic energy $\epsilon_{\mathbf{k}} = \hbar^2 k^2 / 2m^*$. m^* is the mass of the particle. $H_m = IM$ is the molecular field.
- The mean-field equations are given by
$$\overline{n} = \frac{N}{V} = \frac{1}{V}\sum_{\mathbf{k}} \langle n_{\mathbf{k},1} + n_{\mathbf{k},0} + n_{\mathbf{k},-1} \rangle \quad (1)$$
$$H_m = I\frac{1}{V}\sum_{\mathbf{k}} \langle n_{\mathbf{k},1} - n_{\mathbf{k},-1} \rangle \quad (2)$$
- The relation between the chemical potential a and the coupling I_m for small I_m
$$a \approx \frac{4\pi}{(3\zeta[\frac{3}{2}])^{\frac{2}{3}}} I_m^2$$
  where a and I_m are rescaled as
$$a = -(\mu + H_m)/(k_B T), \qquad I_m = In^{1/3} m^* / (2\pi\hbar^2)$$

It means that an infinitesimal FM coupling can induce a FM phase transition at a finite temperature above the BEC critical point

Phenomenological analysis [3]

- Free energy density of *isotropic* spin-F Bose-Einstein condensate
$$f_b = \frac{\hbar^2}{2m^*} \nabla\Psi^\dagger \nabla\Psi + \alpha'(T - T_c^0)|\Psi|^2 + \frac{\beta_0}{2}|\Psi|^4 + \frac{\beta_s}{2} \Psi_\sigma^* \Psi_{\sigma'}^* \mathbf{F}_{\sigma\gamma} \cdot \mathbf{F}_{\sigma'\gamma'} \Psi_{\gamma'} \Psi_\gamma$$
- Free energy density of the NORMAL ferromagnetic phase
$$f_m = c|\nabla \mathbf{M}|^2 + a'(T - T_f)\frac{|\mathbf{M}|^2}{2} + b\frac{|\mathbf{M}|^4}{4}$$
- Coupling between two phases
$$f_c = -g|\mathbf{M}|\Psi_\sigma^* \mathbf{F}_{\sigma\gamma} \Psi_\gamma$$
- The total Ginzburg-Landau free energy is $f_t = f_b + f_m + f_c$. Minimizing f_t with respect to Ψ^* and M, one gets
$$T_c = T_c^0 + \frac{g}{\alpha'} M = T_c^0 + \frac{g}{\alpha'}\sqrt{\frac{a'}{b}[T_f - T_c]}$$
- At $I \to 0$, both $\delta T_c = T_c - T_0$ and $\delta T_f = T_f - T_0$ tend to zero. So we get
$$(\delta T_c) = \delta T_f \left(1 - \frac{\delta T_f}{T^*}\right)$$
  with $T^* = (g/\alpha')^2 a'/b$
- Suppose $\delta T_f = CI$ when $I << 1$,
$$\delta T_c = CI \left(1 - \frac{CI}{T^*}\right)$$

Thus the phase diagram is reproduced.

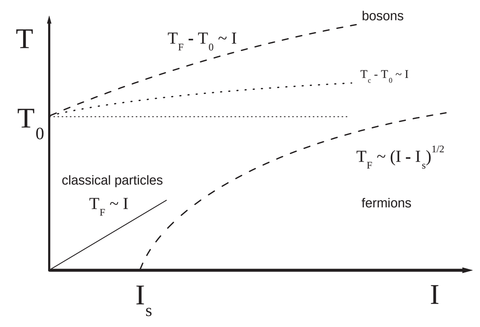
Spin waves [3]

- Considering fluctuations: $\mathbf{M} = (\delta M_x, \delta M_y, M + \delta M_z)$
  $\mathbf{\Psi}^\dagger = (\Psi_1^* + \delta\psi_1^*, \delta\psi_0^*, \delta\psi_{-1}^*)$
- We get
  - The density excitation
$$f^I(\delta\Psi_1^*, \delta\Psi_1) = \sum_{\mathbf{k}}{}' \left(\delta\Psi_1^*(\mathbf{k}) \; \delta\Psi_1(-\mathbf{k})\right) \times \begin{pmatrix} \epsilon_k + \beta\Phi_1{}^2 & \beta\Phi_1{}^2 \\ \beta\Phi_1{}^2 & \epsilon_k + \beta\Phi_1{}^2 \end{pmatrix} \begin{pmatrix} \delta\Psi_1(\mathbf{k}) \\ \delta\Psi_1^*(-\mathbf{k}) \end{pmatrix},$$
  with the spectrum
$$\hbar\omega_1 = \sqrt{\epsilon_k^2 + 2\beta\Phi_1{}^2 \epsilon_k}$$
  - The spin excitation
$$f^I(\delta M_+, \delta M_-; \delta\Psi_0^*, \delta\Psi_0) = \sum_{\mathbf{k}} \left(\delta M_+(\mathbf{k}) \; \delta\Psi_0^*(\mathbf{k})\right) \times \begin{pmatrix} ck^2 + \frac{g\Phi_1{}^2}{2M_0} & -\frac{\sqrt{2}}{2} g\Phi_1 \\ -\frac{\sqrt{2}}{2} g\Phi_1 & \epsilon_k + gM_0 \end{pmatrix} \begin{pmatrix} \delta M_-(\mathbf{k}) \\ \delta\Psi_0(\mathbf{k}) \end{pmatrix}$$
  which suggest the spin waves in normal and condensed components are coupled, with the mixed spin-wave spectrum
$$\hbar\omega_0 \approx c_s k^2, \quad \text{with} \quad c_s = c + \frac{1}{2}\frac{\Phi_1^2}{M_0^2}\frac{\hbar^2}{2m}$$

1. The spectrum has the same momentum-dependance as in usual ferromagnets. 2. The spin-wave stiffness contains two parts, implying "two fluids" feature of the system.

Discussions and outlooks

Phase diagram of ferromagnets



Experimental background

Our results are relevant to current experiments on quantum degenerate atomic bosons.

- Atomic bosons with FM scattering: ^{87}Rb [4]
$$V = \frac{c_0}{2} n \cdot n + \frac{c_s}{2} \mathbf{S} \cdot \mathbf{S}, \qquad c_s < 0$$

- Magnetic dipolar Bose gases [5]
$$U_{md} = \frac{\mu_0}{4\pi r^3}[\mathbf{m}_1 \cdot \mathbf{m}_2 - 3(\mathbf{m}_1 \cdot \hat{\mathbf{r}})(\mathbf{m}_2 \cdot \hat{\mathbf{r}})]$$

The dipolar interaction causes ferromagnetism:
the dipolar ferromagnetism

Though the FM interaction is very week, we expect that the FM transition or dipolar ferromagnetism can be observed at a relatively high temperature.

Spinor Bose gases under experimental conditions

In experiments, the system is away from the thermodynamic limit.

- Total spin conservation
- Small particle number

Can the spin-rotational symmetry be broken spontaneously?

- If yes, [2,3,6]
  - FM transition corresponding to domain formation
  - Domains formed before BEC
  - F_z=1 and -1 domains spatially separated
  - F_z=0 domains much smaller
- If not, [6,7]
  - F_z=1,0 and -1 particles miscible
  - F_z=0 bosons condensed earlier than $\pm$1 bosons
  - F_z=0 component dominating over others

Outlooks

How does the ground state manifest in experiments:
the 3 components miscible or spatially separated?

To examine this, one can

- investigate spin dynamics [8]
- determine fractions of all the components

† with K. Bongs, K. Sengstock and R.A. Klemm

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