Controlled coherent and dissipative dynamics in optical lattices

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QIP IRC

People

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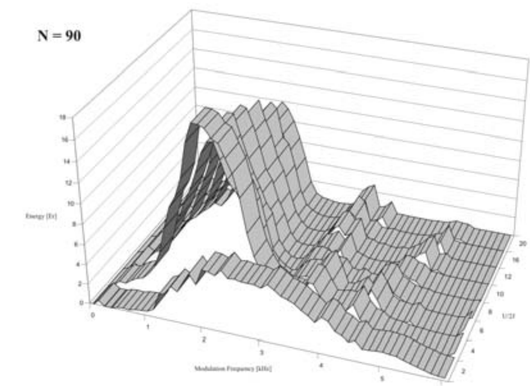
Innsbruck:

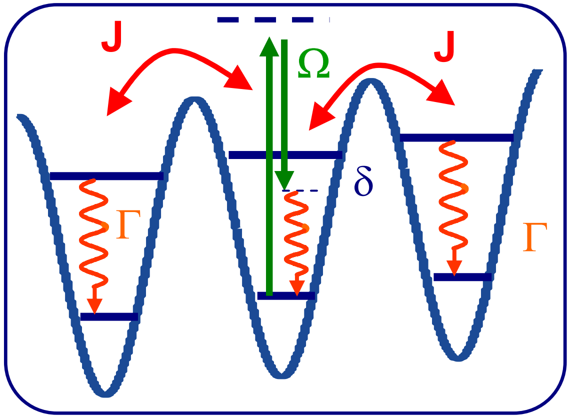
- A. Daley
- A. Griessner (recent Oxford visitor)
- P. Zoller

Overview

- Part I: Controlled coherent dynamics of a strongly correlated atomic system
  - ⇨ Slow and fast ramping of an optical lattice
  - ⇨ Depth oscillations in the lattice
  - ⇨ Experimental dynamics described by the BHM
  - ⇨ Numerical experiments in strongly correlated systems
  - ⇨ Guides to analytical studies
  - ⇨ Suitability for QC → Entanglement properties



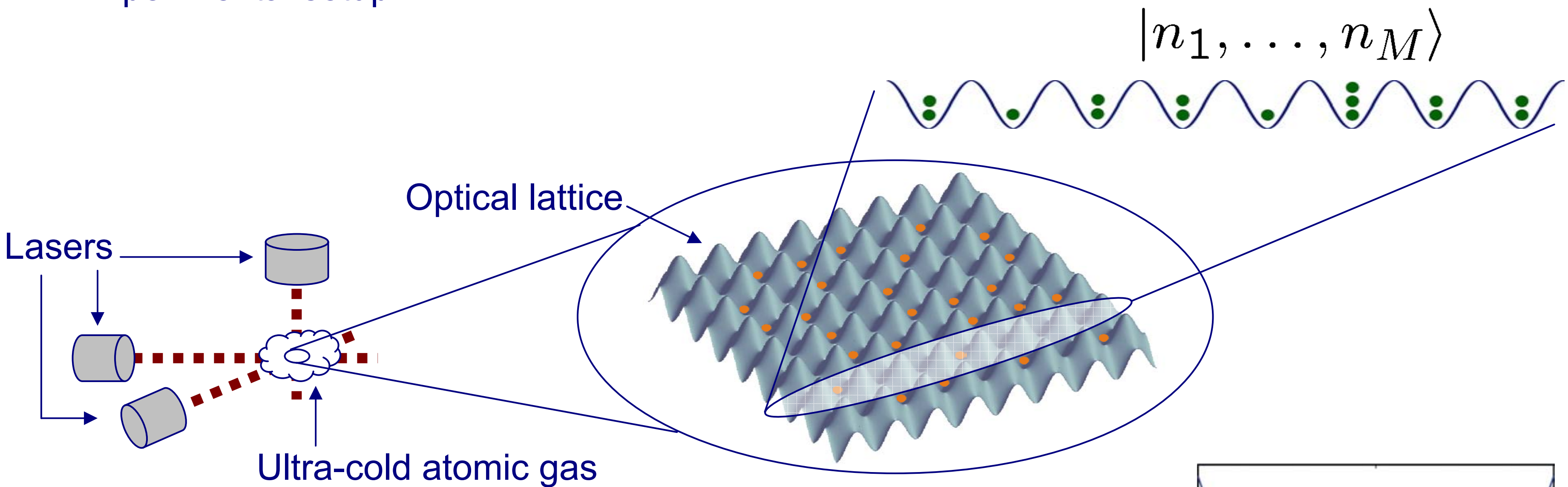
- Part II: Loading and cooling of atoms by spontaneous emissions of phonons
  - ⇨ Loading a lattice by irreversible processes
  - ⇨ Cooling atoms in a lattice
  - ⇨ Quantum optics with phonons → Cavity QED (?)
  - ⇨ Preparation of a quantum register
  - ⇨ Quantum simulation at extremely small temperatures
  - ⇨ Cooling in the lowest Bloch band



Physical system and numerics

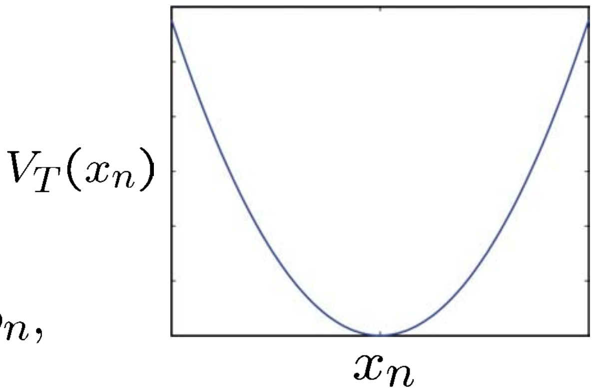
Optical lattice in 1D

- Experimental setup



- Described by the 1D BHM

$$H = -J\sum_n (b_n^\dagger b_{n+1} + \text{h.c.}) - \mu_n b_n^\dagger b_n + \frac{U}{2} b_n^\dagger b_n^\dagger b_n b_n,$$

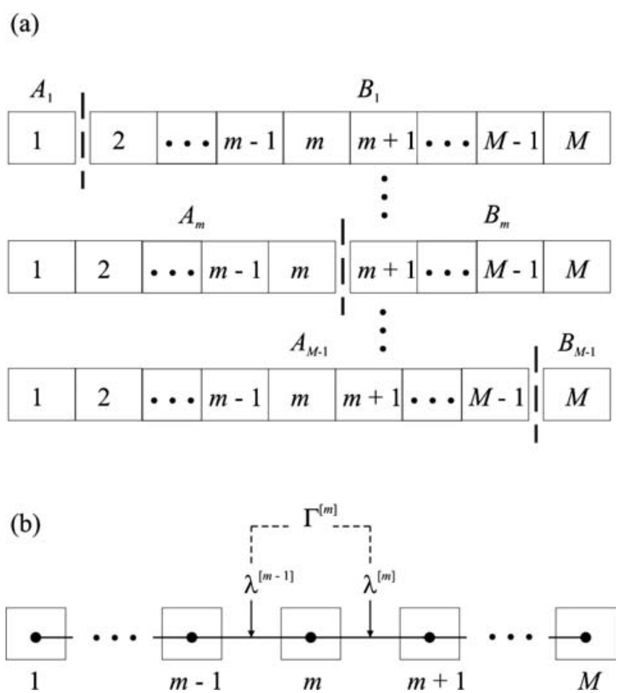


With local chemical potential $\mu_n = \mu - V_T(x_n)$

Numerical method

- We use the TEBD algorithm Vidal (2003), Verstraate & Cirac (2004), Werner (1990)
- System described by a state $|\psi\rangle = \sum_{n_1=0}^{\infty} \cdots \sum_{n_M=0}^{\infty} c_{n_1 \cdots n_M} |n_1, \ldots, n_M\rangle$

  and fix the maximum occupation as $n_{\text{max}} = 5$



Perform successive SD of the system

$$|\psi\rangle = \sum_{\alpha=1}^{\chi_m} \lambda_\alpha^{[m]} \left|\phi_\alpha^{A_m}\right\rangle \left|\phi_\alpha^{B_m}\right\rangle$$

Truncate these to a maximum rank χ

Use the SDs to form tensors Γ and λ

This gives an expansion in matrix product states

$$c_{n_1 \cdots n_M} = \sum_{\alpha_1, \ldots, \alpha_{M-1}} \Gamma_{\alpha_1}^{[1]n_1} \lambda_{\alpha_1}^{[1]} \Gamma_{\alpha_1\alpha_2}^{[2]n_2} \lambda_{\alpha_2}^{[2]} \cdots \lambda_{\alpha_{M-1}}^{[M-1]} \Gamma_{\alpha_M}^{[1]n_M}$$

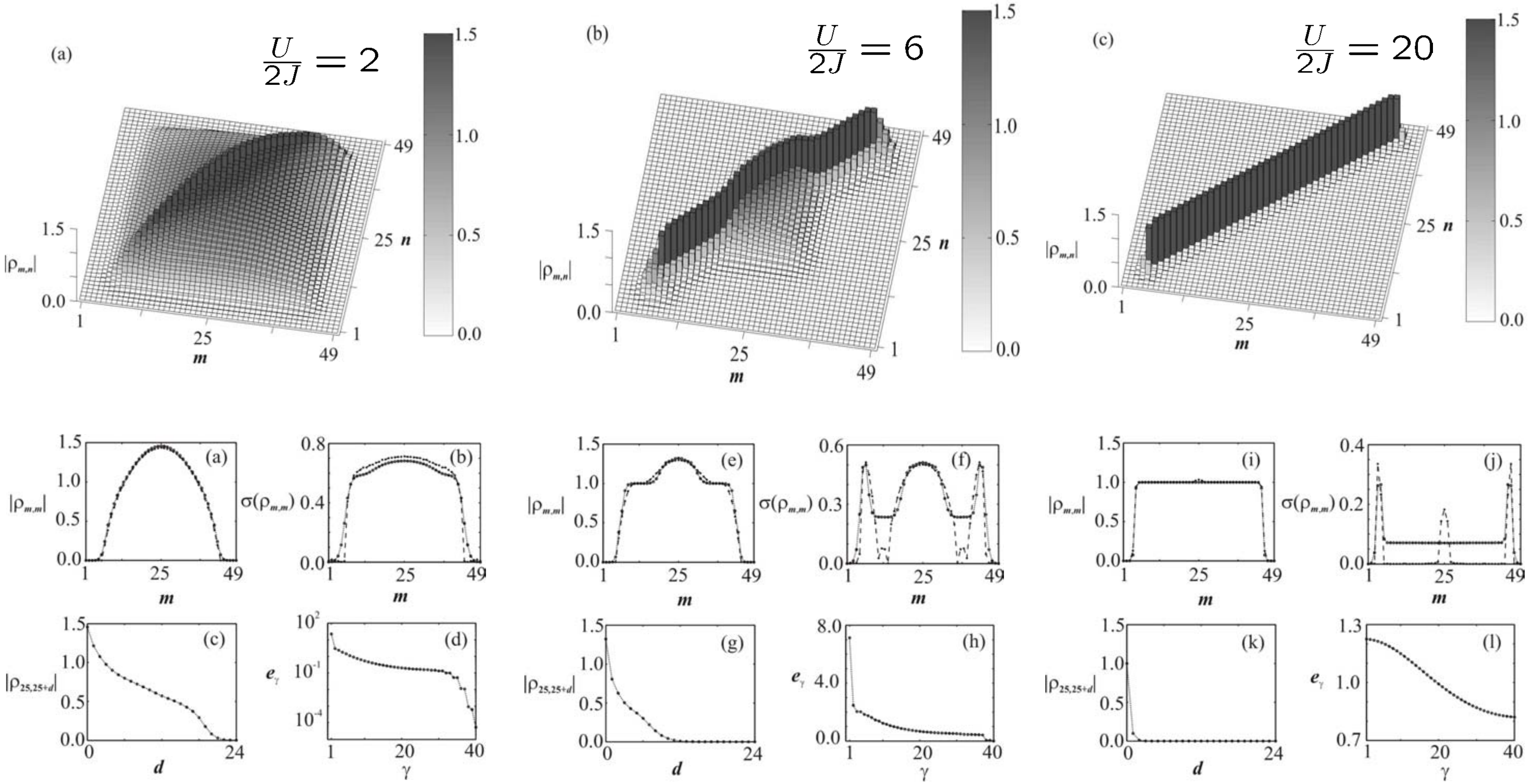
Part I: Ramping the optical lattice

Ground states

S.R. Clark and D.J, PRA to be published

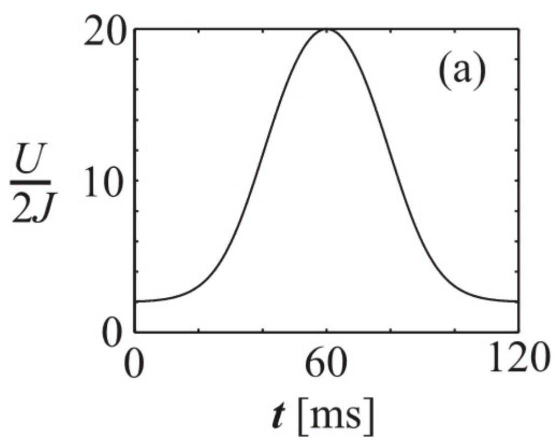
- For M=49 sites including trapping we plot the single particle density matrix $|\rho_{n,m}|$

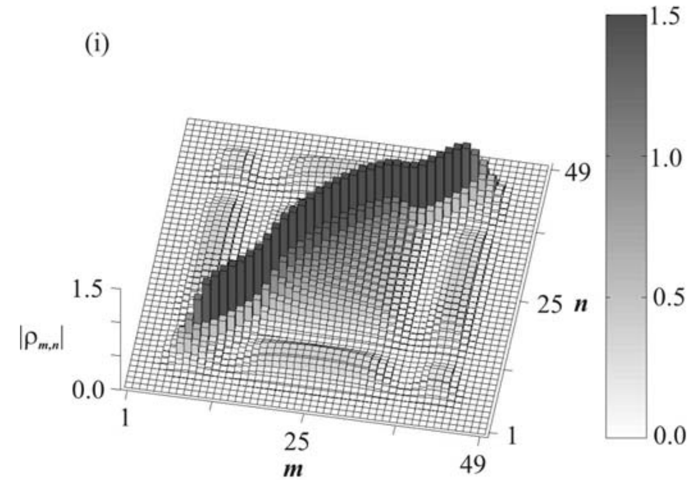
SF | Intermediate | MI



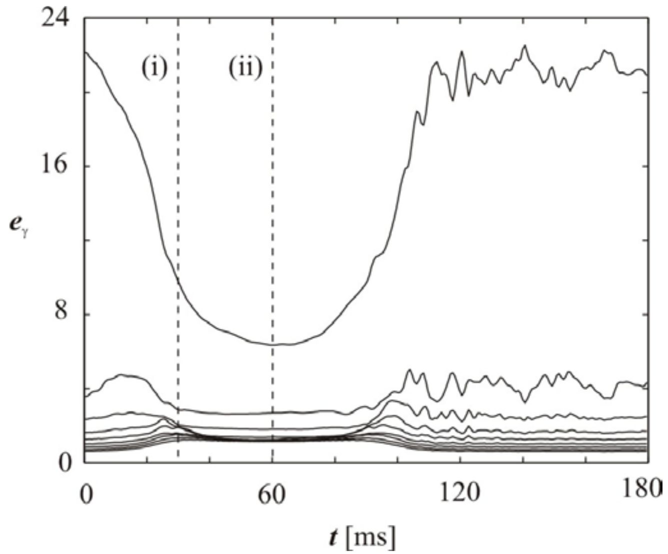
Slow dynamics

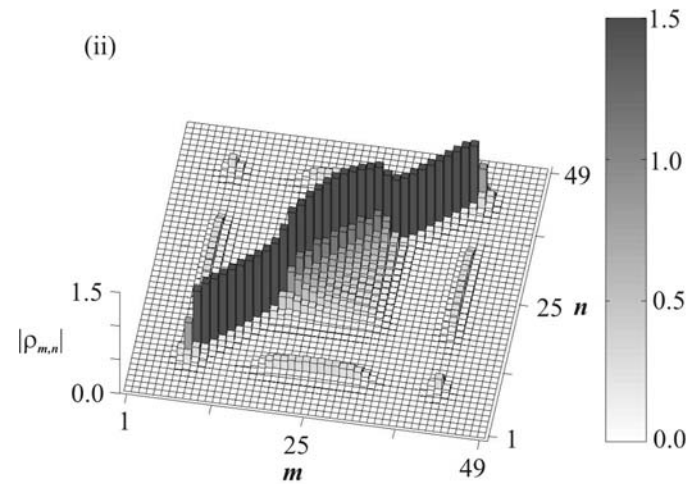
- We consider "slowly" ramping the lattice for $M = 49$





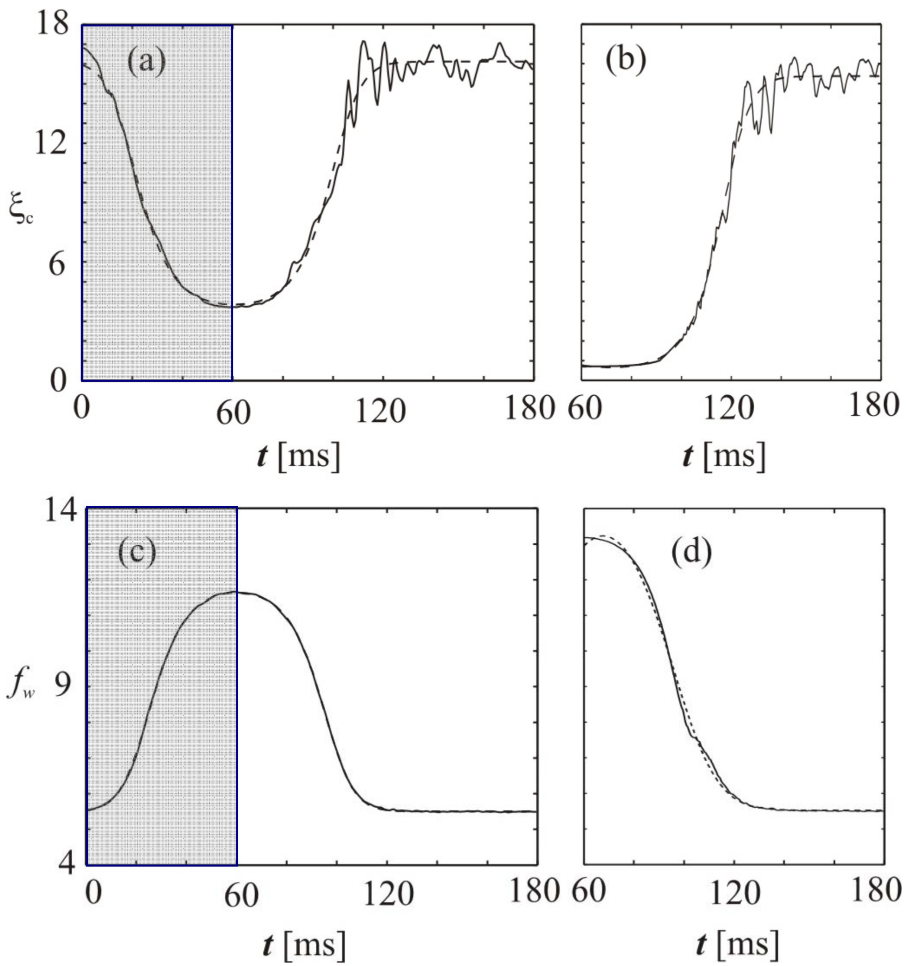
- Eigenvalues of the single particle density matrix





Slow dynamics cont.

- Correlation length cut-off length and momentum distribution width



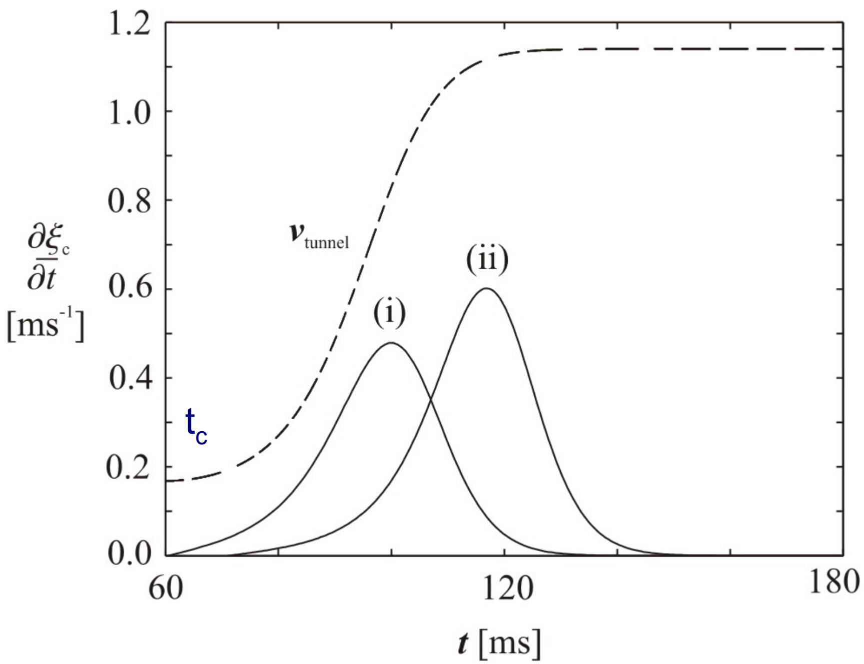
- Define a correlation cut-off length ξ_c
  $|\rho_{25,25+\xi_c}| = 1/e \approx 0.37$

- And also consider the momentum distribution width f_w

- Starting from the MI ground state at t=60ms yields similar results.

Slow dynamics cont.

- We also compute the correlation cut-off speed $\frac{\partial \xi_c}{\partial t}$ for $t > t_c$

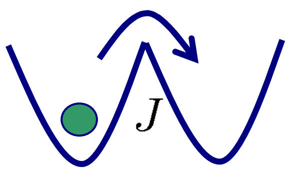


(i) = dynamically driven

(ii) = MI ground-state

$$\tau_{\text{tunnel}}(t) = \frac{\pi}{2J(t)}$$

$$v_{\text{tunnel}}(t) = \frac{1}{\tau_{\text{tunnel}}(t)}$$

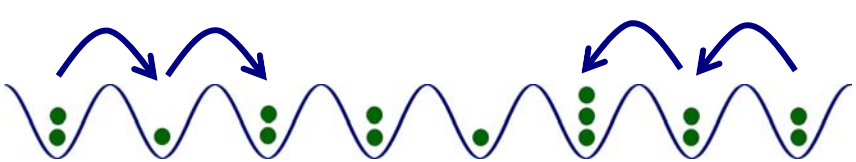


- Note that neither correlation speed exceeds the instantaneous tunnelling speed.

- Consistent with the growth of correlations being caused by a single atom hopping to the centre.

Greiner *et al* Nature (2002)

Batrouni *et al* PRL (2002)



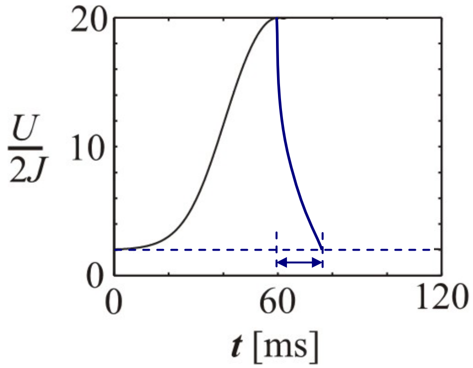
$$t_{\text{restore}} = \frac{M\tau_{\text{tunnel}}}{2} \approx 23 \text{ ms}$$

Fast dynamics

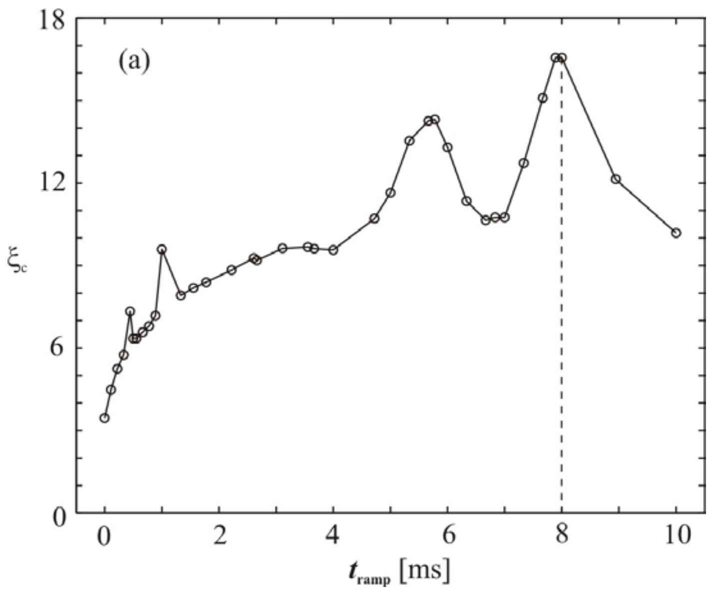
- Replace the latter half $t > t_c$ with rapid linear ramping of the form

$$V_0(t) = V_{\mathrm{MI}} - \frac{(V_{\mathrm{MI}} - V_{\mathrm{SF}})}{t_{\mathrm{ramp}}}(t - t_c)$$

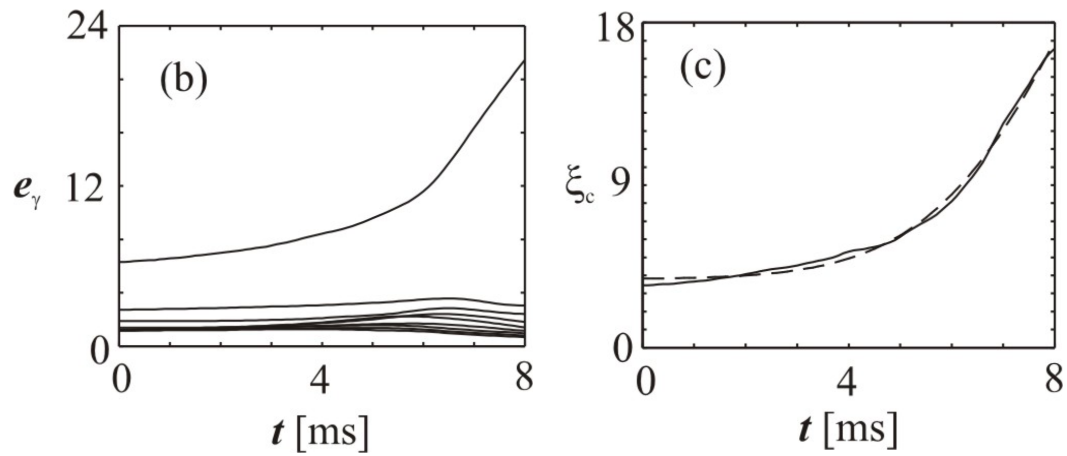
where t_{ramp} is the total ramping time.



- We considered t_{ramp} between 0.1 ms and 10 ms.



- Focussing on $t_{\mathrm{ramp}} = 8$ ms



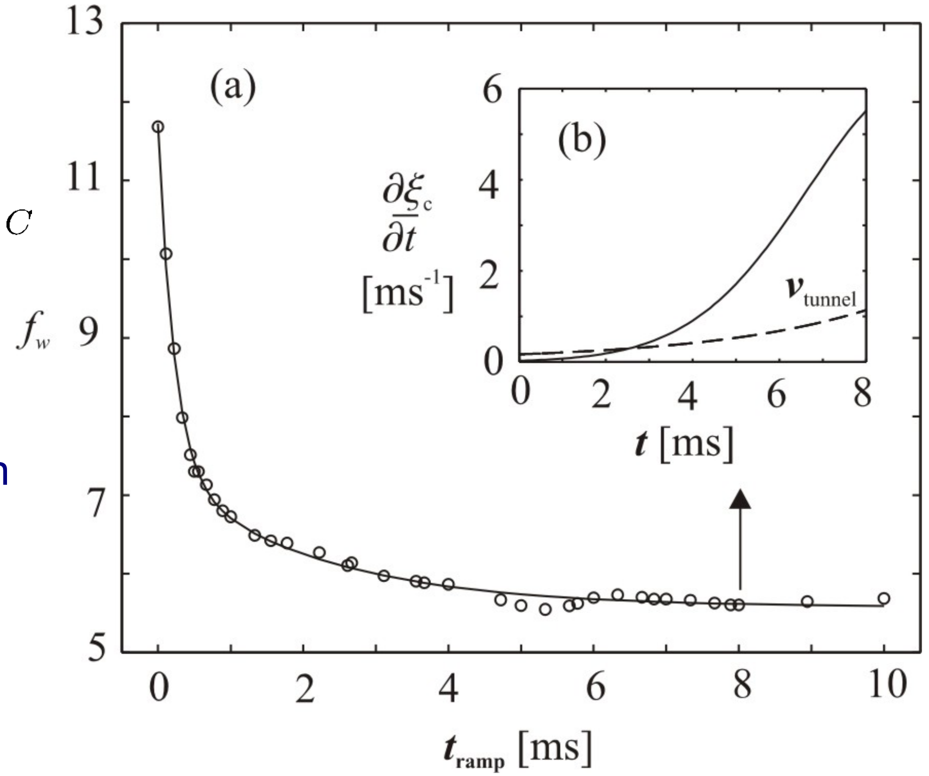
Fast dynamics cont.

- Here we plot (a) the final momentum distribution width f_w for each rapid ramping.

- The fitted curve is a double exponential decay

$$f_w(t_{\rm ramp}) = A_1\ e^{-t_{\rm ramp}/\tau_1} + A_2\ e^{-t_{\rm ramp}/\tau_2} + C$$

with $\frac{\tau_2}{\tau_1} \approx 10$



- The steady state SF width is acquired in approx. 4 ms.

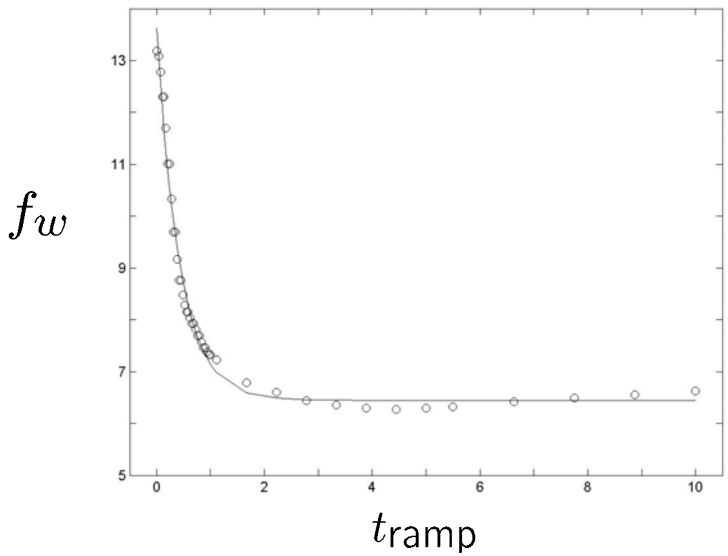
- For the 8 ms ramping we plot (b) the correlation speed.

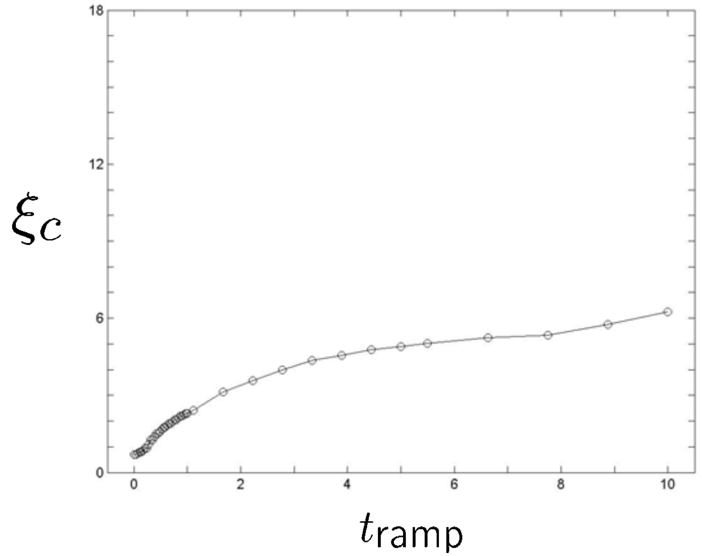
- Rapid restoration explicable with BHM alone, and occurs in 1D.

- Higher order correlation functions are important – **how do they contribute?**.

Starting from the MI ground state

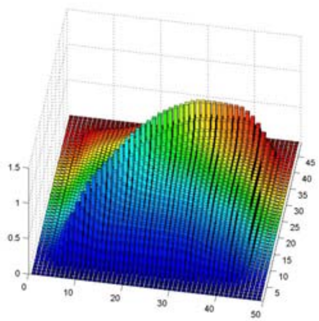
- Ratio of characteristic times roughly one but cut-off much smaller



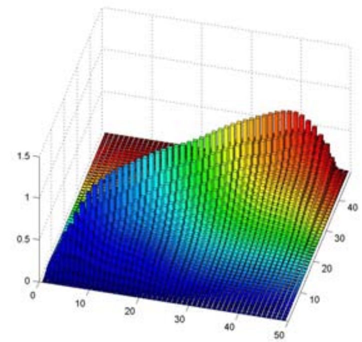


- The one-particle density matrix after 8ms of ramping from the dynamical state.

dynamical



ground

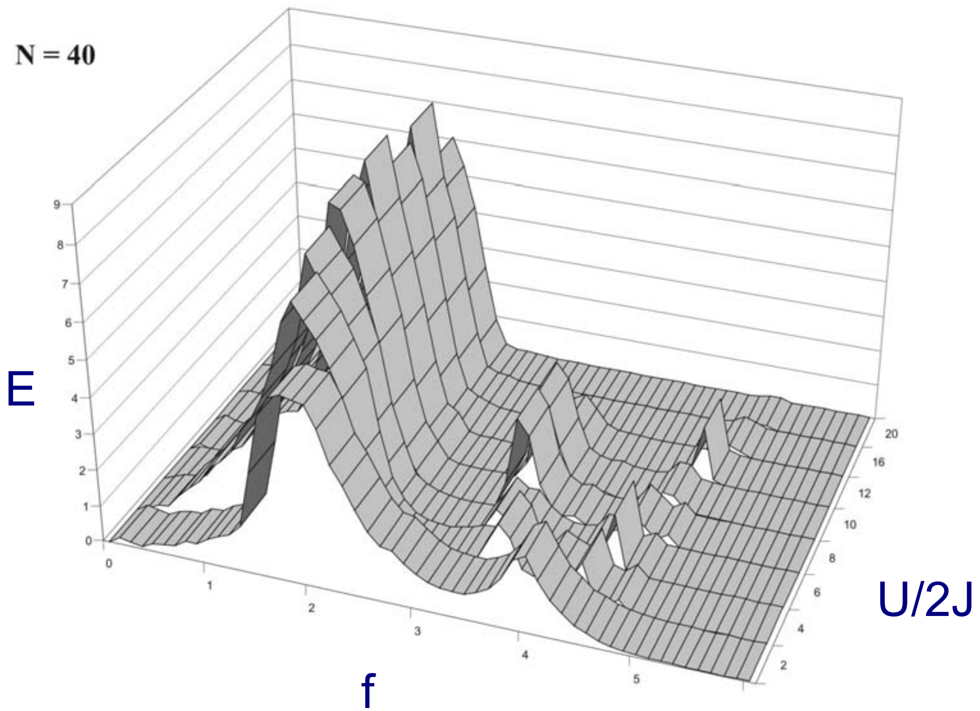


Part I: The excitation spectrum

Probing the excitation spectrum

S.R. Clark and D.J, work in progress

- Using a large $M = 59$ system and computing the excitation spectrum generated by lattice depth modulation : $V_{\text{mod}}(x,t) = \left(V_0 + A_{\text{mod}} \sin\left(2\pi\nu_{\text{mod}} t\right)\right) \sin^2\left(kx\right)$



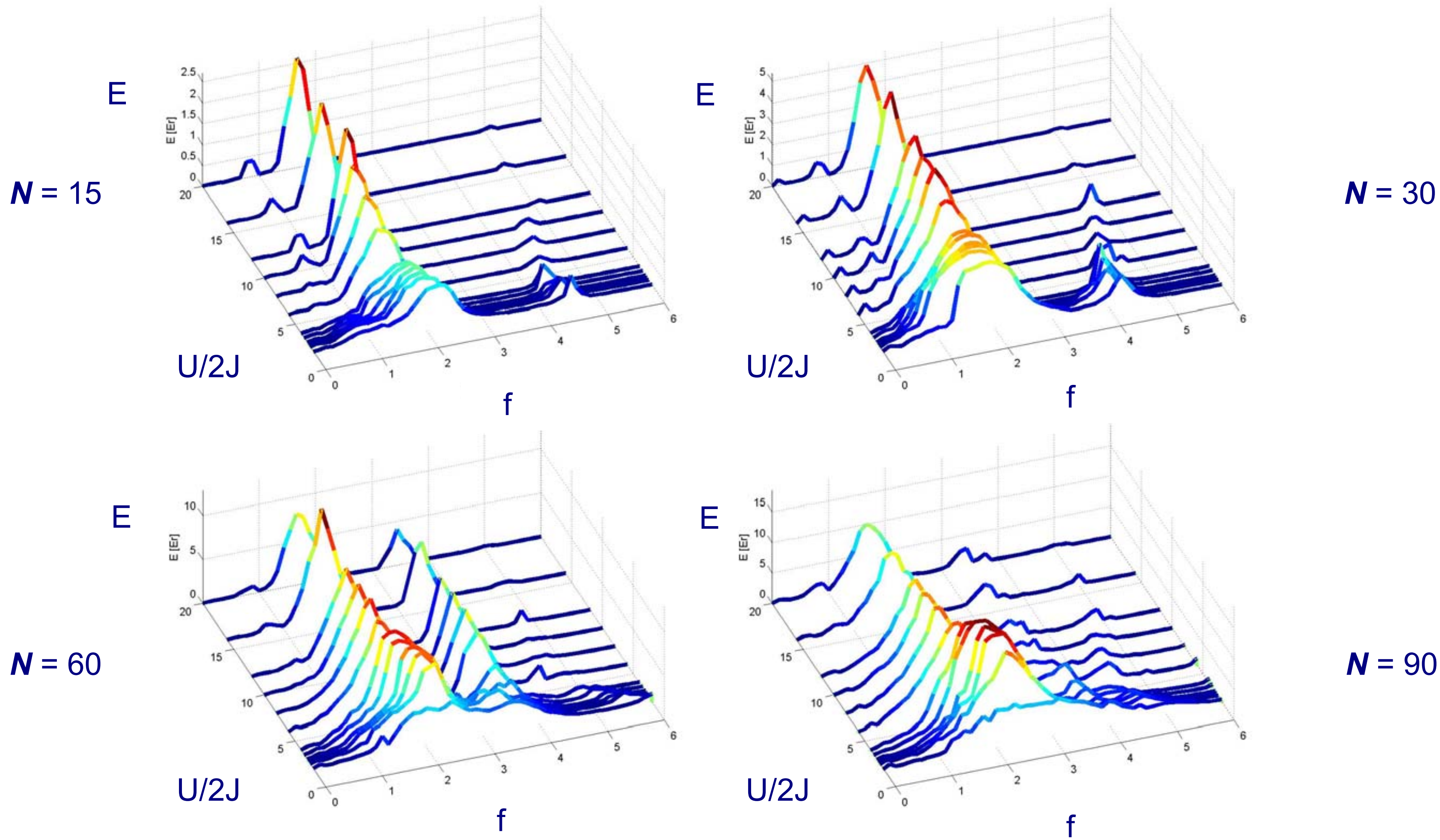
- Look at final energy versus modulation frequency, after applying the modulation for **30**ms, for a variety of lattice depths

- ***U*** has a value of around 1.5 kHz over these depths.

as in Stoferle et al PRL (2004)

Dependence on the number of particles

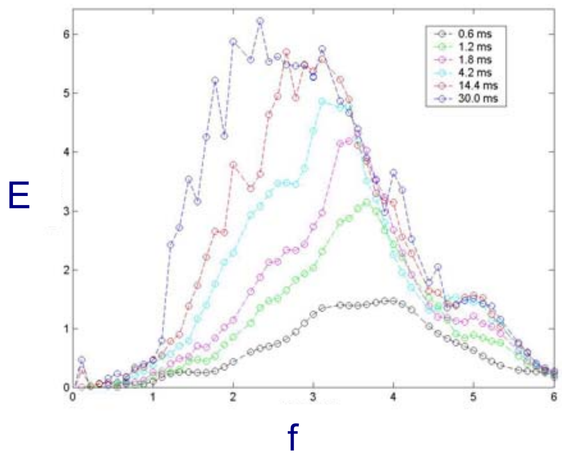
- We consider varying numbers of atoms with showing MI and SF with 1 to 2 particles

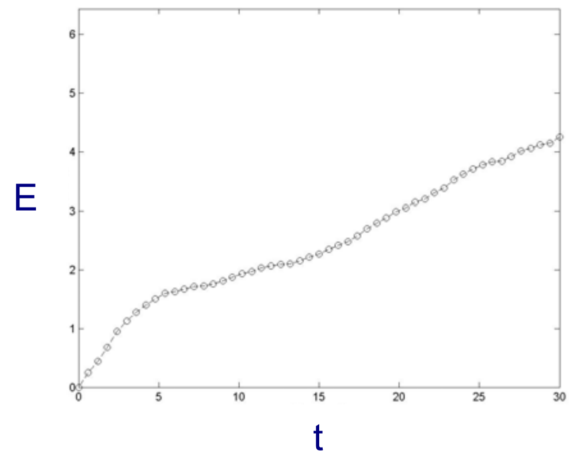


Time dependence

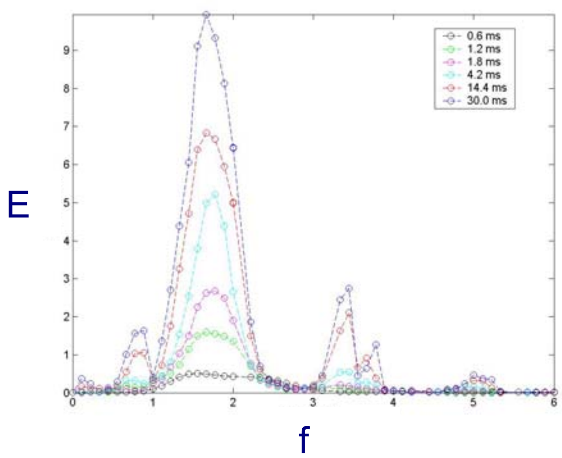
- We look at how the energy is put into the system at ***N***=90

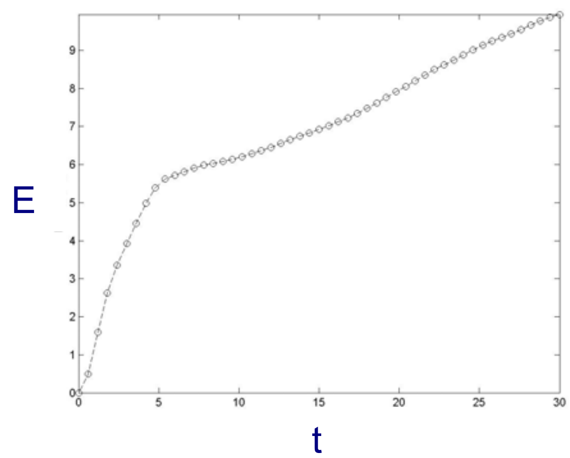
SF





MI

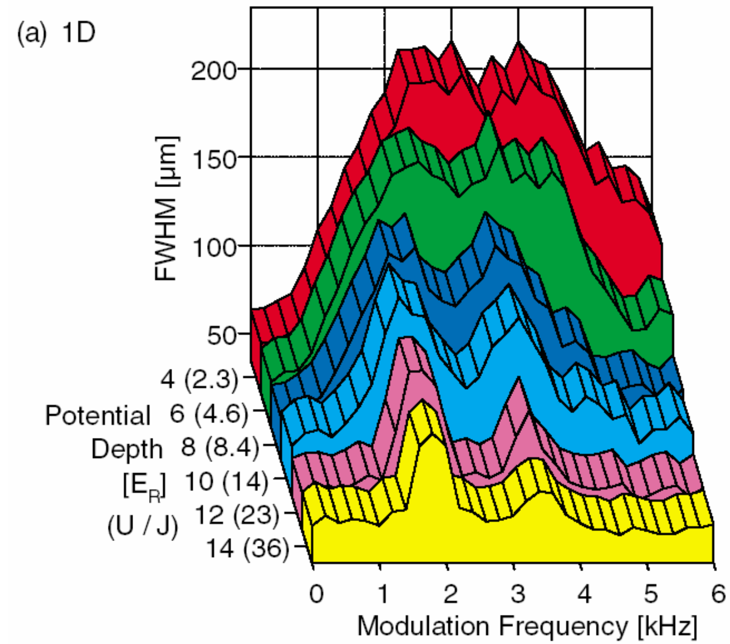


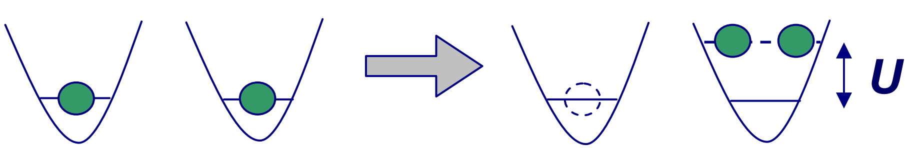


- not linear response!
- details at short times also no linear behavior
- shift of peak in SF region with time
- agreement with exp. good

Probing the excitation spectrum cont.

- We obtain very good agreement with the experimental data
  - ⇨ broad spectrum in the superfluid region
  - ⇨ split up into several peaks when going to the MI regime
  - ⇨ Shift of the MI peaks from U by approximately 10%
- Differences compared to the experiment
  - ⇨ Different relative heights of the peaks
  - ⇨ Transition between SF and MI region at a different value of ***U/J***
- Explanation of peaks
  - ⇨ Gap in the MI
  - ⇨ Hopping between SF and MI due to trap
  - ⇨ Higher order processes





exp. Stoferle et al PRL (2004)

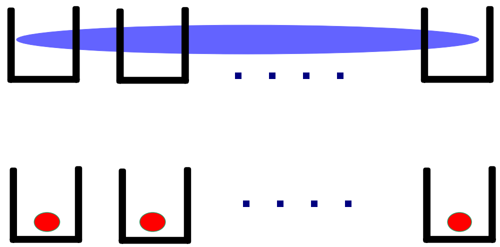
Part II: Irreversible loading schemes

Current loading methods

- Adiabatic loading in one sweep with almost no disorder
  - ⇨ Arrange atoms by repulsion between bosons

Supefluid

Mott insulator



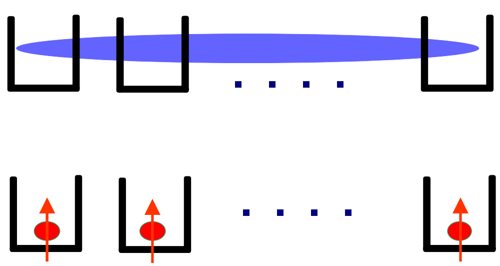
D. J., C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).

M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, and I. Bloch, Nature **415**, 39 (2002).

  - ⇨ Arrange atoms by Fermi blocking

Fermi gas

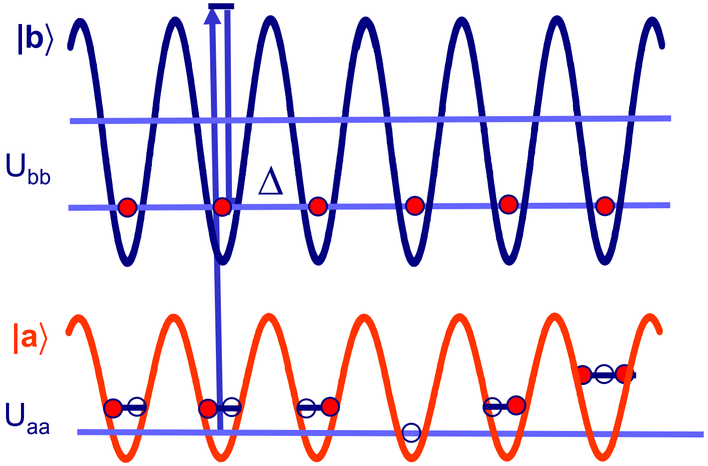
Single occupancy



L. Viverit, C. Menotti, T. Calarco, A. Smerzi, quant-ph/0403178.

Possible Improvements

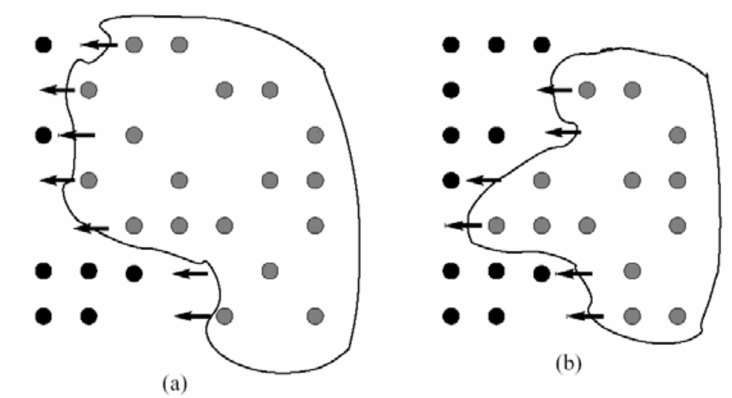
- Defect suppressed lattices



irregular → regular filling, i.e., mixed state → pure state → cooling

P. Rabl, A. J. Daley, P. O. Fedichev, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **91**, 110403 (2003).

- Measurement based schemes



Selective shift operations to close gaps

J. Vala, A.V. Thapliyal, S. Myrgren, U. Vazirani, D.S. Weiss, K.B. Whaley, quant-ph/0307085.

Selective measurements of double occupancies

G.K. Brennen, G. Pupillo, A.M. Rey, C.W. Clark, C.J. Williams, quant-ph/0312069.

Repeatable irreversible loading schemes?

- Can we combine irreversible processes with repulsive and/or Fermi blocking for irreversible loading schemes?

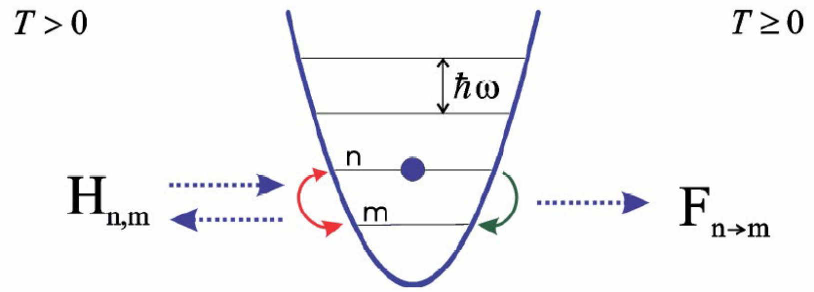
  - ⇨ Spontaneous emission of photons
    - Large momentum kick → heating
    - Large energies → no selectivity
    - Reabsorption → heating

N

PHOTONS

  - ⇨ Other mechanisms
    - Spontaneous emission of phonons
    - Decay of hole excitations
    - Evaporative cooling
    - Photodissociation

A.J. Daley, *et al.* Phys. Rev. A **69**, 022306 (2004).

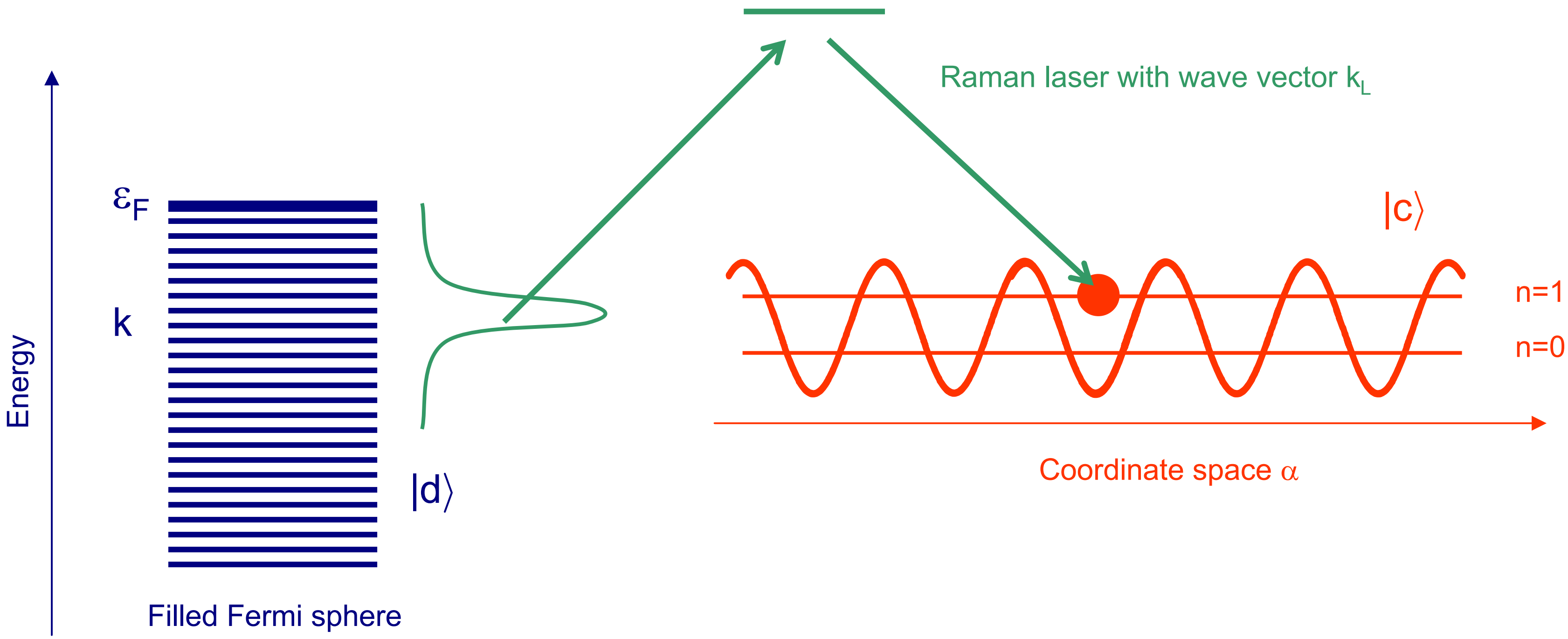


We combine irreversible processes with the blocking mechanisms

Fermionic lattices

A. Griessner *et al.*, in progress

- We consider an optical lattice immersed in an ultracold Fermi gas



Fermi sphere: $H_{\text{res}} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} d^\dagger_{\mathbf{k}} d_{\mathbf{k}}$

Optical Lattice: $H_{\text{sys}} = \sum_{\alpha,n} (\omega \delta_{n,1} + \epsilon_\alpha) c^\dagger_{\alpha,n} c_{\alpha,n}$ No hopping

Raman laser transitions

- Raman Hamiltonian

$$H_{\mathrm{RL}} = \sum_{\mathbf{k}} \sum_{\alpha,n} (R_{\alpha,n,\mathbf{k}} d_{\mathbf{k}} c^{\dagger}_{\alpha,n} + \mathrm{h.c.}) + \sum_{\mathbf{k}} \Delta_1 d^{\dagger}_{\mathbf{k}} d_{\mathbf{k}} + \sum_{\alpha,n} \Delta_2 c^{\dagger}_{\alpha,n} c_{\alpha,n}$$

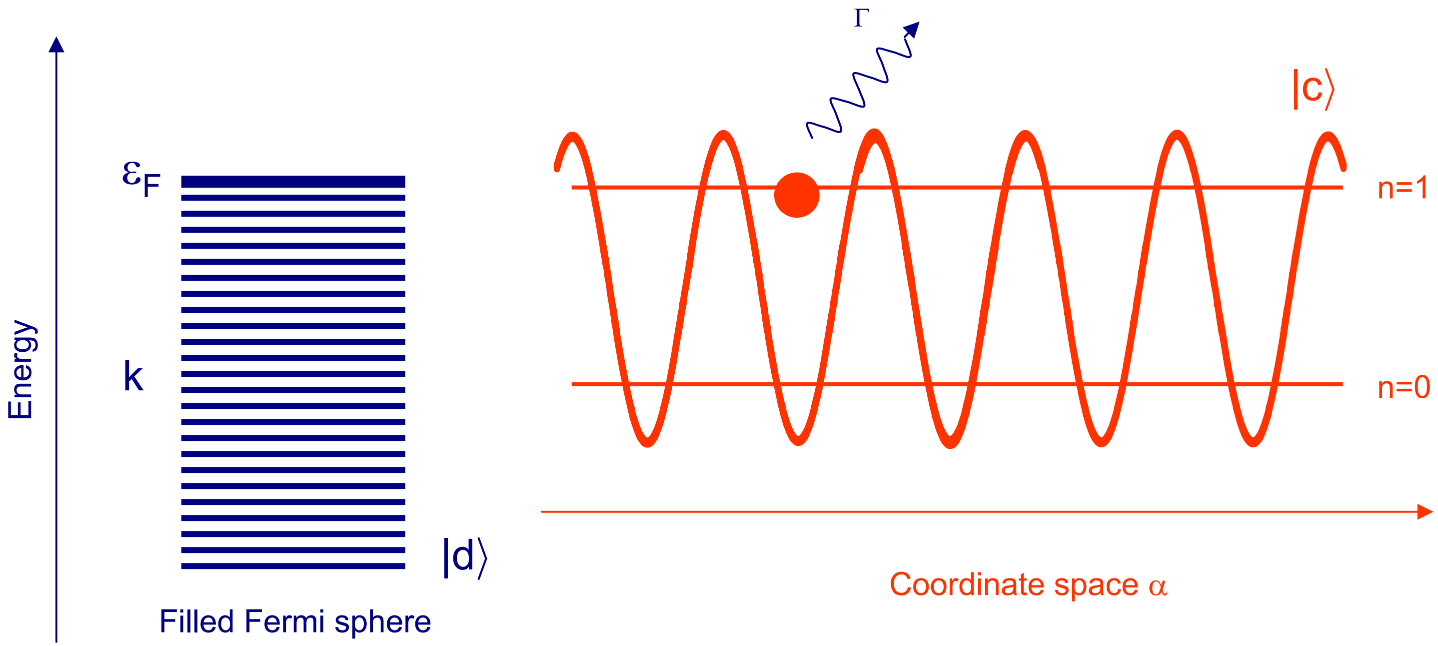
- ⇨ By choosing Δ_2 space dependent pattern loading can be achieved
- ⇨ Different Fermi sphere modes are addressed by varying Δ_1
- ⇨ The Fermi energy ε_F is much smaller then the band separation ω
- ⇨ The coupling between Fermi sea and lattices is determined by $R_{\alpha,k}$ which reads

$$R_{\alpha,n,\mathbf{k}} = \frac{\Omega_{\mathrm{eff}}}{\sqrt{V}} \mathrm{e}^{\mathrm{i}(\mathbf{k}-\mathbf{k}_L)\mathbf{x}_\alpha} \int \mathrm{d}^3 x \mathrm{e}^{-\mathrm{i}(\mathbf{k}-\mathbf{k}_L)\mathbf{x}} \phi_{\mathbf{n}}(\mathbf{x})$$

- ⇨ We consider the case where $|R_{\alpha,k}|^{-1}$ is large compared to the time it takes a Fermi particle to move between the lattice sites $|R_{\alpha,k}|^{-1} \gg \lambda/2v_F = T$, where v_F is the Fermi velocity and λ the optical lattice wave length
  - This allows to “locally” increase the particle density in the loading process
  - We can obtain energy selective loading of the lattice

Loading into the band n=1

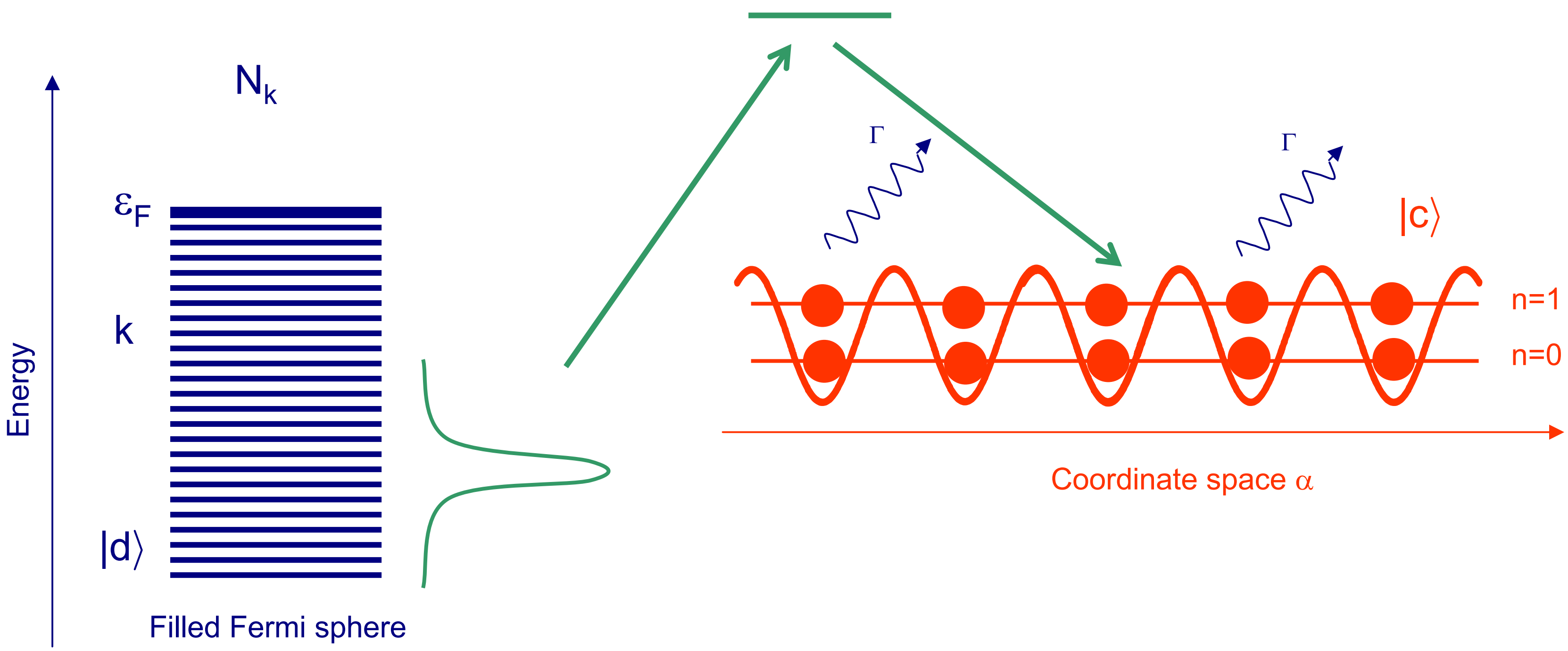
- Consider the interactions of lattice particles with the Fermi sphere



This interaction can cause the creation of particle/hole excitations:

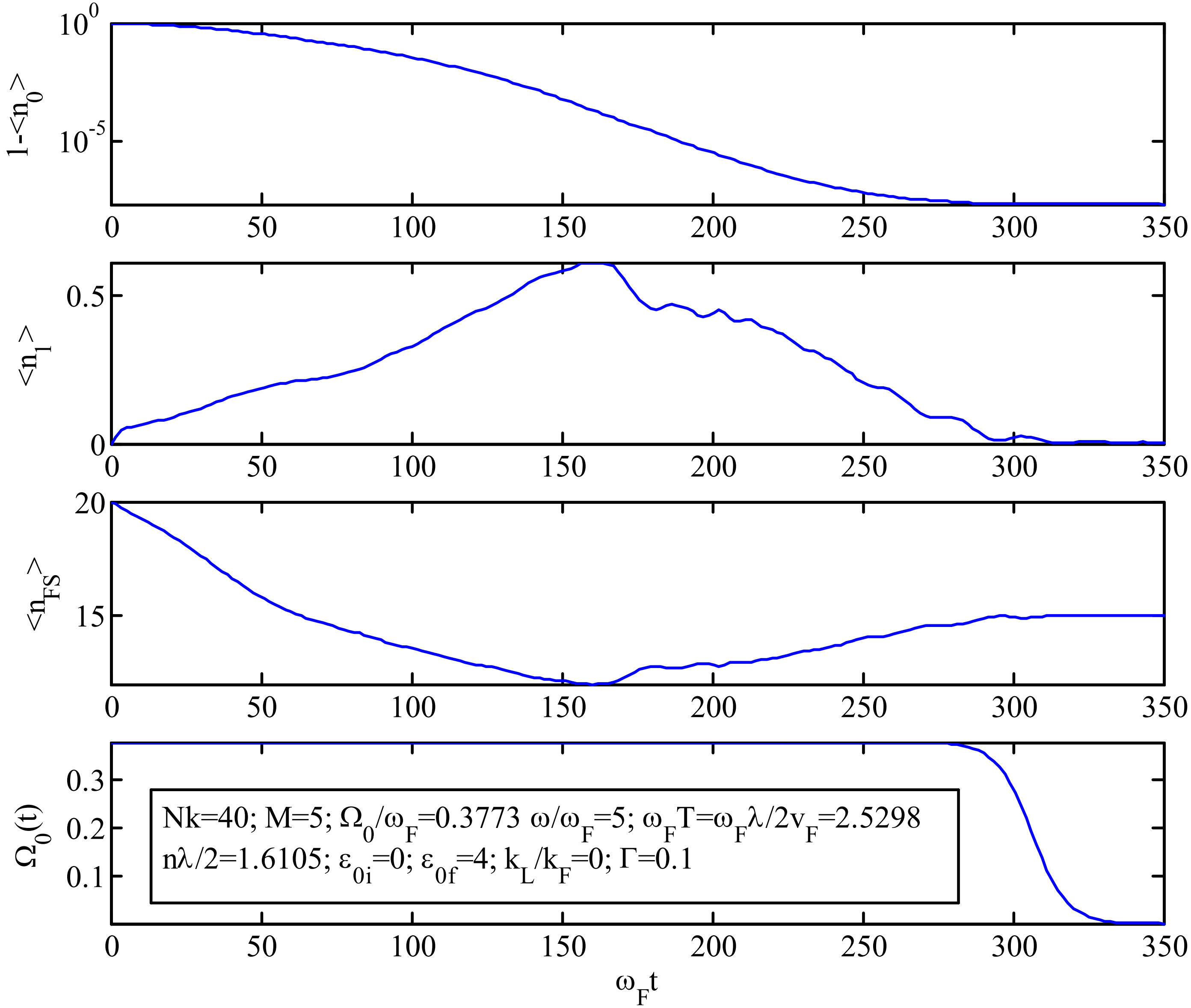
$$H_{\mathrm{int}} = \frac{g}{V}\sum_{\mathbf{k},\mathbf{k}'}\sum_{\alpha,m,n}\int \mathrm{d}^3x\, \mathrm{e}^{\mathrm{i}\mathbf{x}(\mathbf{k}'-\mathbf{k})}\phi_m(\mathbf{x}-\mathbf{x}_\alpha)\phi_n(\mathbf{x}-\mathbf{x}_\alpha)d^\dagger_{\mathbf{k}}d_{\mathbf{k}'}c^\dagger_{im}c_{in}$$

Linear sweep to load the whole lattice



- We change the laser detuning dynamically
  1. Sweep through the Fermi sea to load the first band
  2. Spontaneously emit phonons
  3. empty remaining atoms by tuning above the Fermi sea

Numerical results



Particle hole excitations - numbers

- Creation of a particle hole excitations at a rate $\Gamma \approx a_s^2 \rho_0 \omega^{3/2}$

  ⇨ For typical experimental parameters $\Gamma \approx 2\pi \times 1$kHz

- Excitation life time τ limited by (important for reabsorption)

  ⇨ p-wave collisions very small $\sigma_p \propto T^2$ giving typically $\tau \approx 2$s
  ⇨ Sympathetic cooling with another species yields $\tau \approx 1$ms
  ⇨ Evaporation at a distance of $l \approx 1\mu$m in $\tau \approx 30\mu$s

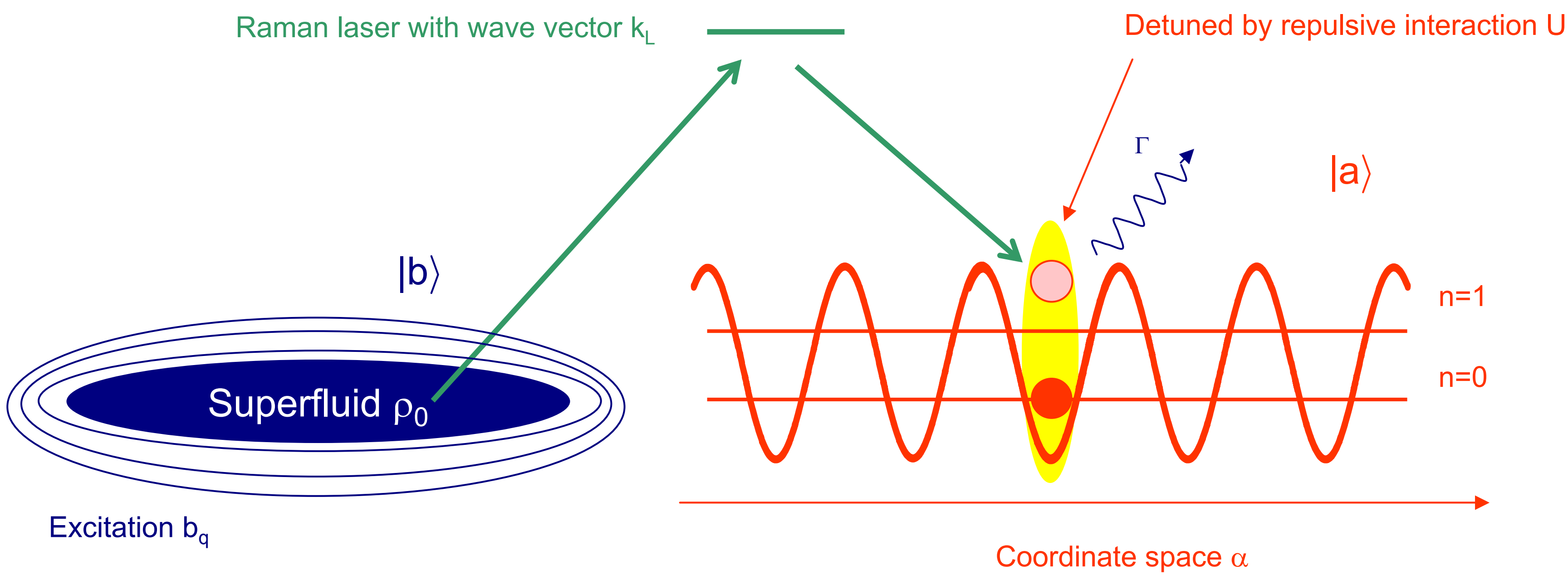
- Reabsorption does not cause heating but slowing of the cooling process

  ☒ Typical parameters: $\rho_0 \approx 10^{13}$ cm^{-3}, $T_F \approx 0.5\mu$K, $T \approx 0.04 T_F$, $\sigma_s \approx 10^{-11}$ cm^2 for ^{40}K.

- Quantum optical master equations to describe these systems

Irreversible loading of bosons

- The Fermi sphere is replaced by a superfluid immersion



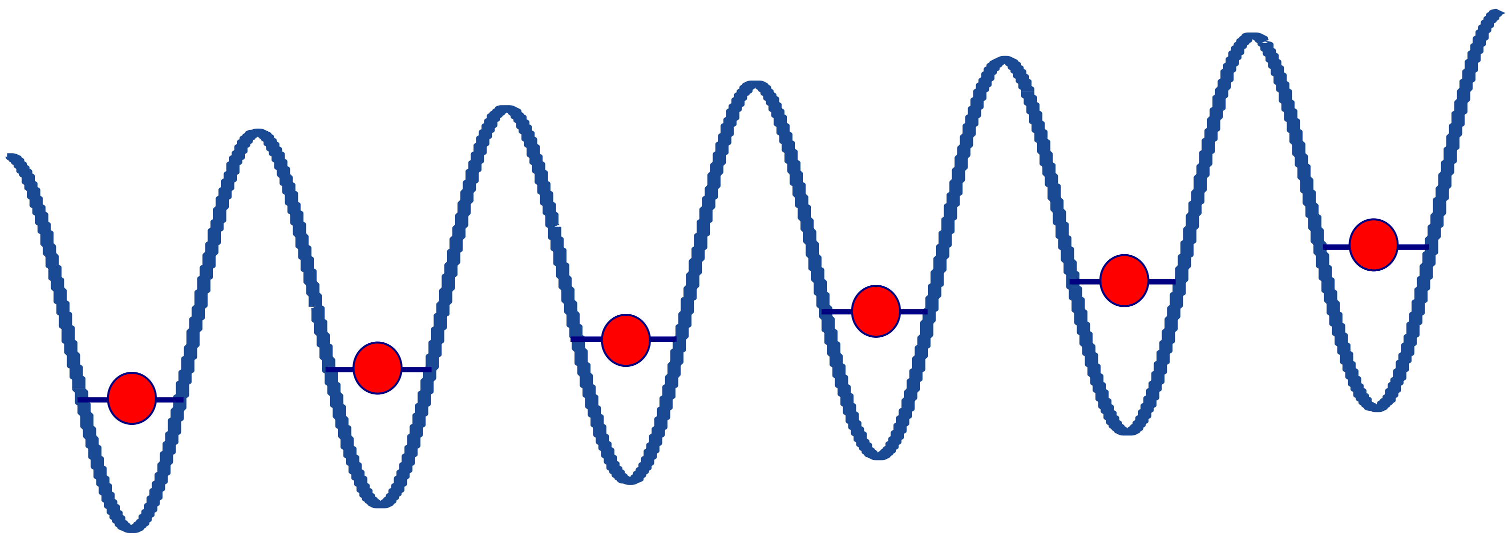
- The scheme works similar as for fermions but
  - ⇨ A superfluid acts as the particle reservoir (c-number) and as the bath
    - A.J. Daley, et al. Phys. Rev. A **69**, 022306 (2004).
  - ⇨ Fermi blocking is replaced by interaction blocking due to U

Part II: Cooling atomic patterns

Cooling atoms to the lowest energy sites

A.J. Daley *et al.*, in progress

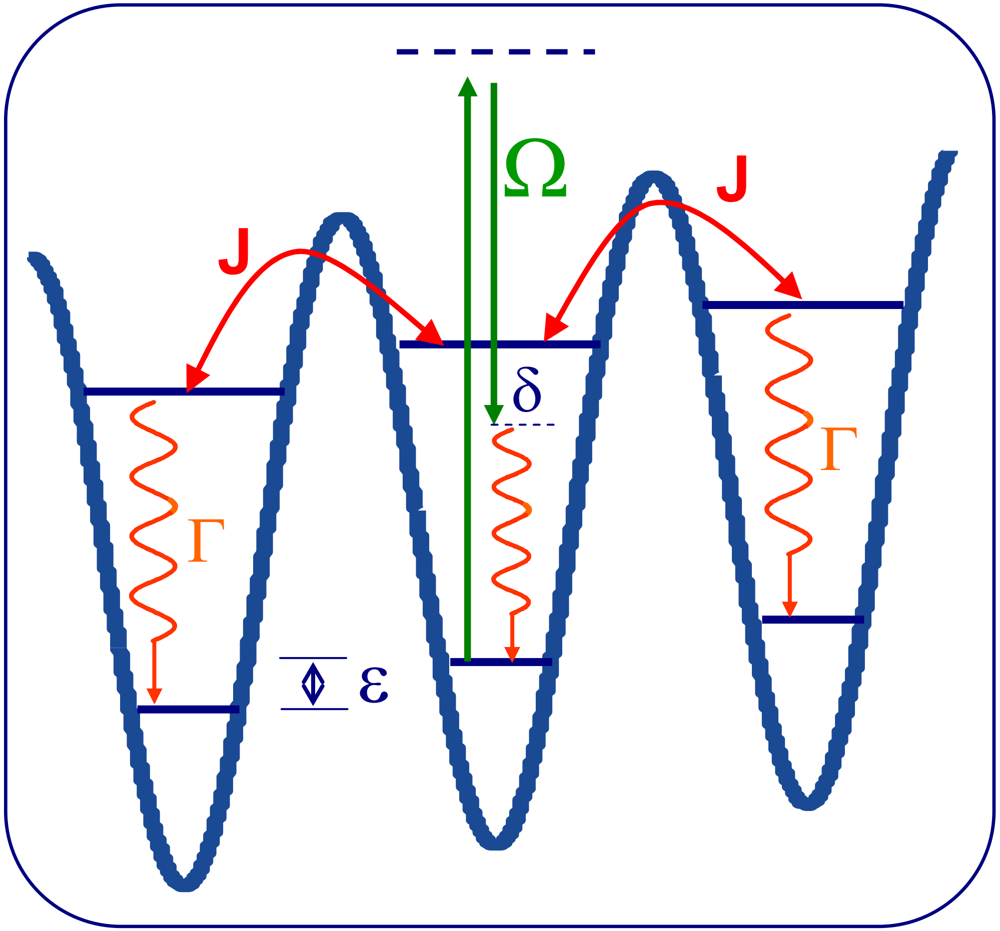
- These cooling ideas can also be used to redistribute existing atoms, cooling them to the lowest energy sites:



→ The existing atoms are compacted, removing the empty sites.

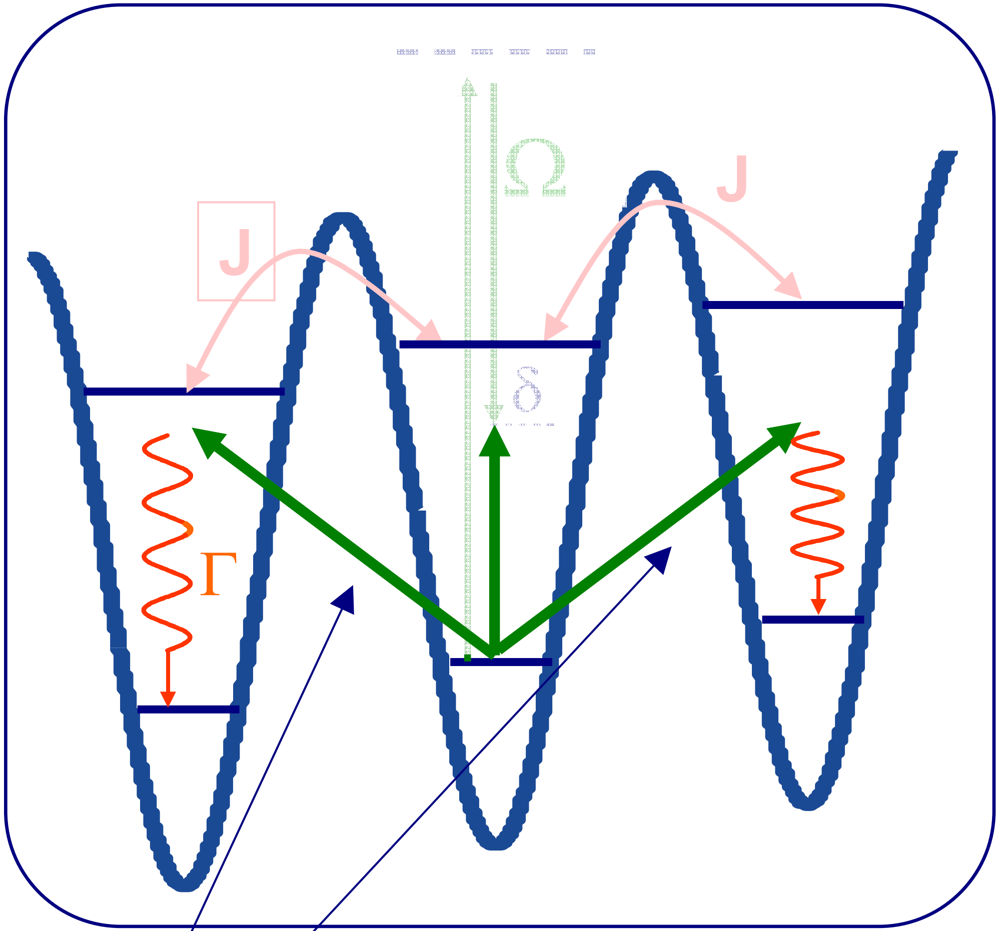
Cooling mechanism

- Sites offset by ε
- Off-resonant Raman process couples lowest Bloch band to upper band
- Tunnelling in upper band J (tunnelling in lowest band small)
- Cooling from excited motional states via phonon emission
- We must choose the Ω and δ appropriately to avoid populating the excited motional states.



Cooling Mechanism

- Sites offset by ε
- Off-resonant Raman process couples lowest Bloch band to upper band
- Tunnelling in upper band J (tunnelling in lowest band small)
- Cooling from excited motional states via phonon emission
- We must choose the Ω and δ appropriately to avoid populating the excited motional states.

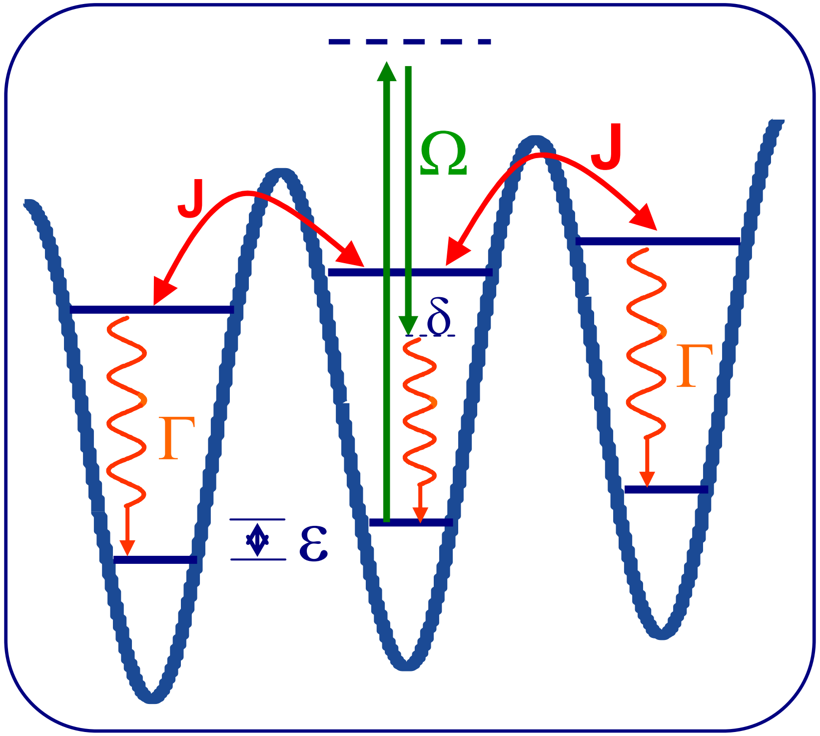


e.g., Ω, J $\ll \varepsilon,\delta$
$\Omega \ll$ Gamma:
Sideband Cooling!!!

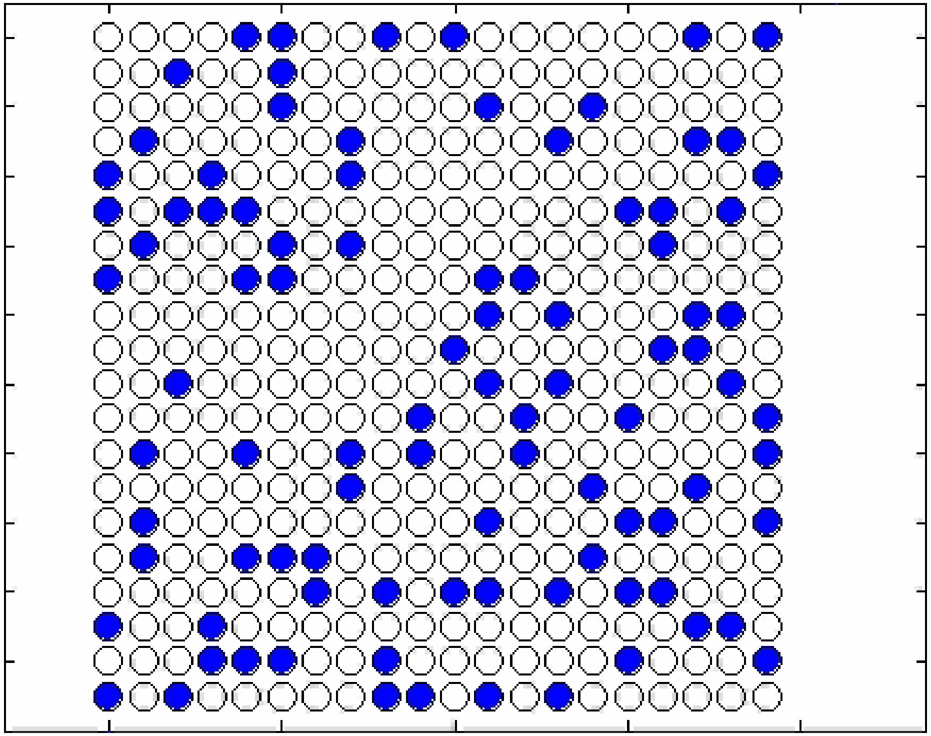
Cooling $\propto \Gamma (\Omega J/\delta)^2 / [(\varepsilon-\delta)^2+ \Gamma^2/4)]$

Heating $\propto \Gamma (\Omega J/\delta)^2 / [(\varepsilon+\delta)^2+ \Gamma^2/4)]$

Cooling Mechanism



- Further Analysis:
  - ➡ Quantum Boltzmann Master Eq.
  - ➡ Quantum Trajectories Simulation [see DJ *et al.* PRA **56**, 575 (1997)]



- Timescale Limitations:
  - ➡ $1/\tau \ll J$
  - ➡ <u>BUT</u> J is tunelling in **<u>upper</u>** band

Outlook

- Part I:
  - ⇨ Dynamics of multi component BHM in 1D optical lattices
  - ⇨ Extension to dissipative dynamics
  - ⇨ Extension to 2D optical lattices
  - ⇨ Entanglement creation in a Tonks gas
  - ⇨ Ramping a lattice in a superfluid environment
  - ⇨ Cooling in the lowest Bloch band
- Part II:
  - ⇨ Photodissociation of a molecular BEC to fill the lattice
  - ⇨ Probing of excitations in the Fermi sphere (Life times)
  - ⇨ CW phonon atom laser by stimulated emission of phonons
  - ⇨ Controlled loading of several bosons
  - ⇨ Extension to the case of finite hopping $J \neq 0$
    - Study the dissipative dynamics of strongly correlated systems