Nonlinear effects of Bose Condensates in optical lattices

Istituto Nazionale per la Fisica della Materia

M. Jona-Lasinio, O. Morsch, M. Cristiani, M. Anderlini, E. Courtade and E. Arimondo

Dipartimento di Fisica Enrico Fermi, Università di Pisa,
via F. Buonarroti 2, I-56127 Pisa, Italy

UNIVERSITÀ DI PISA

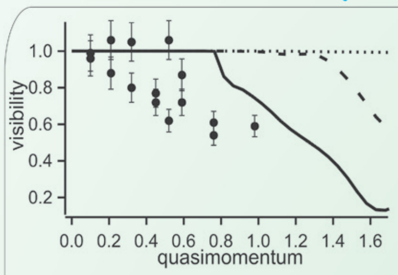
Introduction and motivations

The realization of Bose-Einstein Condensate (BECs) as macroscopic quantum objects combined with the realization of stable optical lattices, has made possible new and interesting experiments to test the validity of simple theoretical quantum models.

Linear Landau-Zener tunneling is a well known effect and a complete mathematical apparatus exists. An explicit formula for the tunneling probability was given by Landau and Zener in 1932. It has been suggested that the presence of a self interacting term in the evolution equation greatly modify the linear behaviour. We have studied these modifications both theoretically and experimentailly. Most notably, we have found that the nonlinearity leads to an asymmetry in the Landau-Zener tunneling between the two lowest energy bands.

Another consequence of the nonlinearity in a Bose condensate is the occurrence of dynamical instabilities. We have studied these by accelerating the lattice in a controlled way, thus scanning the entire Brillouin zone. By measuring the visibility of the interference pattern after switching off the lattice, we could identify the region of the Brillouin zone in which unstable behaviour occurs and identify a timescale on which the instabilities grow.

Instability of a BEC in an optical lattice

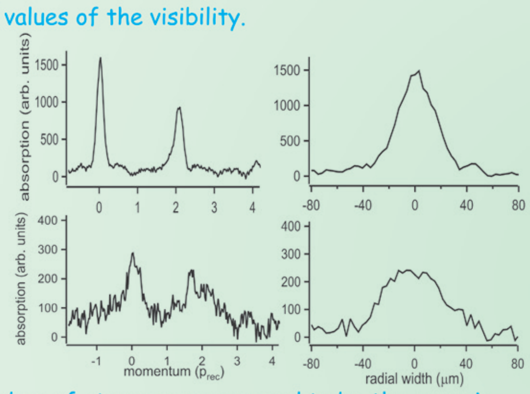


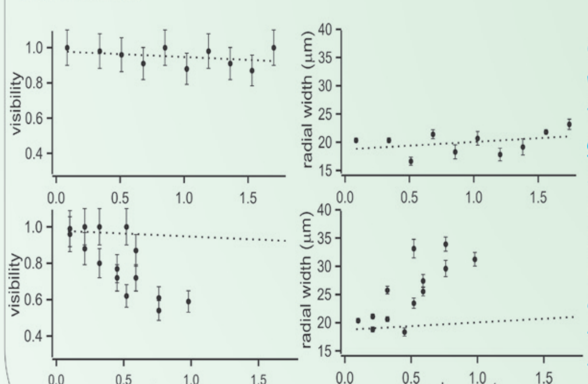
By integrating the profile in a direction perpendicular to the optical lattice direction, we obtain a two-peaked curve for which we can define a visibility (in analogy to spectroscopy) reflecting the phase coherence of the condensate:

$$\text{visibility} = (h_{peak} - h_{middle})/(h_{peak} + h_{middle})$$

Here h_{peak} is the mean value of the two peaks (both averaged over 1/10 of their separation symmetrically about the positions of the peaks); h_{middle} is obtained by averaging the longitudinal profile over 1/3 of the peak separation symmetrically about the midpoint between the peaks. Above, results of a 1D simulation for different values of the nonlinear *C* parameter and acceleration a=0.3 m s^{-2}. Solid, dashed and dotted line correspond to *C*=0.008 (the value of our experiment), *C*=0.004 and *C*=0 respectively. The band edge is at quasimomentum 1.0. The closed symbols are the experimental values of the visibility.

Integrated longitudinal and transverse profiles of the interference pattern of a condensate, are shown on the right. The condensate is released from an optical lattice after acceleration to a quasimomentum 0.9 and a subsequent time-of-flight of 21ms. In the upper graphs the acceleration was 5 m s^{-2}, whereas in the lower graphs it was 0.3 m s^{-2}. Note the different vertical axis scales (by a factor 4) for the upper and lower graphs. The total number of atoms was measured to be the same in both cases.





On the left, the visibility and radial width as a function of quasimomentum for acceleration 5.0 m s^{-2} (upper graphs) and 0.3 m s^{-2} (lower graphs). As the acceleration is lowered, instabilities close to quasimomentum 1.0 lead to a decrease in visibility and increase in radial width. For comparison, in each graph the (linear) fits to the visibility and radial width for the a = 5.0 m s^{-2} data are included.

Nonlinear Landau-Zener effect: 1D model

The motion of a Bose Einstein Condensate in an accelerated periodic potential can be modeled by the nonlinear Schroedinger equation, namely Gross-Pitaevskii equation (GPE):

$$i\frac{\partial\psi}{\partial t} = \frac{1}{2}\left(-i\frac{\partial}{\partial x} - \alpha t\right)^2\psi + v\cos(x)\psi + c|\psi|^2\psi$$

where all quantities are dimensionless. The applied force is in the vector potential representation..
We can assume that in the neighborhood of the Brillouin zone edge (k=1/2) only the ground state and the first excited state are populated. Then the wavefunction is well approximated by

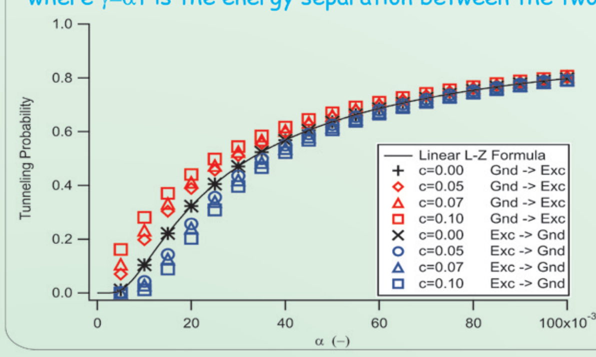
$$\psi(x,t) = a(t)e^{ikx} + b(t)e^{i(k-1)x}$$

where $|a|^2 + |b|^2 = 1$

Substituting this ansatz in the GPE, linearizing the kinetic term, dropping a constant energy and comparing the coefficients of the two plane waves, we are left with a simple two level model. The GPE equation for the plane waves coefficients now reads:

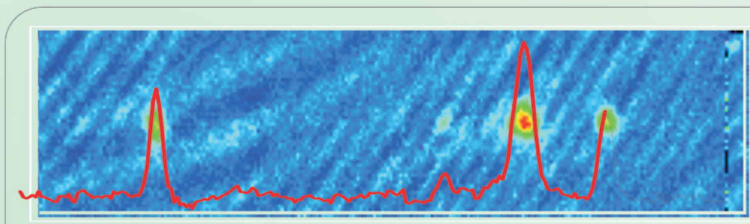
$$i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \gamma + c(|b|^2 - |a|^2) & v \\ v & -\gamma - c(|b|^2 - |a|^2) \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

where $\gamma=\alpha t$ is the energy separation between the two levels.

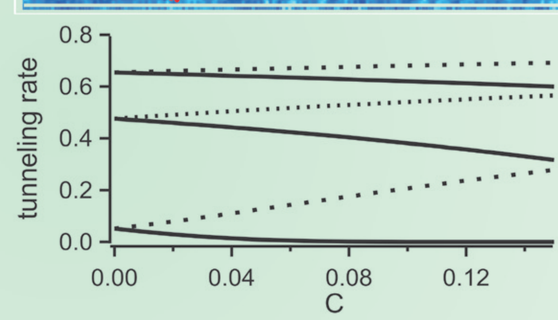


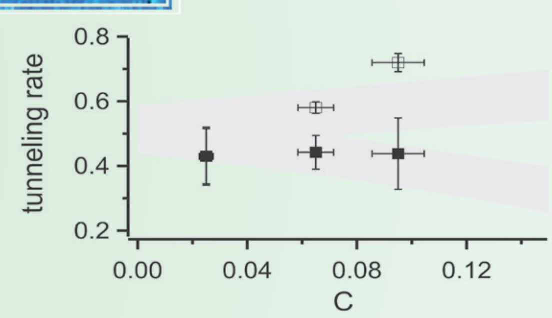
LZ tunneling rate within the two-level model as a function of the acceleration α for different *C* values evaluated by numerical integration. Complete symmetry exists between the two bands in the linear case (*C*=0); the asymmetry appears as soon as *C* differs from zero. The value α=0.0364 (2.9 m s^{-2}) is the experimental value and experimental data cover from *C*=0.04 to *C*=0.11.

Nonlinear Landau-Zener effect: the experiment



An experimental profile, corresponding to a transition between ground and excited state, is shown. Only the highest peaks are visible.





Left: LZ tunneling rate within the two-level model as a function of the nonlinear *C* parameter for different accelerations. The dashed (solid) lines correspond to tunneling from the ground (excited) band. The rates coincide in the linear case, whereas they significantly differ in the nonlinear case. Results for v=0.134 (2.2 E_{rec}) and acceleration 0.8, 3.2, 5.6 m s^{-2} from bottom to top.

Right: Experimental results are denoted by open symbols for the ground band to the excited band tunneling and by filled symbols for the opposite direction. The shaded area represents the confidence region for the prediction of the two-level model, taking into account the uncertainty in the measurement of the lattice depth.

It must be stressed that the simple Wu-Niu model does not take into account the presence of the harmonic trap, which indeed perturbes the experimental results.

Experimental setup

Instability: Experimental procedure

After creating BECs with ~10^4 atoms into a magnetic trap, we adiabatically lower the trapping frequency. Thereafter, the intensity of the lattice beams is ramped up from 0 to a value corresponding to a lattice depth of ~2 E_{rec}.: the ramping time is of the order of several milliseconds in order to ensure adiabaticity. Once the final lattice depth has been reached, the lattice is moved with a constant acceleration a for a time t. The final quasimomentum is a function of t: for large accelerations it is simply a t while for small accelerations the presence of the harmonic trap should be taken into account. In this case, we derived the quasimomentum value from a numerical integration of the semiclassical equations of motion of the condensate in the presence of the periodic potential and of the magnetic trap. Finally, both the magnetic trap and the optical lattice are switched off, and the condensate is observed after a time-of-flight of 21 ms by absorption imaging.
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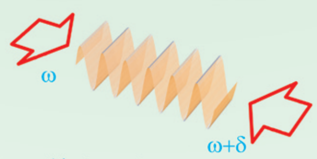
L-Z: Experimental procedure

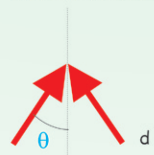
A Rb BEC of about 10^4 atoms is prepared into a magnetic harmonic trap. A periodic potential is created by mean of two counterpropagating laser beams. The lattice depth is adiabatically raised from 0 to 2.25 E_{rec}. During this procedure the lattice is at rest resulting in a condensate loaded in the ground state. At this time the lattice is suddenly accelerated for about 1 ms. At the end, the lattice depth is increased (tipically until 4.5-5.0 E_{rec}) while the acceleration is lowered: the ground state separates from the excited states. Finally both the periodic potential and harmonic trap are switched off and the condensate evolves freely (Time of flight phase) for typically 20 ms. At the end a CCD camera takes a picture of the condensate.
In order to load the condensate in the excited band all this procedure remains valid except for the reversed acceleration sign and for the lattice starting velocity: the lattice starts with 1.5 V_{rec}.

1-D Optical lattice



It is possible to create a periodic potential using two intersecting laser beams

Varying the differential detuning of the two beams we can change the velocity of the lattice with respect to the Lab frame of reference.



The lattice step **d** depends on the angle between the two beams according to:

$$d = \frac{\lambda}{2\cdot \text{Sin}(\theta)} \qquad k_L = \frac{\pi}{d}$$

$$E_{recoil} = \frac{\hbar^2 k_l^2}{2m} \qquad V_{recoil} = \frac{\hbar k_l}{m}$$

Experimental value was d=1.18 μm
m is the Rubidium mass

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Instability

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