Mesoscopic Phenomena in Ultracold Matter:
From Single Atoms to Coherent Ensembles, October 11-15, 2004

Tunable scattering resonances: What are they like?

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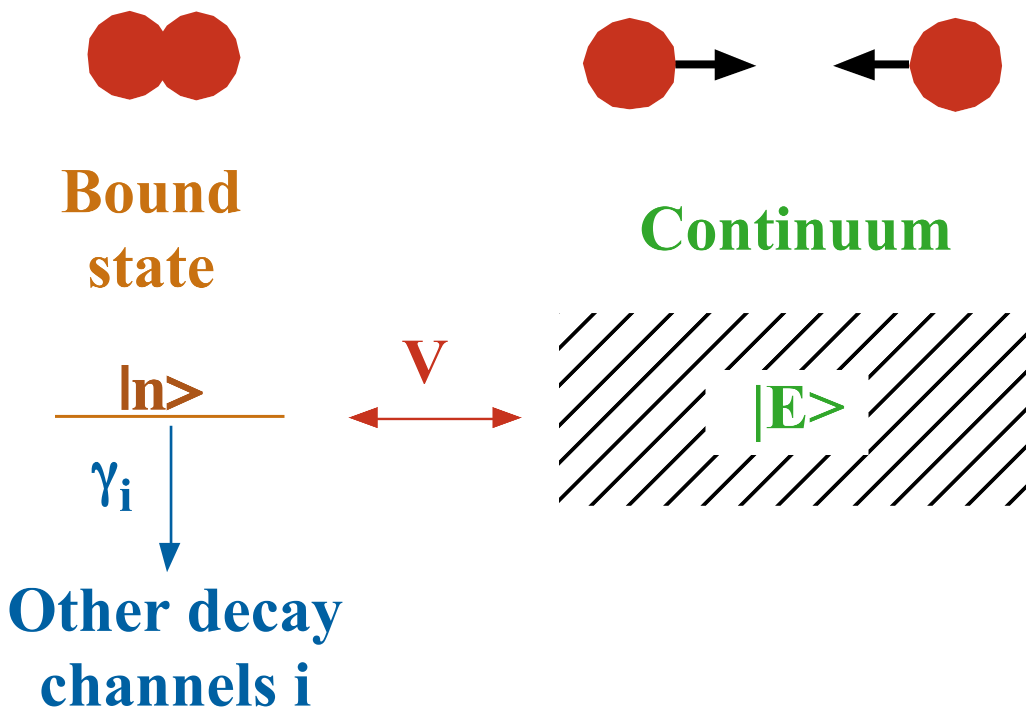
Cheng Chin (Innsbruck)

Experiments:
Grimm group at Innsbruck
Weiman group at JILA
Ketterle group at MIT

Resonant Scattering Picture

(U. Fano, Phys. Rev. 124, 1866 (1961))



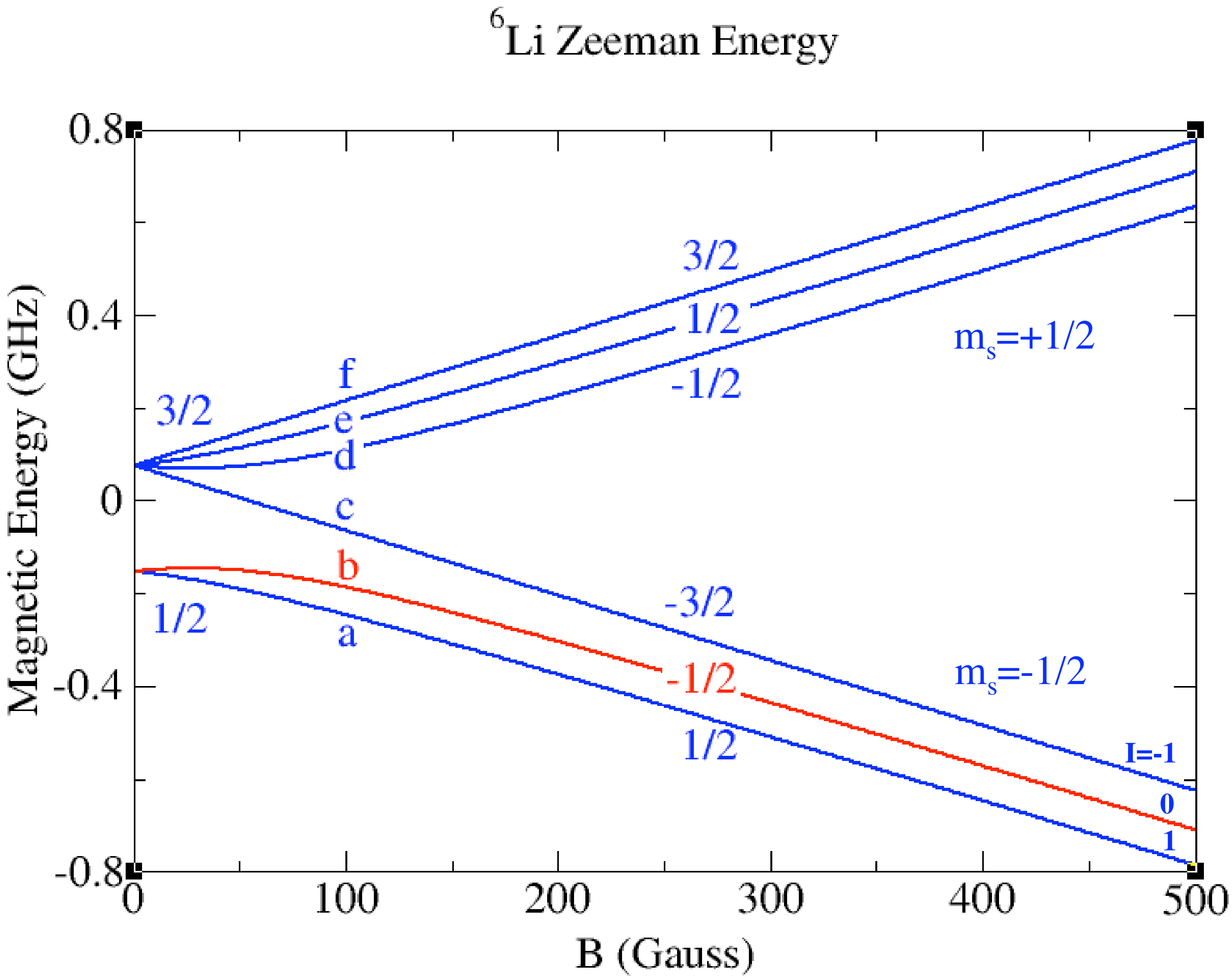
Width = $\Gamma_n = 2\pi\, |\langle n|\, V\, |E_n\rangle|^2$

Shift

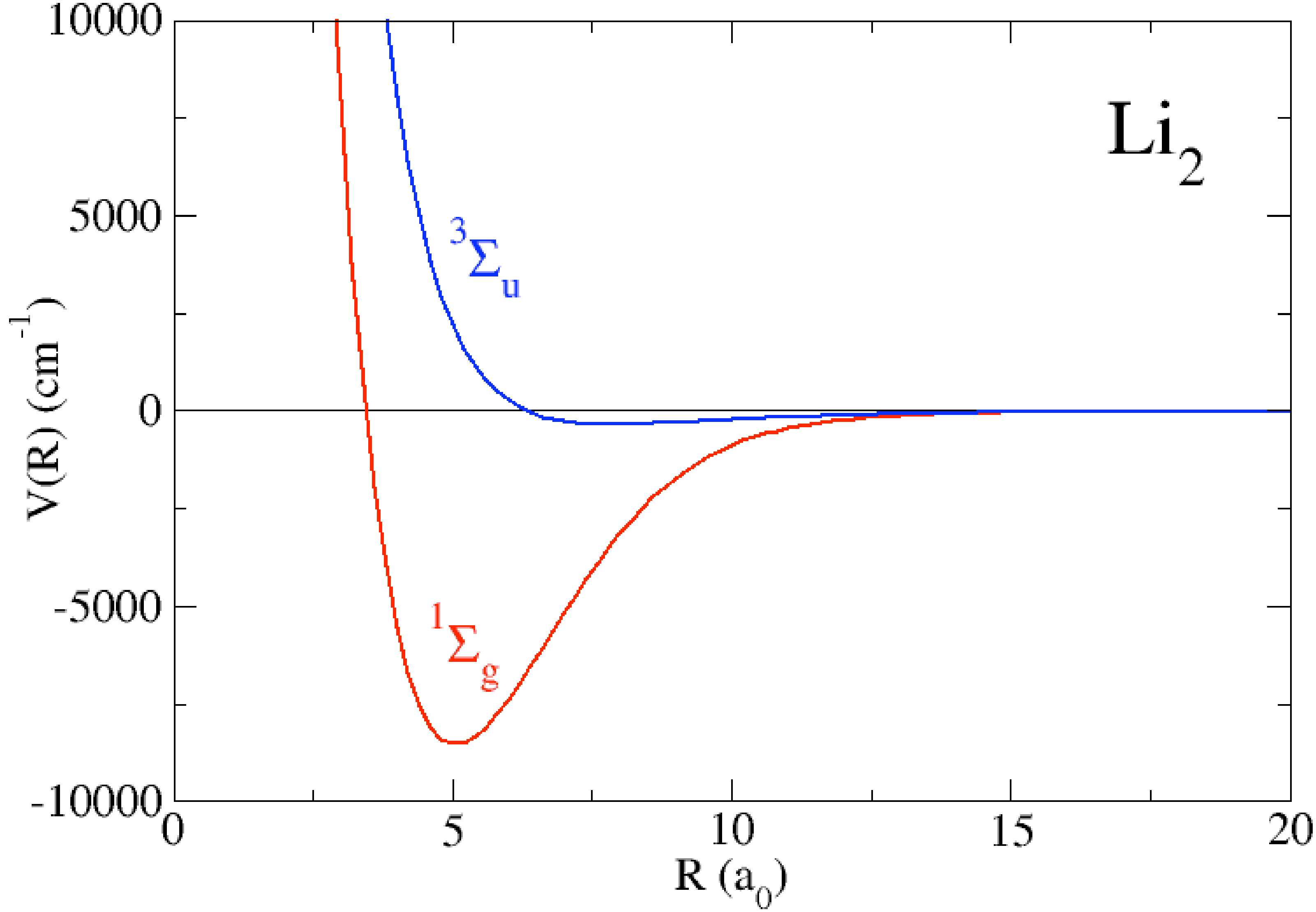
Characteristic Lengths

De Broglie wavelength	$\lambda = \dfrac{h}{p}$	~ 20000 a_0 (1 μm)
Van der Waals length	$x_0 = \dfrac{1}{2}\left(\dfrac{2\mu C_6}{\hbar^2}\right)^{1/4}$	30 - 100 a_0 (1.5 - 5 nm)
Chemical bond		< 20 a_0 (< 1nm)
Scattering length	A	$-\infty < A < \infty$
Trap size		> 200000 a_0 (10 μm) Lattice: 1000 a_0 (50 nm)

$$\text{If}\, |A| >> x_0,\ \text{then}\ E_b = -\frac{\hbar^2}{2\mu A^2}\ \text{and}\ \langle b|R|b\rangle = \frac{A}{2}$$



^{6}Li Zeeman Energy
Magnetic Energy (GHz)
B (Gauss)
0.8
0.4
0
-0.4
-0.8
0
100
200
300
400
500
3/2
f
e
d
c
b
a
1/2
3/2
1/2
-1/2
-3/2
-1/2
1/2
m_s=+1/2
m_s=-1/2
I=-1
0
1



Li$_2$
$^3\Sigma_u$
$^1\Sigma_g$
10000
5000
0
-5000
-10000
V(R) (cm^{-1})
0
5
10
15
20
R (a_0)

Theoretical/experimental methodology

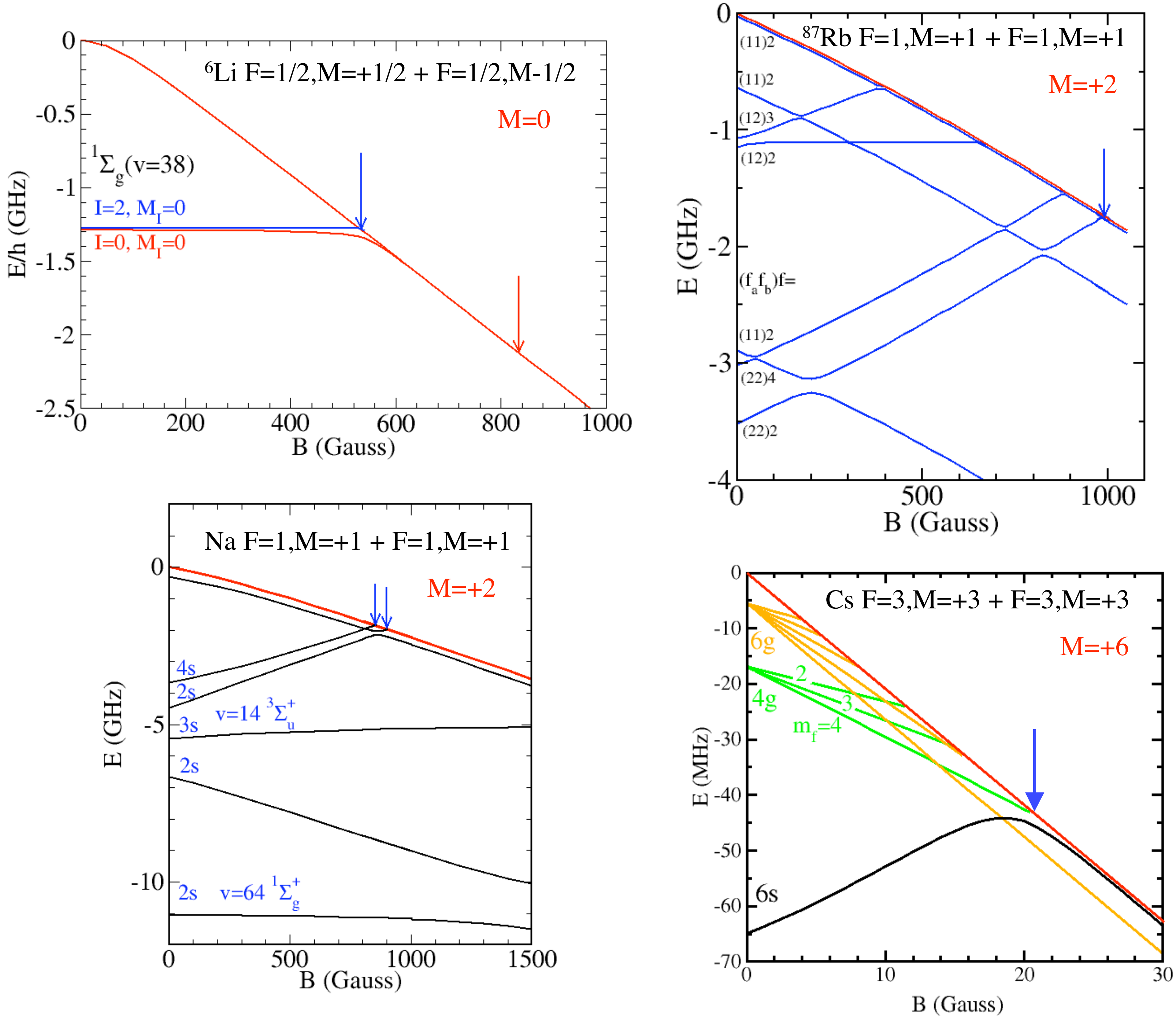
Full quantum scattering calculations, based on *ab initio* or semi-empirical short range potentials and asymptotic atomic properties, parameterized by

- **$^1\Sigma_g^+$ scattering length**
- **$^3\Sigma_u^+$ scattering length**
- **C_6 van der Waals coefficient**

Fit available experimental data

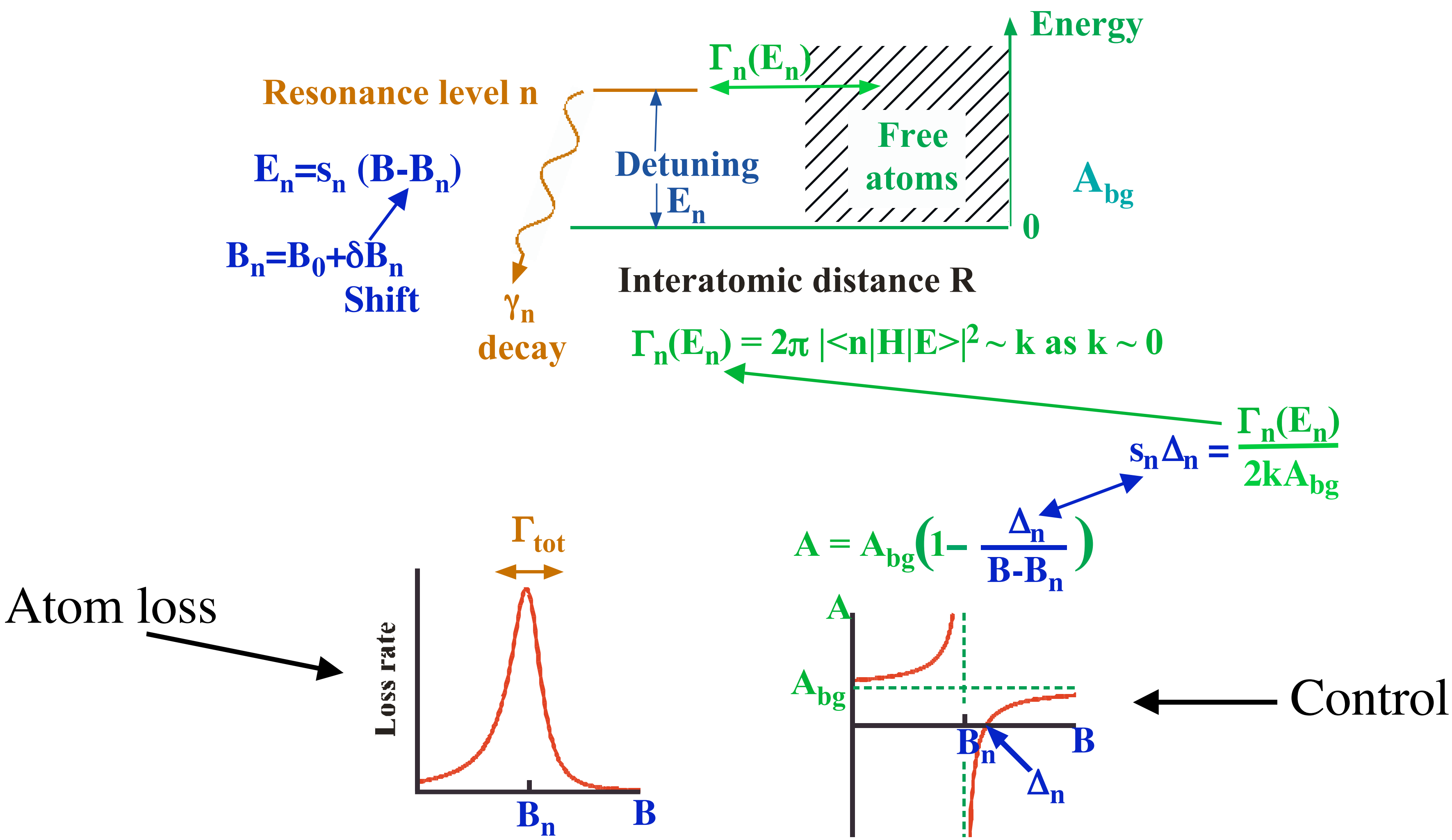
- **Bound states (threshold resonance spectroscopy)**
- **Collisional data**

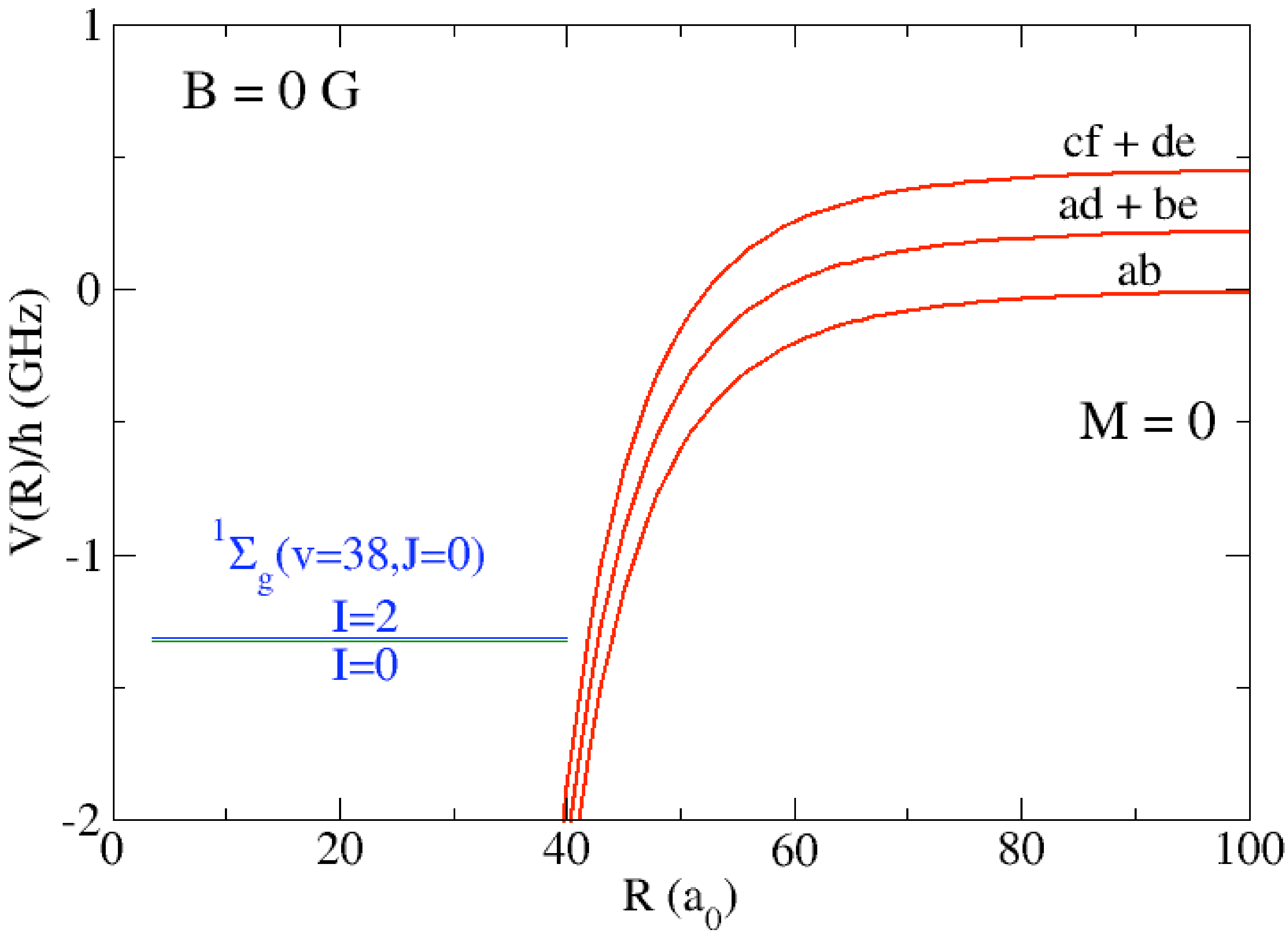
Quantitative, predictive theoretical model for E, B in near threshold domain.



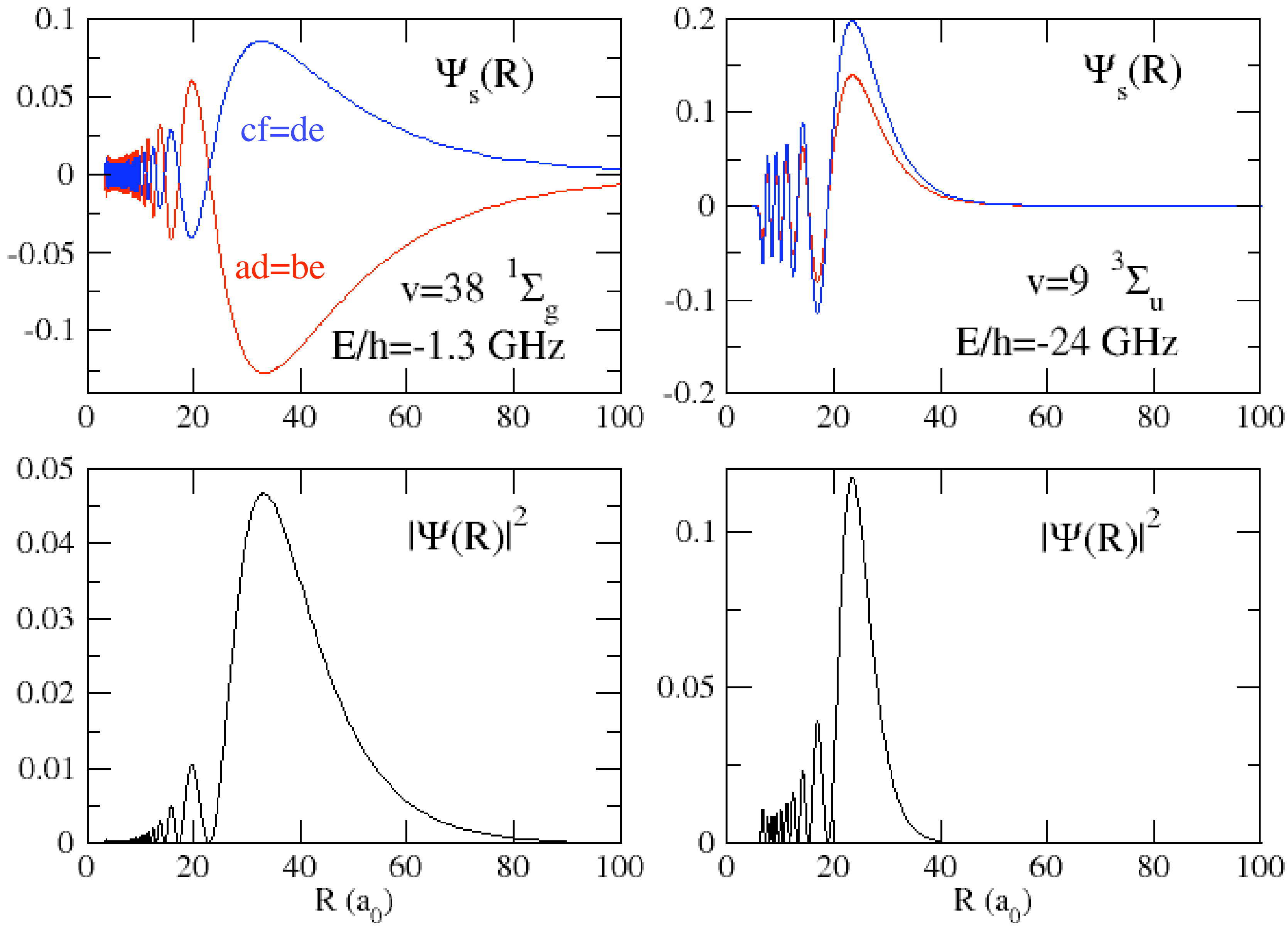
6Li F=1/2,M=+1/2 + F=1/2,M-1/2
M=0
1Σg(v=38)
I=2, MI=0
I=0, MI=0
E/h (GHz)
B (Gauss)
87Rb F=1,M=+1 + F=1,M=+1
M=+2
(11)2
(11)2
(12)3
(12)2
(fafb)f=
(11)2
(22)4
(22)2
E (GHz)
B (Gauss)
Na F=1,M=+1 + F=1,M=+1
M=+2
4s
2s
3s
v=14 3Σu+
2s
2s
v=64 1Σg+
E (GHz)
B (Gauss)
Cs F=3,M=+3 + F=3,M=+3
M=+6
6g
4g
2
3
mf=4
6s
E (MHz)
B (Gauss)

Threshold Resonance Scattering

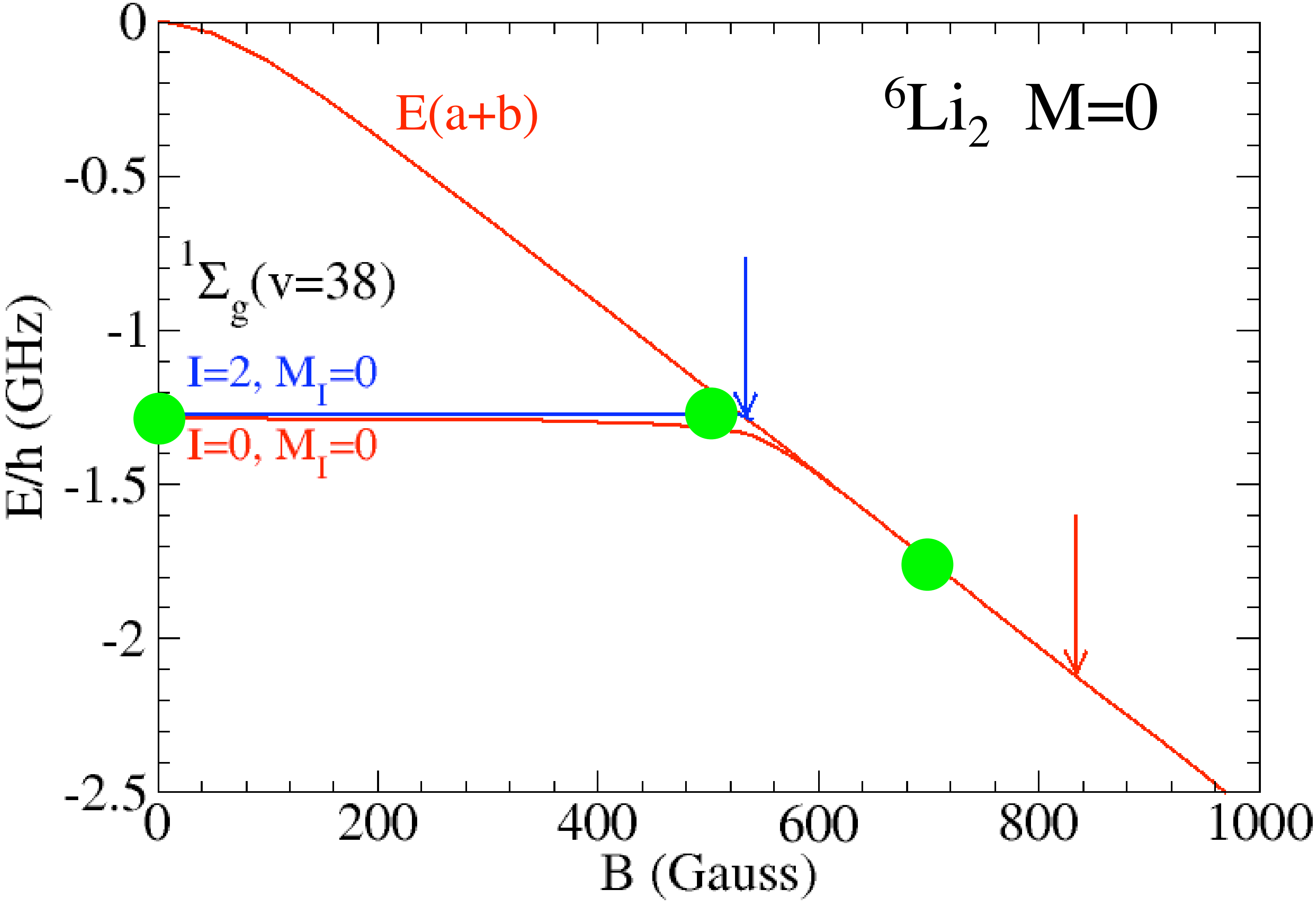




B = 0 G
cf + de
ad + be
ab
M = 0
$^1\Sigma_g$(v=38,J=0)
I=2
I=0
V(R)/h (GHz)
R (a_0)
1
0
-1
-2
0
20
40
60
80
100

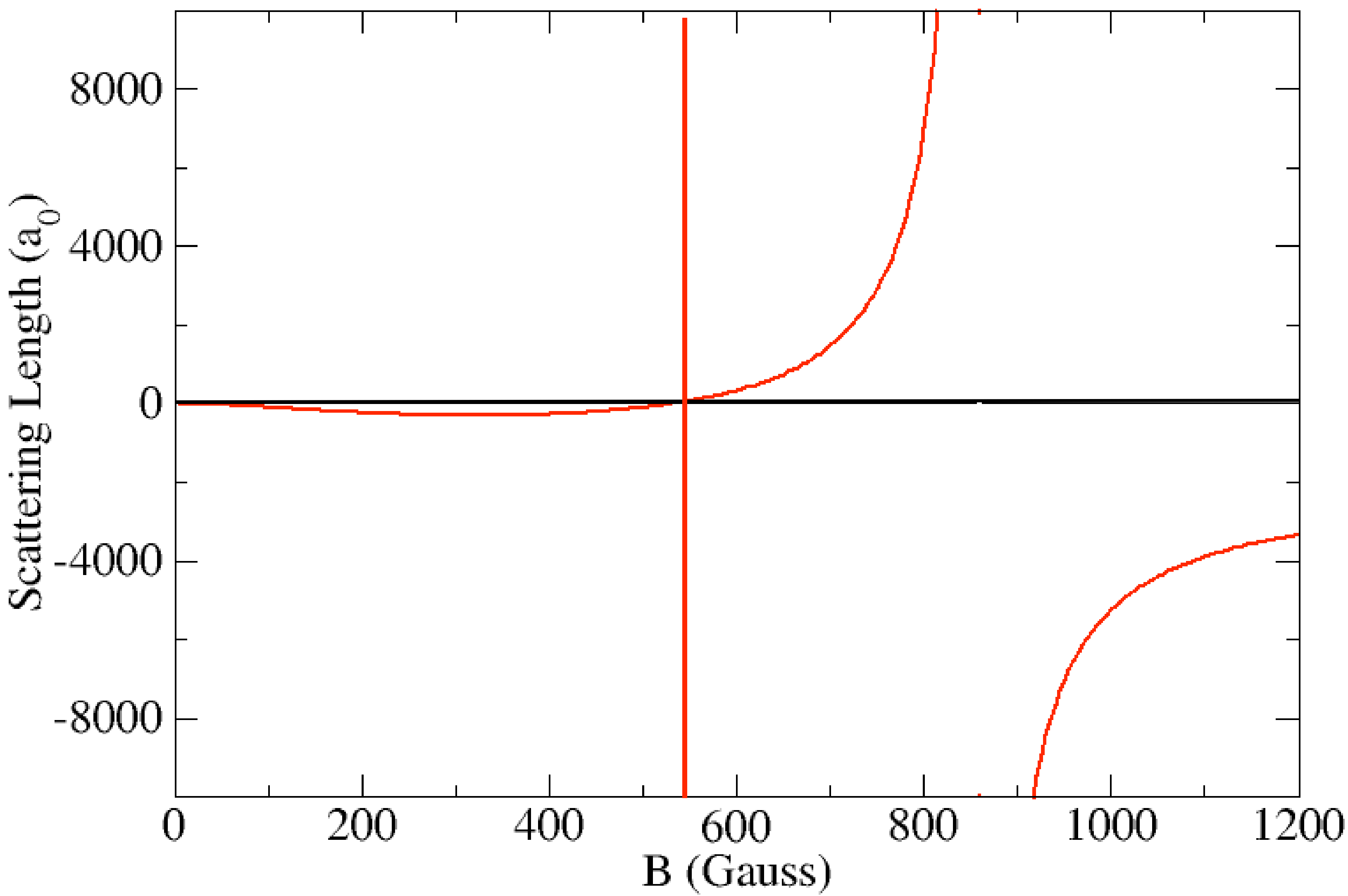


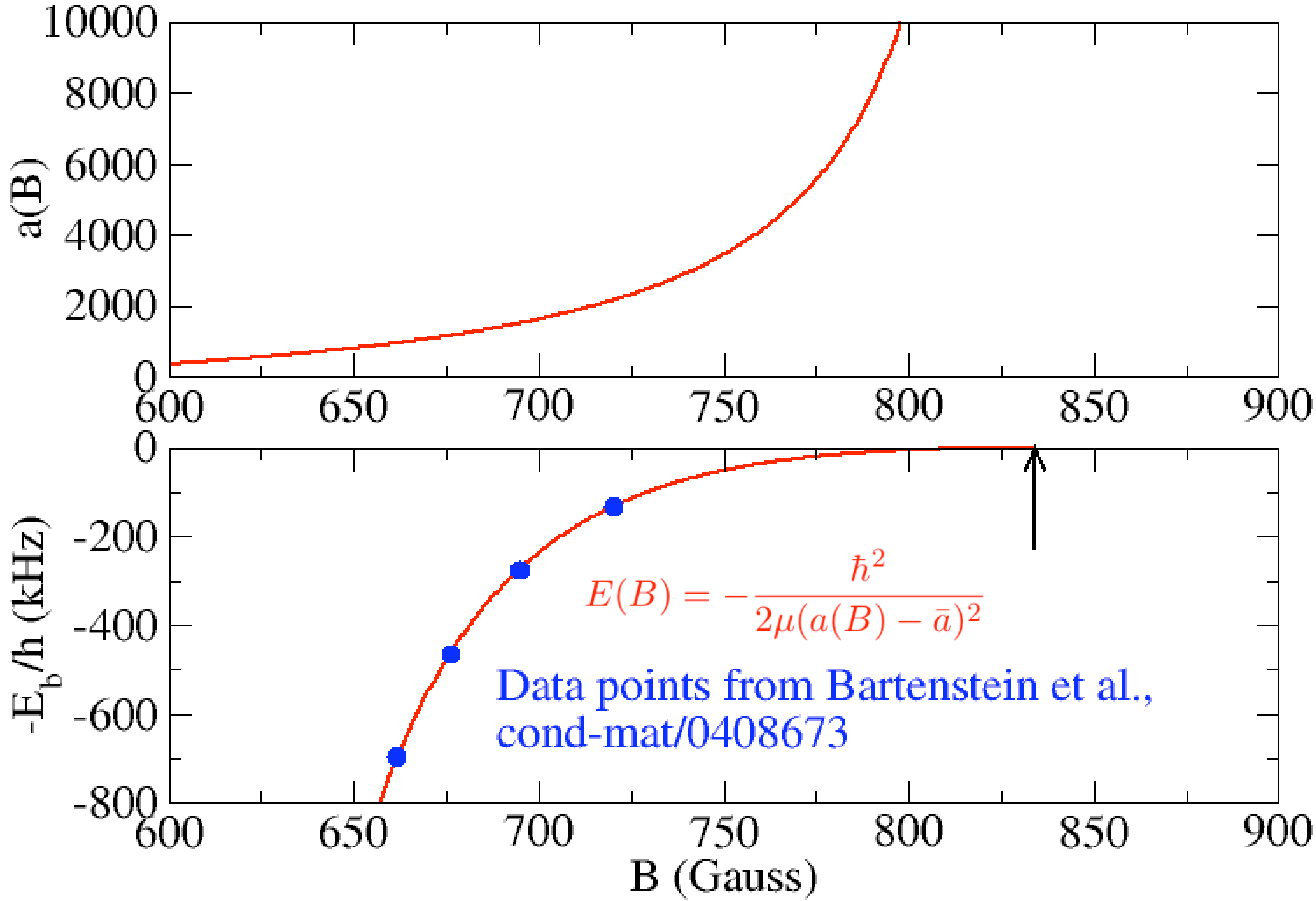
Ψs(R)
cf=de
ad=be
v=38 1Σg
E/h=-1.3 GHz
Ψs(R)
v=9 3Σu
E/h=-24 GHz
|Ψ(R)|2
|Ψ(R)|2
R (a0)
R (a0)

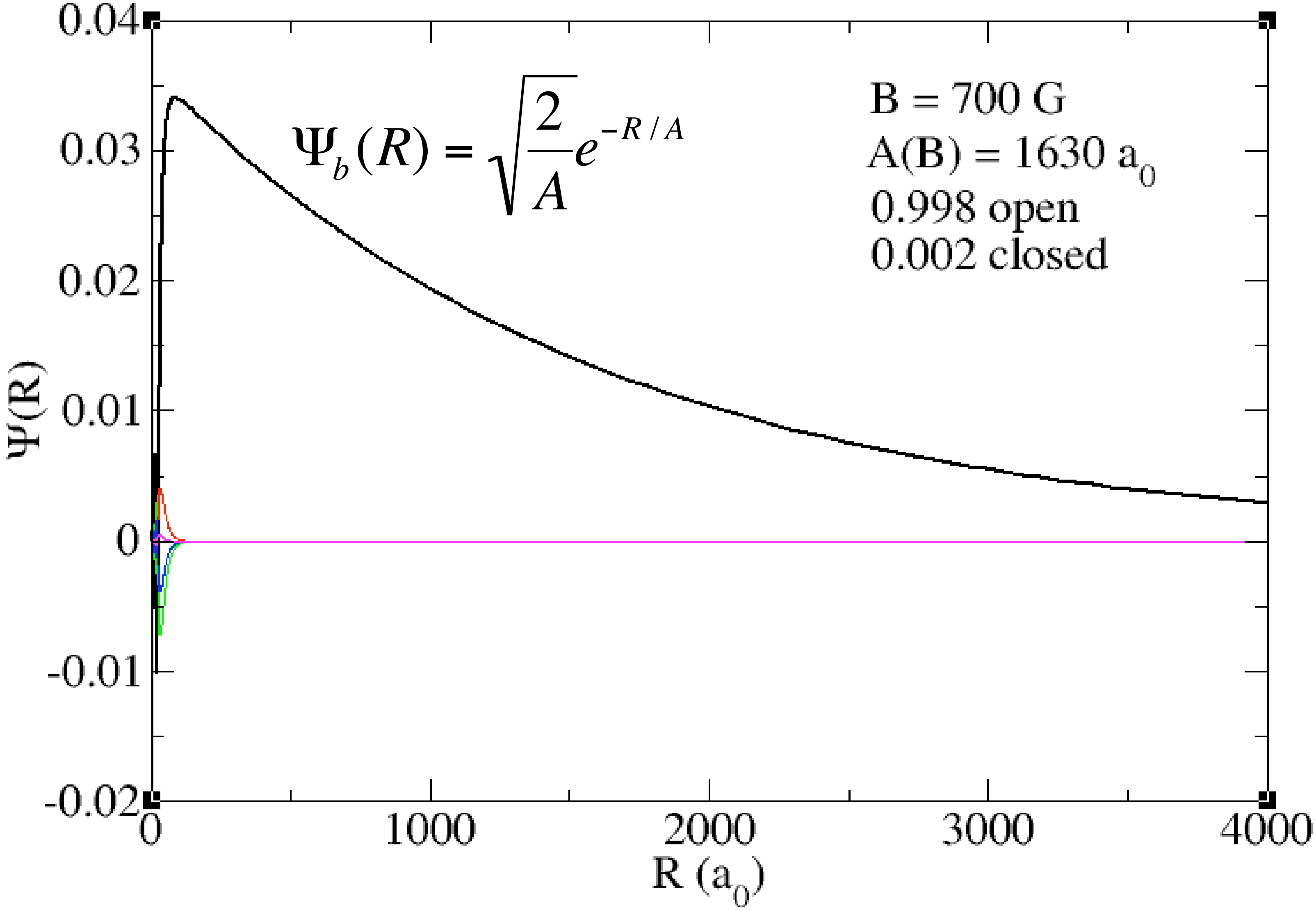


0
-0.5
-1
-1.5
-2
-2.5
0
200
400
600
800
1000
E/h (GHz)
B (Gauss)
E(a+b)
6Li2 M=0
1Σg(v=38)
I=2, MI=0
I=0, MI=0

^{6}Li a+b Scattering Length vs. B

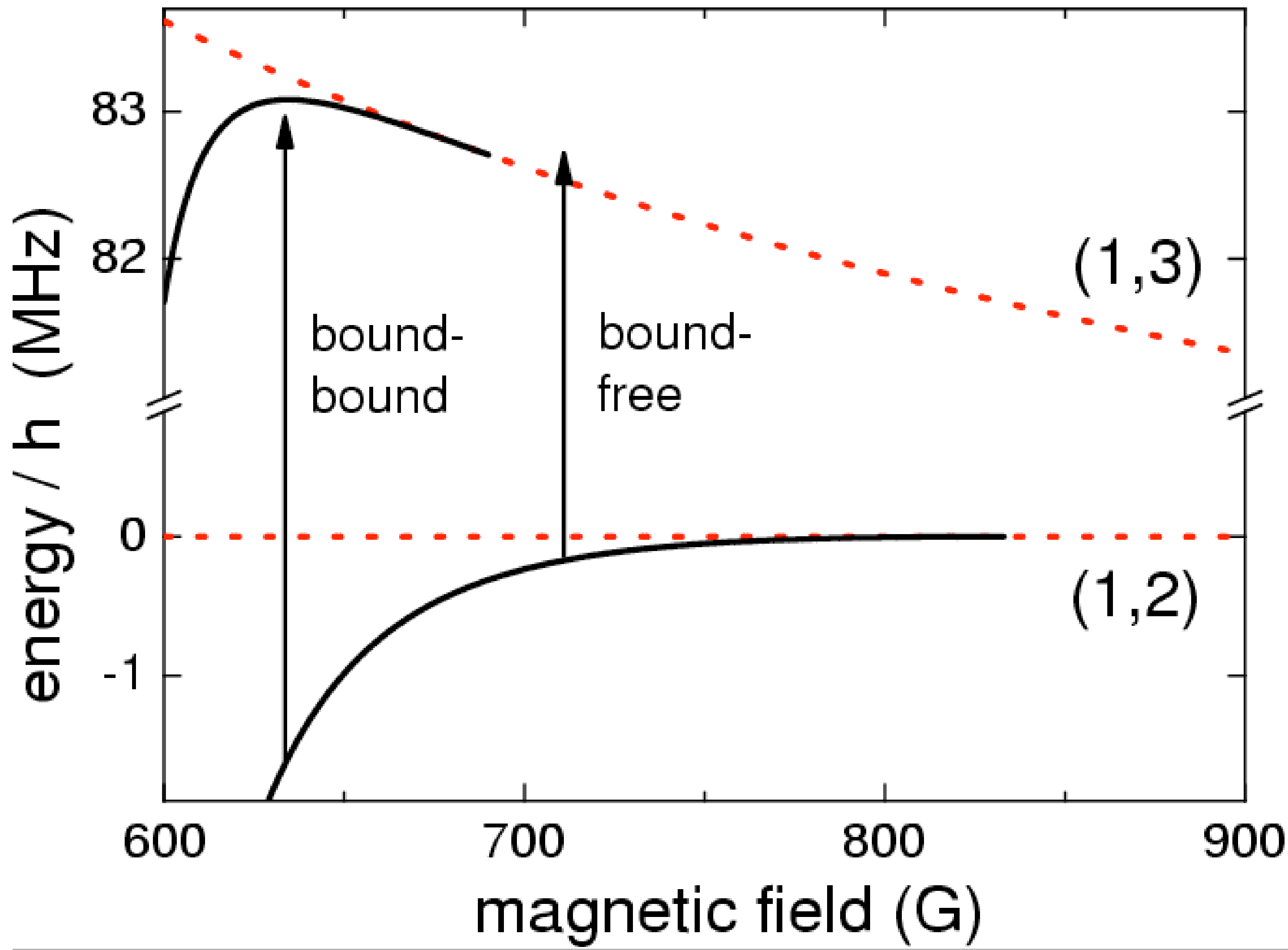



10000
8000
6000
4000
2000
0
a(B)
600
650
700
750
800
850
900
0
-200
-400
-600
-800
$-E_b/h$ (kHz)
$E(B) = -\frac{\hbar^2}{2\mu(a(B)-\bar{a})^2}$
Data points from Bartenstein et al.,
cond-mat/0408673
B (Gauss)



$\Psi_b(R) = \sqrt{\frac{2}{A}} e^{-R/A}$
B = 700 G
A(B) = 1630 a0
0.998 open
0.002 closed
Ψ(R)
R (a0)
0.04
0.03
0.02
0.01
0
-0.01
-0.02
0
1000
2000
3000
4000

High resolution rf spectroscopy, Grimm Group, Innsbruck

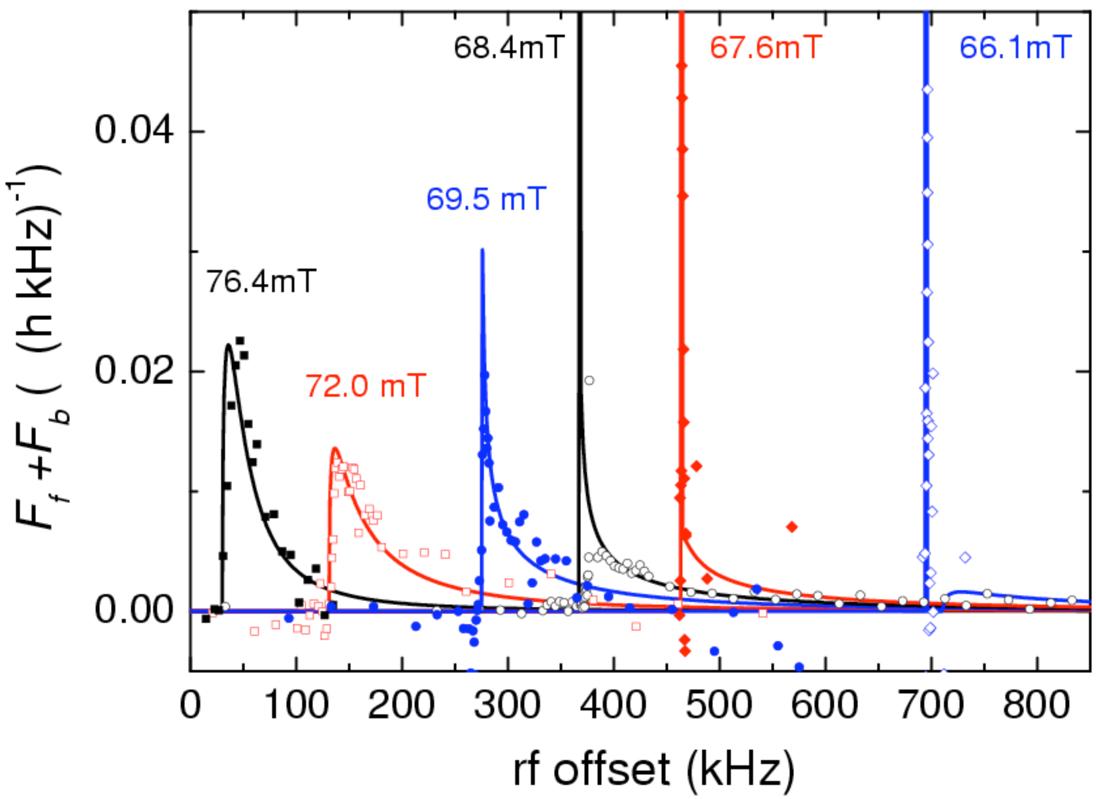


Bound-Free Franck-Condon Factor

$$\Psi_b(R) = \sqrt{\frac{2}{A}} e^{-R/A}$$

$$\Psi_f(R,E) = \sqrt{\frac{m}{\pi\hbar^2 k}} \sin(k(R - A_f))$$

$$\mathrm{FCF} = \left| \langle \Psi_b \mid \Psi_f \rangle \right|^2$$



Chin and Julienne
cond-mat/0408254

Atoms (MHz)	B (mT)	Molecules (MHz)	Theory (MHz)
82.96808(20)	66.1436(20)	83.6645(3)[a]	83.6640(10)
82.83184(30)	67.6090(30)	83.2966(5)[a]	83.2973(10)
82.66686(30)	69.4826(40)	82.9438(20)[b]	82.9422(13)
82.45906(30)	72.0131(40)	82.5928(20)[b]	82.5910(13)

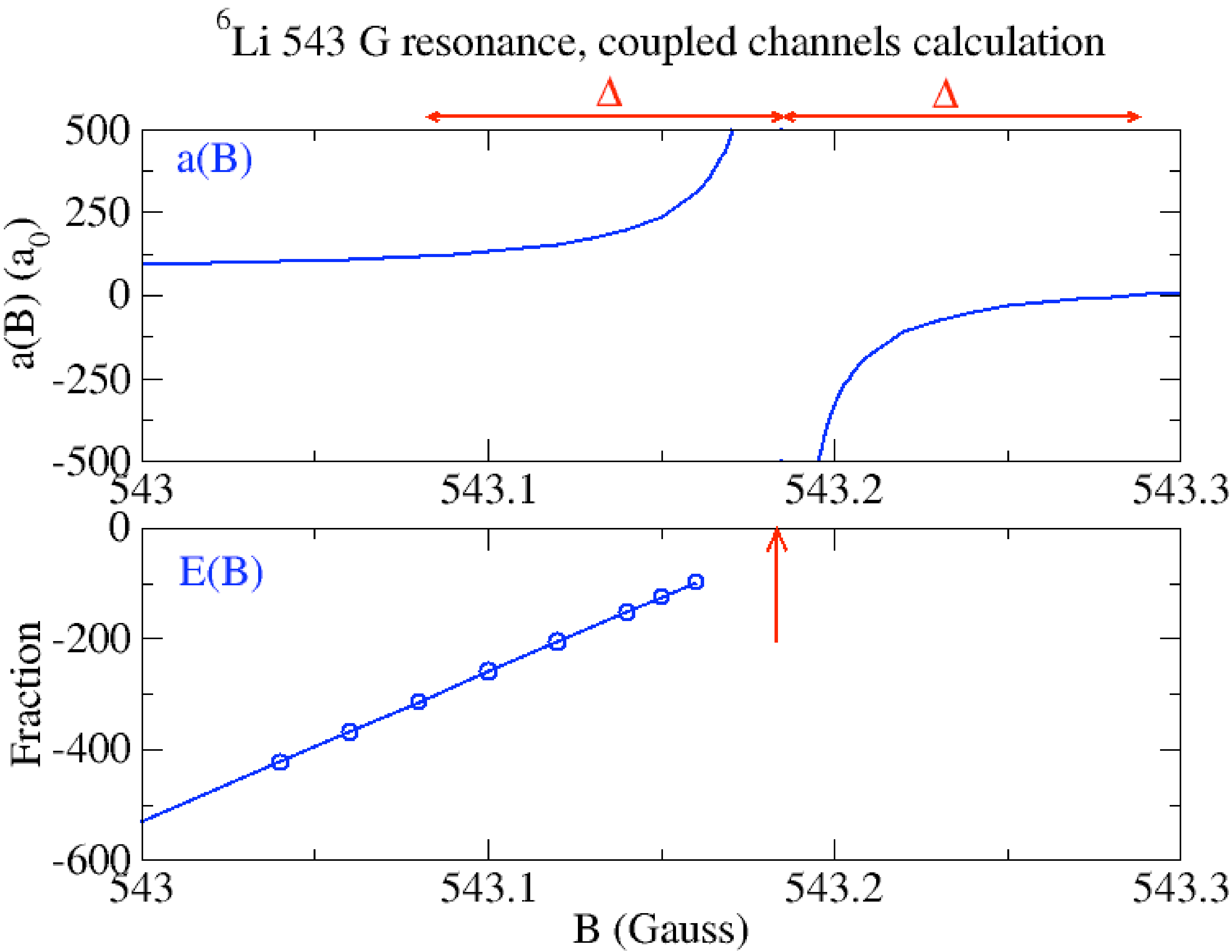
[a] bound-bound transition frequency.

[b] bound-free transition threshold.

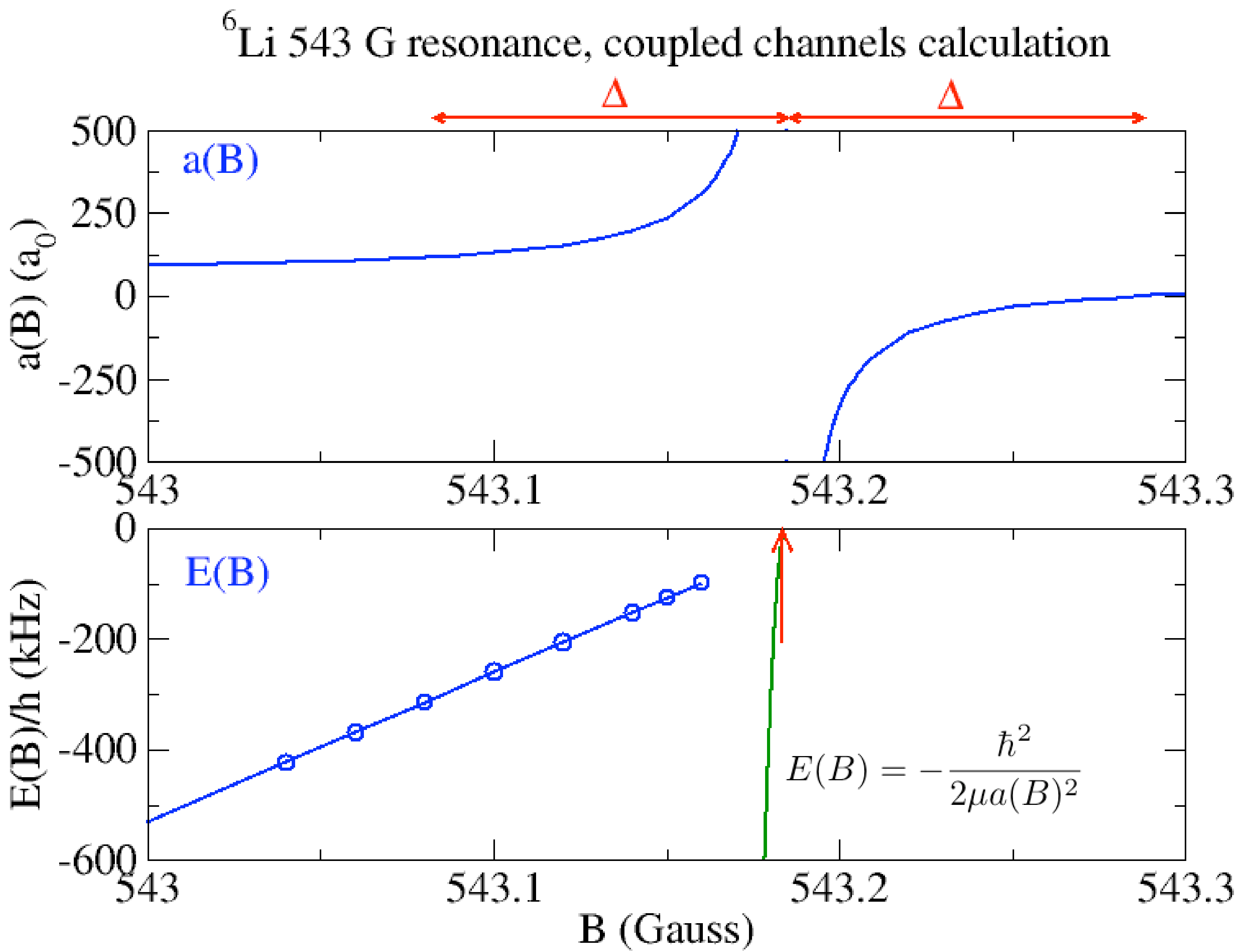
$^1\Sigma_g^+$ scattering length: 45.167(8) a_0
$^3\Sigma_u^+$ scattering length: -2140(18) a_0

(1,2) resonance: 834.1(1.5) Gauss
(1,3) resonance: 690.4(5) Gauss

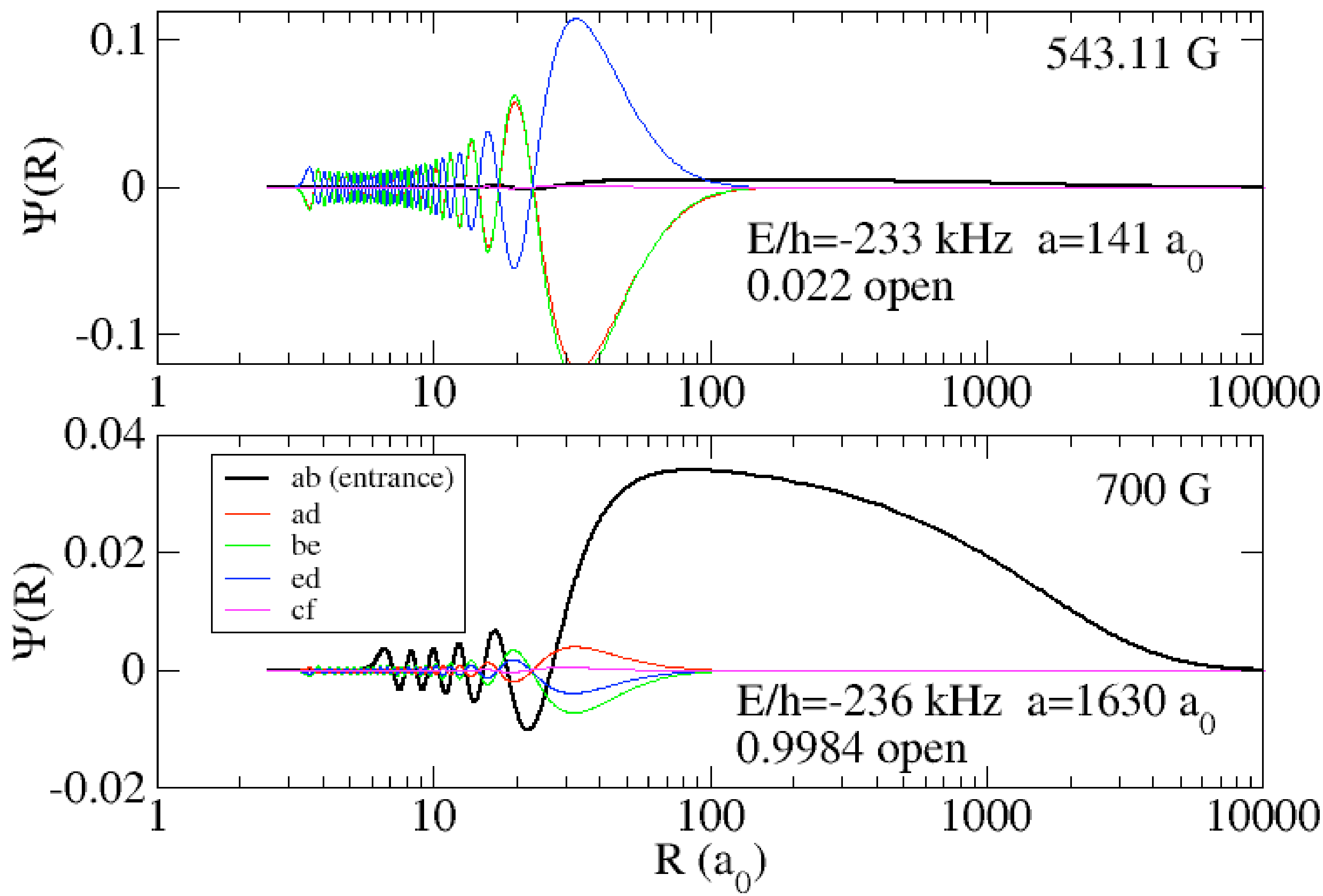
M. Bartenstein, A. Altmeyer, S. Riedl, R. Geursen, S. Jochim, C. Chin, J. Hecker Denschlag, R. Grimm, A. Simoni, E. Tiesinga, C. J. Williams, and P. S. Julienne, cond-mat/408673



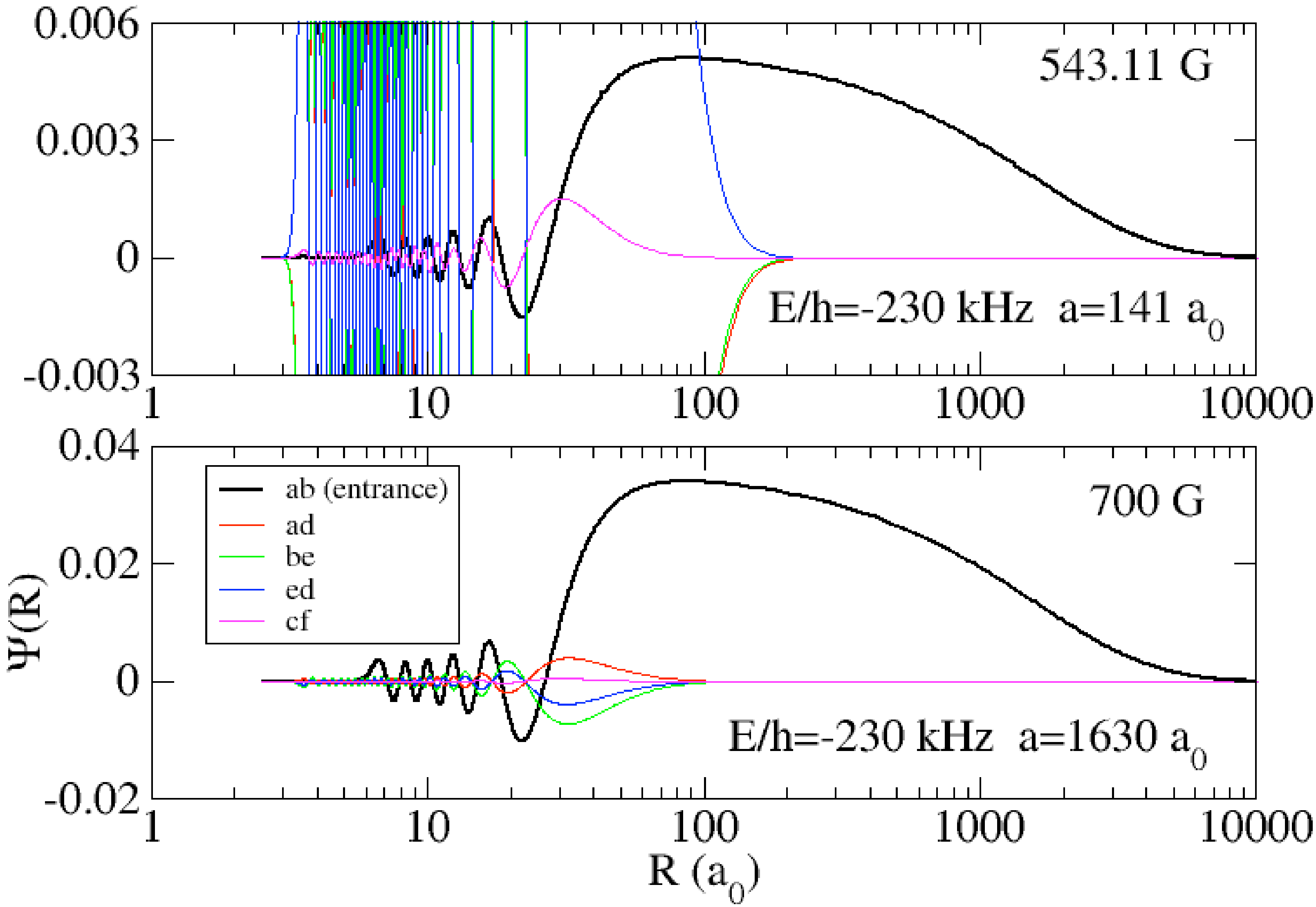
6Li 543 G resonance, coupled channels calculation
Δ
Δ
a(B)
a(B) (a0)
500
250
0
-250
-500
543
543.1
543.2
543.3
E(B)
Fraction
0
-200
-400
-600
543
543.1
543.2
543.3
B (Gauss)



^{6}Li 543 G resonance, coupled channels calculation
Δ
Δ
a(B)
a(B) (a_0)
500
250
0
-250
-500
543
543.1
543.2
543.3
E(B)
E(B)/h (kHz)
0
-200
-400
-600
$E(B) = -\frac{\hbar^2}{2\mu a(B)^2}$
B (Gauss)



543.11 G
E/h=-233 kHz a=141 a_0
0.022 open
Ψ(R)
0.1
0
-0.1
1
10
100
1000
10000
700 G
E/h=-236 kHz a=1630 a_0
0.9984 open
ab (entrance)
ad
be
ed
cf
0.04
0.02
-0.02
R (a_0)



0.006
0.003
0
-0.003
543.11 G
E/h=-230 kHz a=141 a0
1
10
100
1000
10000
0.04
0.02
0
-0.02
ab (entrance)
ad
be
ed
cf
700 G
E/h=-230 kHz a=1630 a0
Ψ(R)
R (a0)

Open channel range: $$\left|\frac{B-B_n}{\Delta_n}\right| \ll \frac{2\mu a_{bg}^2}{\hbar^2}\frac{s_n\Delta_n}{2}$$

(from Eqs. 34 and 109 of Goral *et al*, J. Phys. B 37, 3457 (2004))

Open channel dominated resonances: $|(B-B_n)/\Delta_n| \sim 1$

^{6}Li ab 834 G

^{40}K ab 200 G

^{85}Rb ee 155 G

^{133}Cs aa 48 G

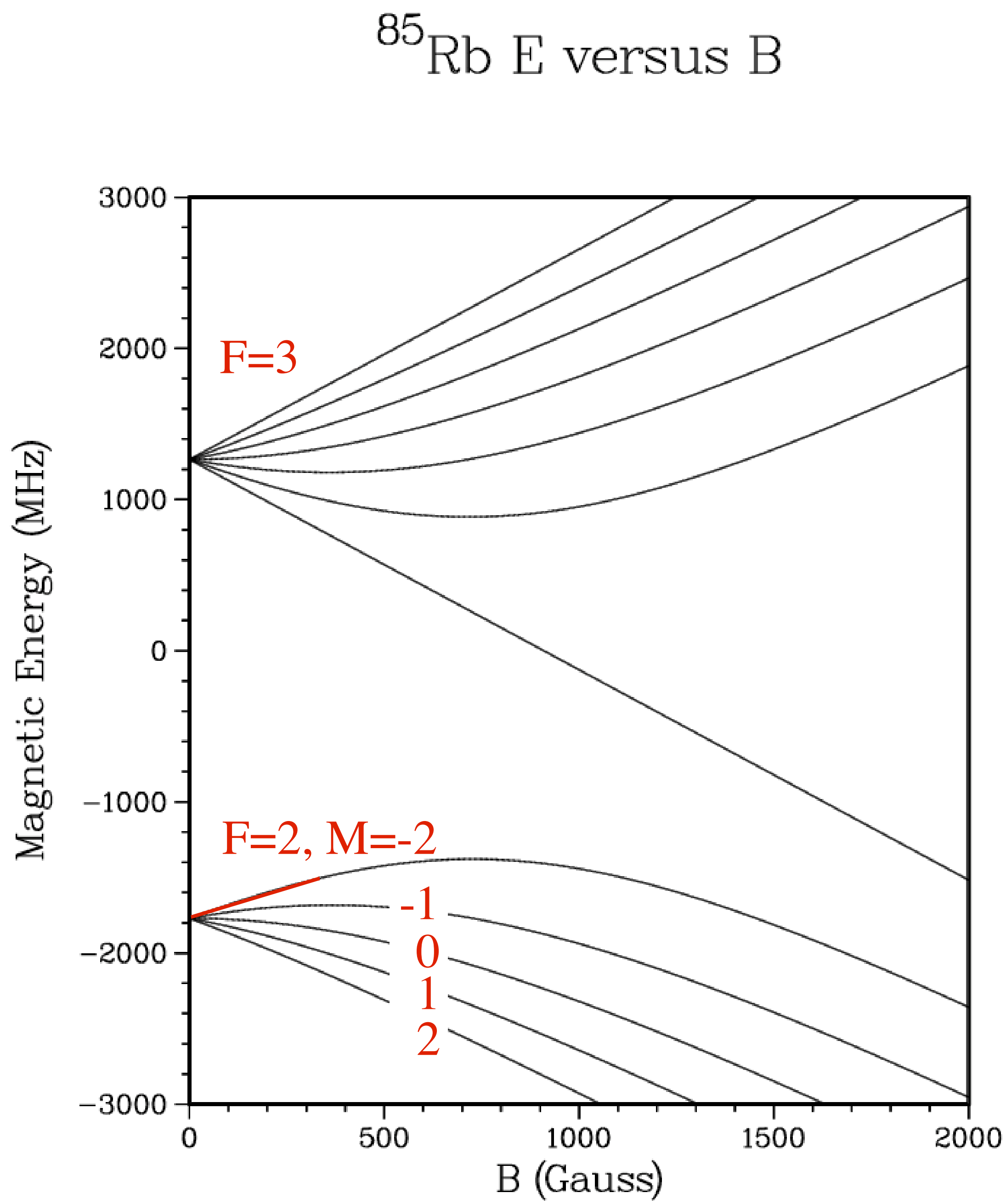
Closed channel dominated resonances: $|(B-B_n)/\Delta_n| << 1$

^{6}Li ab 543 G

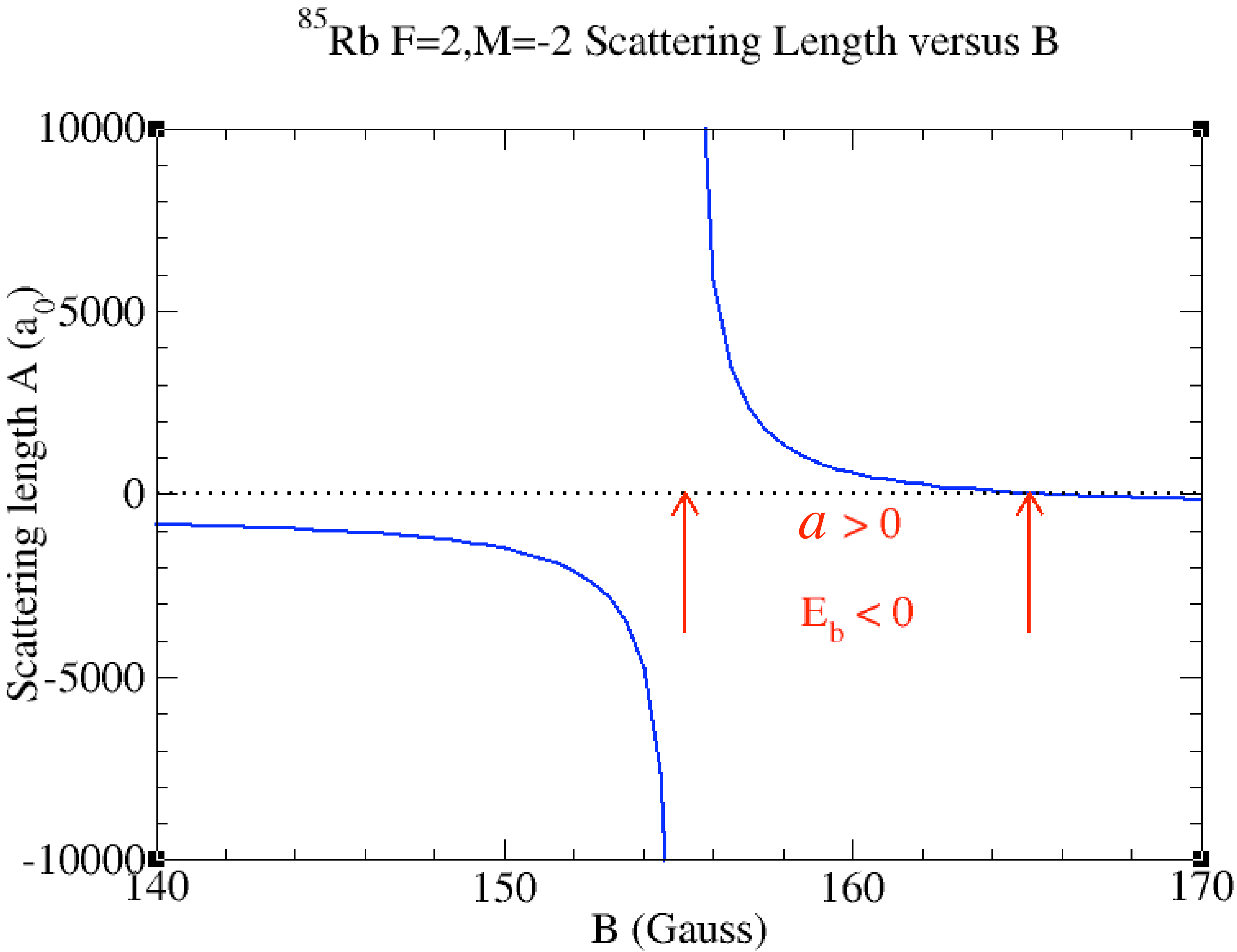
^{23}Na aa 852 and 907 G

^{87}Rb aa 1007 G

^{133}Cs aa 17 G

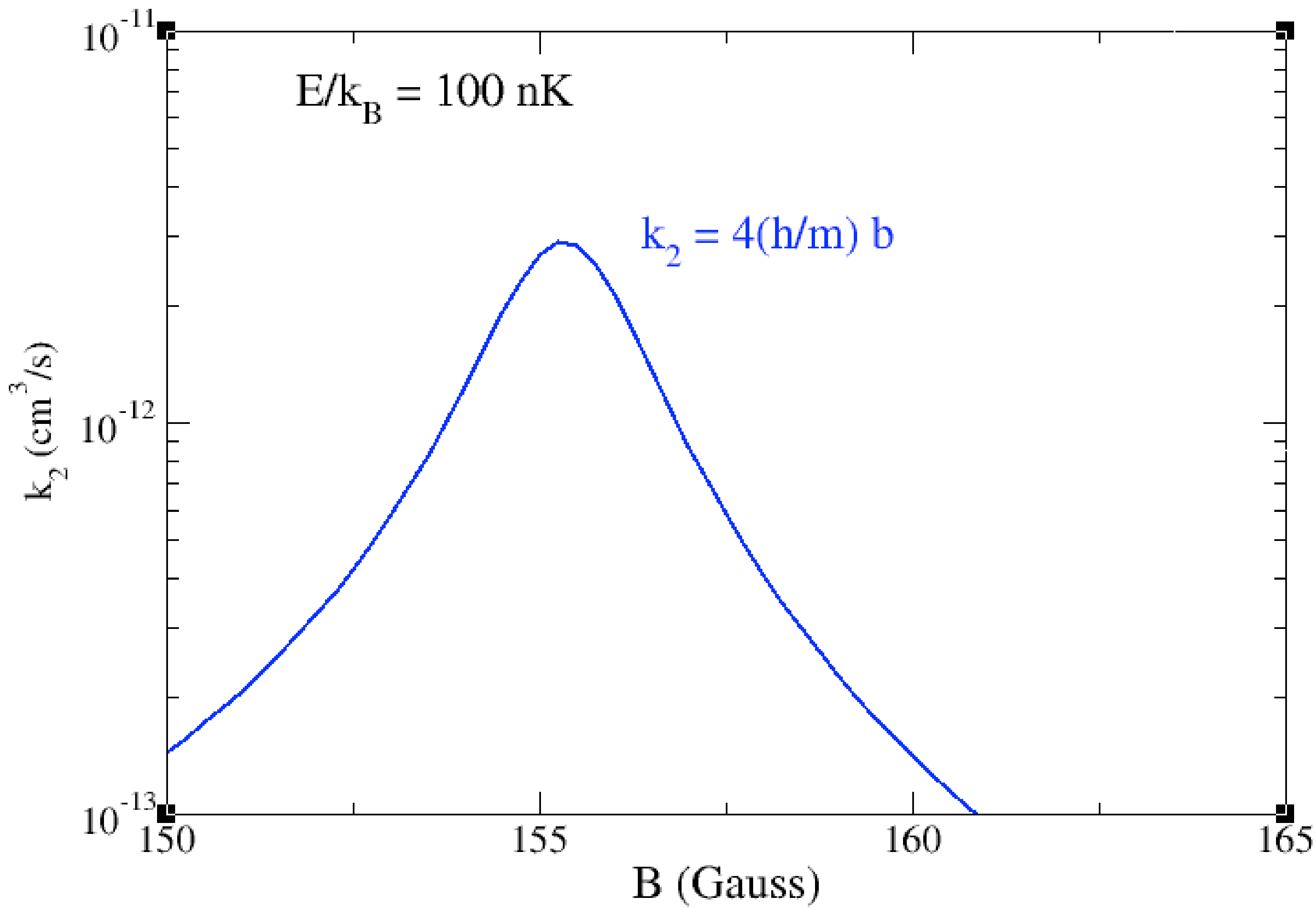


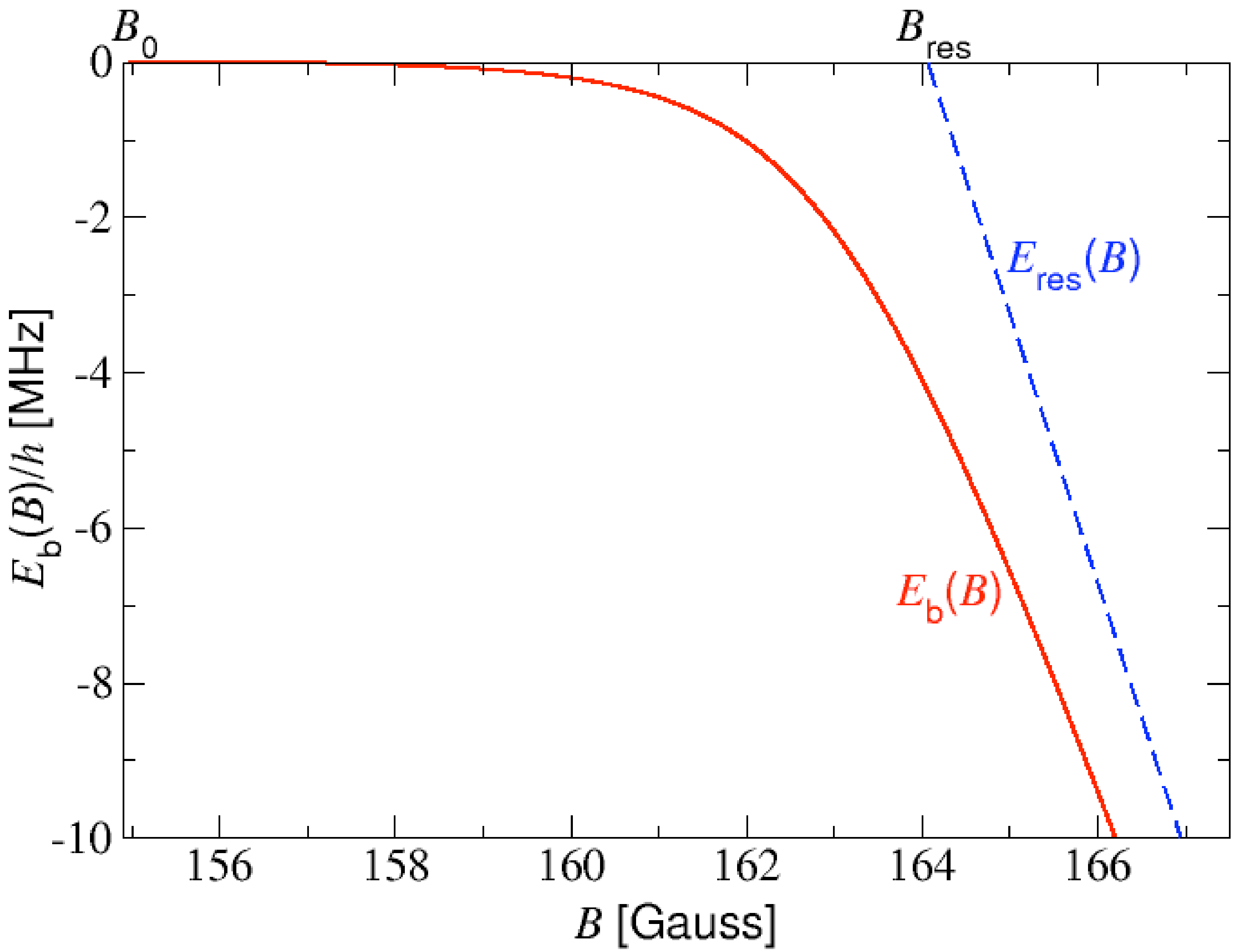
85Rb E versus B
F=3
F=2, M=-2
-1
0
1
2
Magnetic Energy (MHz)
B (Gauss)
3000
2000
1000
0
−1000
−2000
−3000
0
500
1000
1500
2000



85Rb F=2,M=-2 Scattering Length versus B
10000
5000
0
-5000
-10000
Scattering length A (a0)
140
150
160
170
B (Gauss)
a > 0
Eb < 0

2-body inelastic decay rate constant

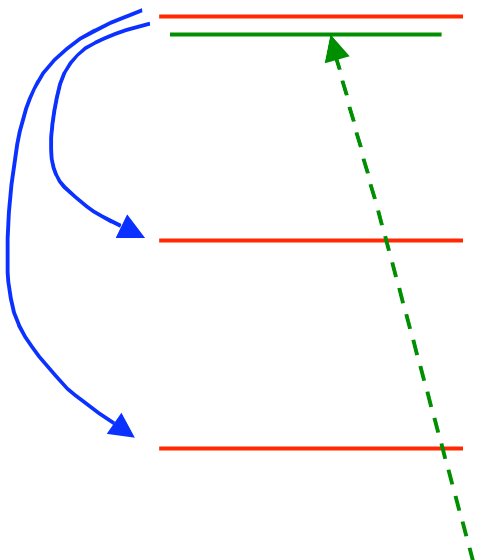




B_0
B_res
E_res(B)
E_b(B)
E_b(B)/h [MHz]
B [Gauss]
0
-2
-4
-6
-8
-10
156
158
160
162
164
166

Lifetime of coupled-channels bound state

2,-2 + 2,-2 $\xrightarrow{k_2}$ products



2,-2 + 2,-2 channel undergoes spin-dipolar relaxation

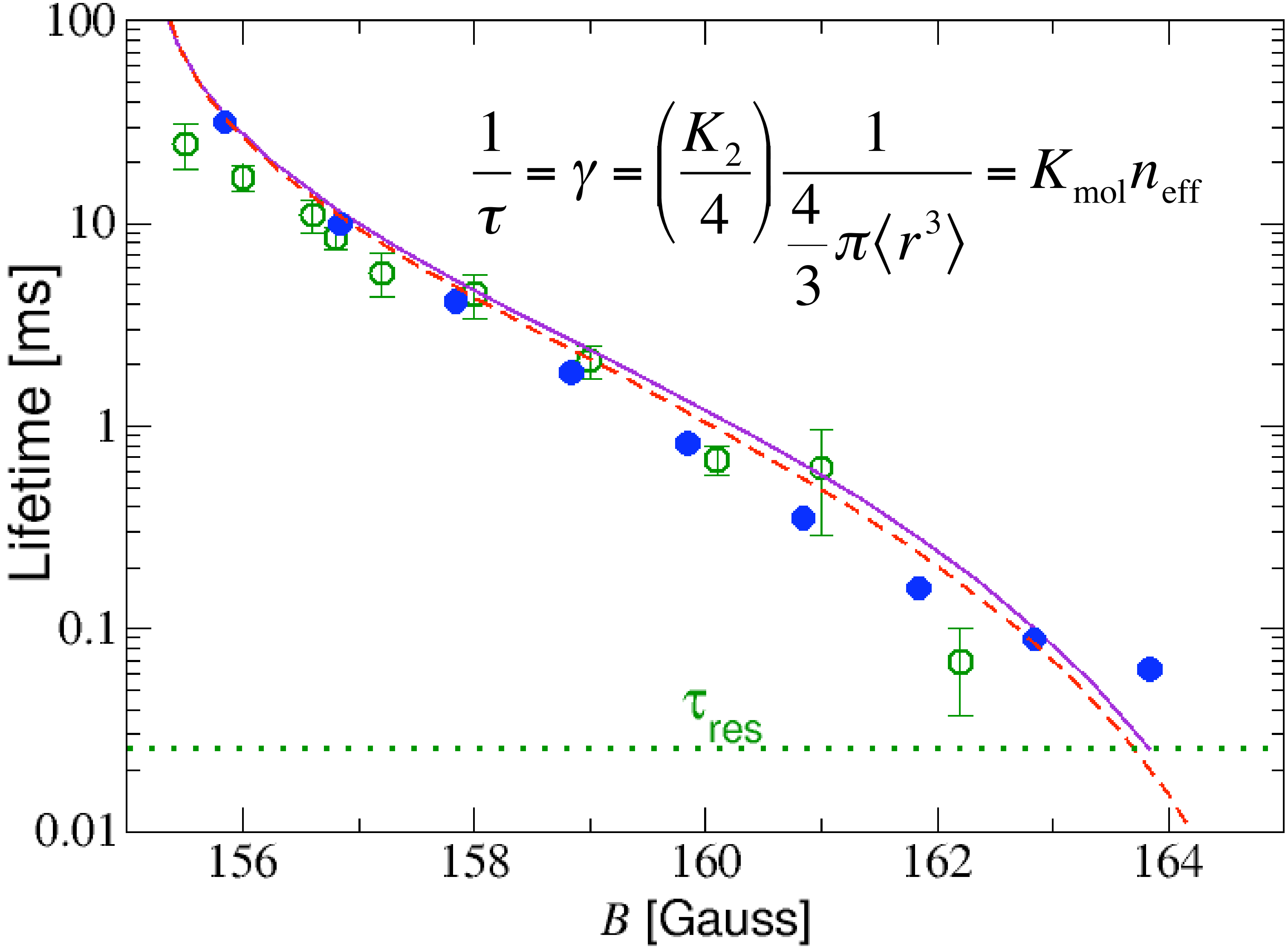
Bound $\xrightarrow[1/\tau]{}$ products

2,-2 + 2,-1 + 4 mK (at 164 G)

2,-2 + 2,0 + 8 mK

2,-1 + 2,-1 + 8 mK

Last bound state of 2,-2 + 2,-2 channel decays to same channels via dipolar relaxation.



Theory: Koehler, Tiesinga, Julienne, cond-mat/0408387
Data: Thompson, Hodby, Wieman, cond-mat/0408144

Making cold molecules by time-dependent resonances in a trap

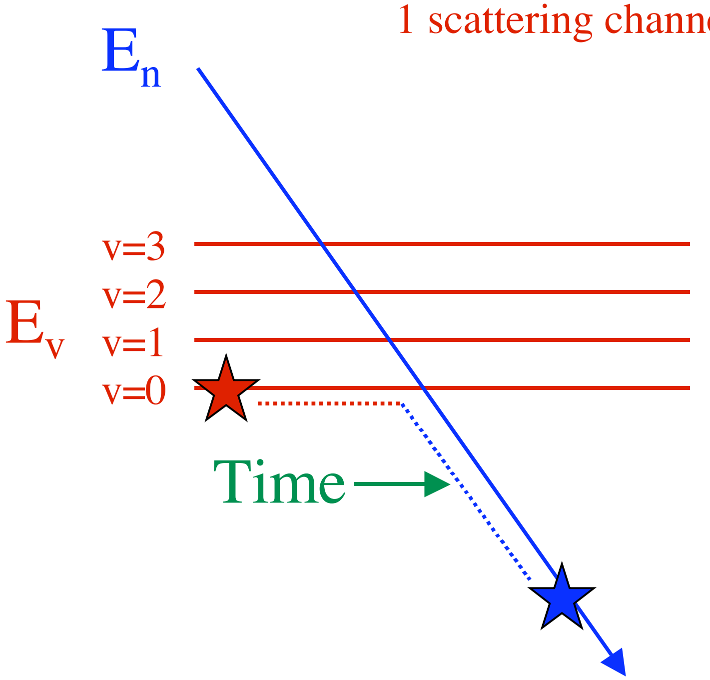
F. H. Mies, E. Tiesinga, P. S. Julienne, *Phys. Rev.* A **61**, 022721 (2000)

P. S. Julienne, E. Tiesinga, T. Köhler, cond-mat/0312492; J. Mod. Opt. **51**, 1787(2004)

Adapt coupled channels free-space scattering to a trap:

1 closed channel with 1 bound (resonance) state

1 scattering channel with discrete trap (scattering) levels

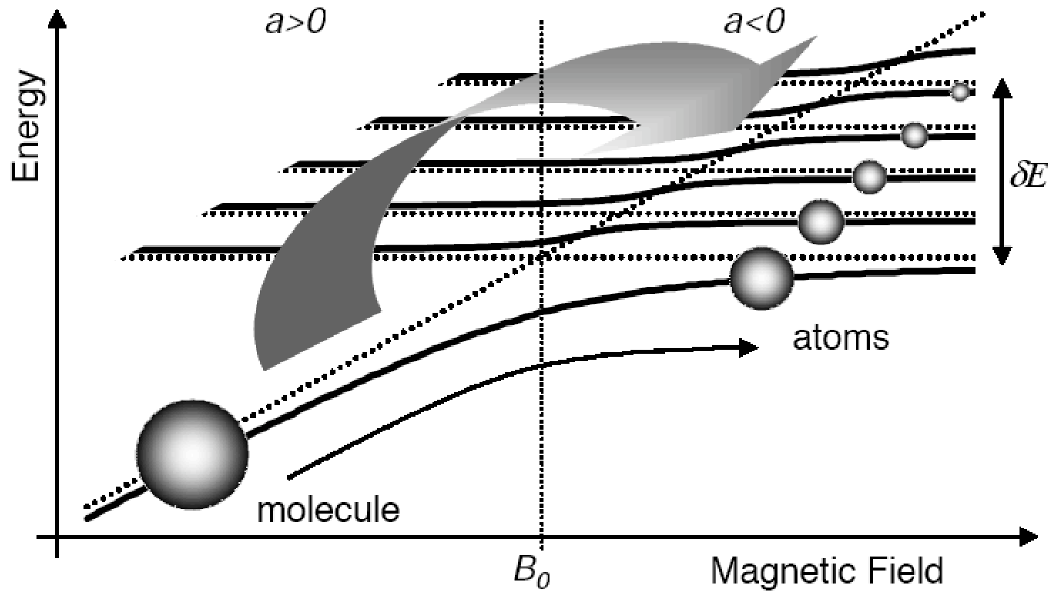


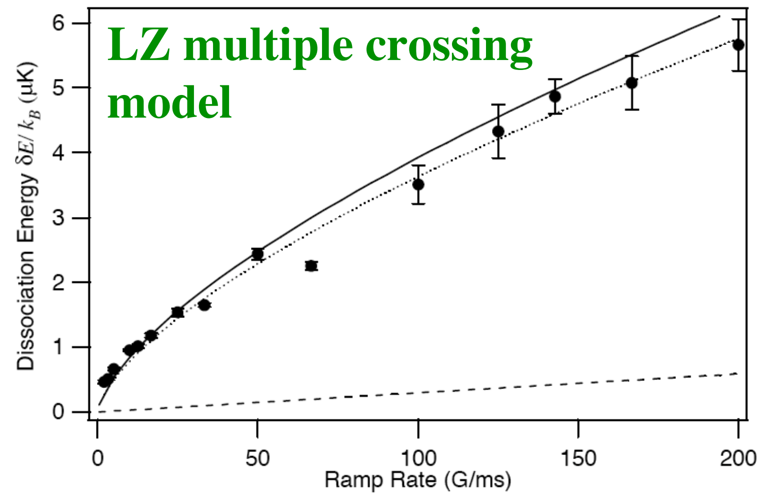
$$\frac{V_{nv}}{\hbar\omega} = \left(\frac{2\sqrt{3}}{\pi}\right)^{\frac{1}{2}} \left(\frac{A_{bg}}{d}\right)^{\frac{1}{2}} \left(\frac{s_n \Delta_n}{\hbar\omega}\right)^{\frac{1}{2}} \left(1 + \tfrac{4}{3}\mathrm{v}\right)^{\frac{1}{4}}$$

Landau-Zener curve crossing:

$$A_{LZ} = \frac{2\pi}{\hbar} \frac{|V_{nv}|^2}{s_n \dot{B}} = \frac{4\hbar}{\mu d^3} \left| \frac{A_{bg}\Delta_n}{\dot{B}} \right|$$

Fraction converted: $1 - e^{-A_{LZ}}$





Time --->

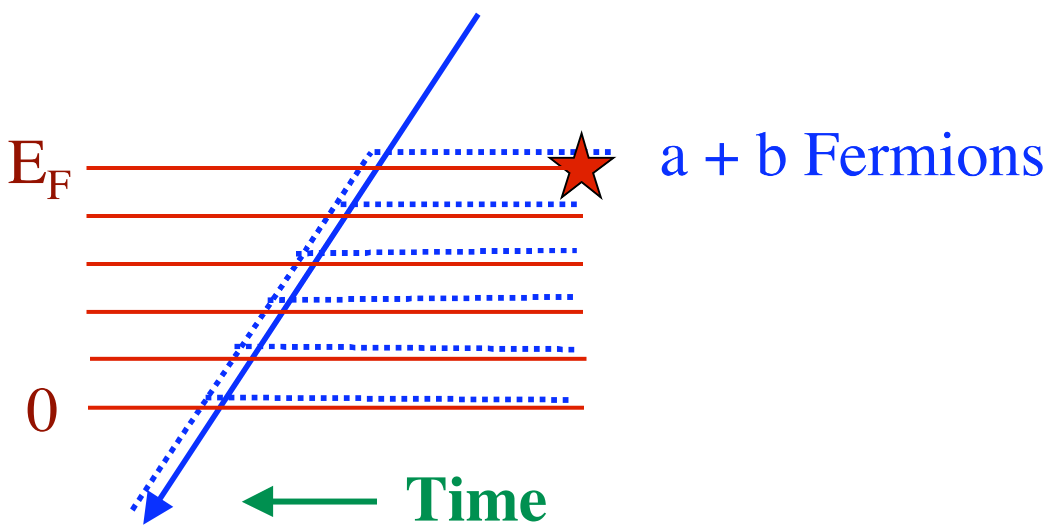
From Mukaiyama et al., PRL 92, 180402 (2004)

See also Goral et al, J. Phys. B 37 (2004) 3457-3500.

Short time, single collision model

Landau-Zener curve crossing: $A_{LZ} = \frac{1}{V} 4\pi \frac{h}{m} \left| \frac{A_{bg}\Delta_n}{\dot{B}} \right|$ independent of E

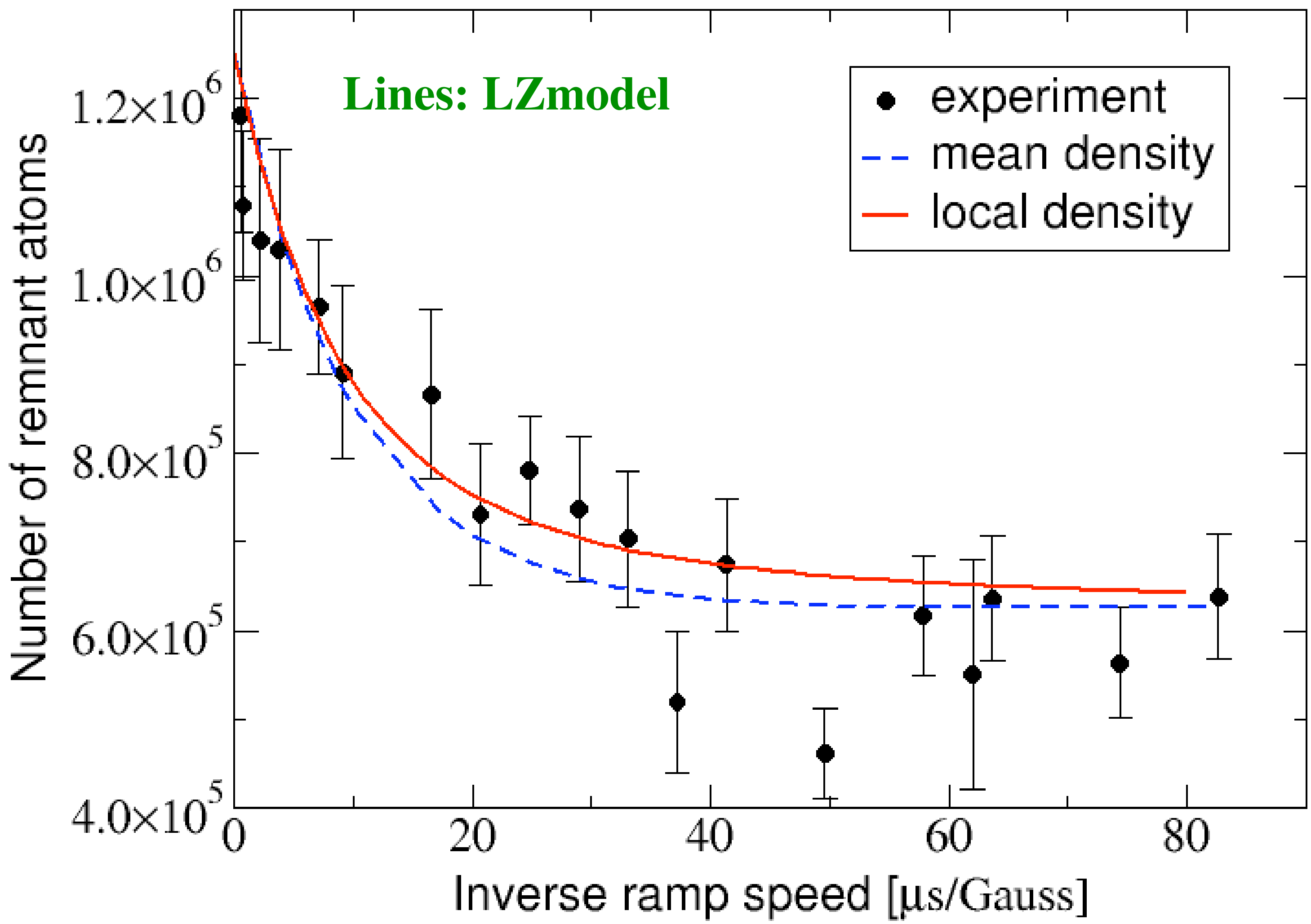
Fraction converted: $2\alpha(1-\alpha)\left[1-\exp\left(-n\frac{4\pi h}{m}\left|\frac{A_{bg}\Delta_n}{\dot{B}}\right|\right)\right]$



Jan Chwenenczuk, Krzysztof Goral, Thorsten Köhler, PSJ, cond-mat/0409192

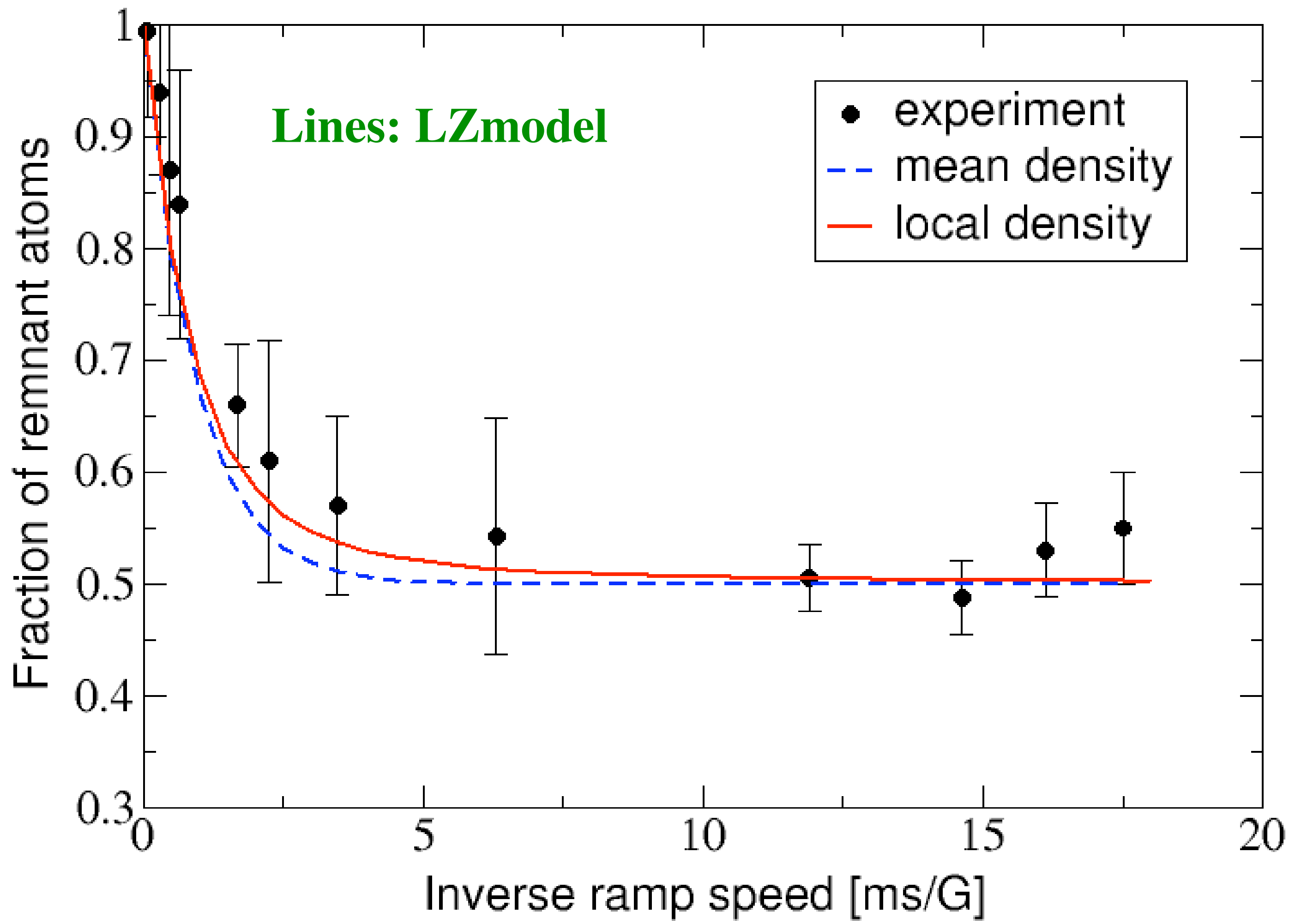
$^{40}K_2$ F=9/2, M=-9/2 + F=9/2, M=-5/2

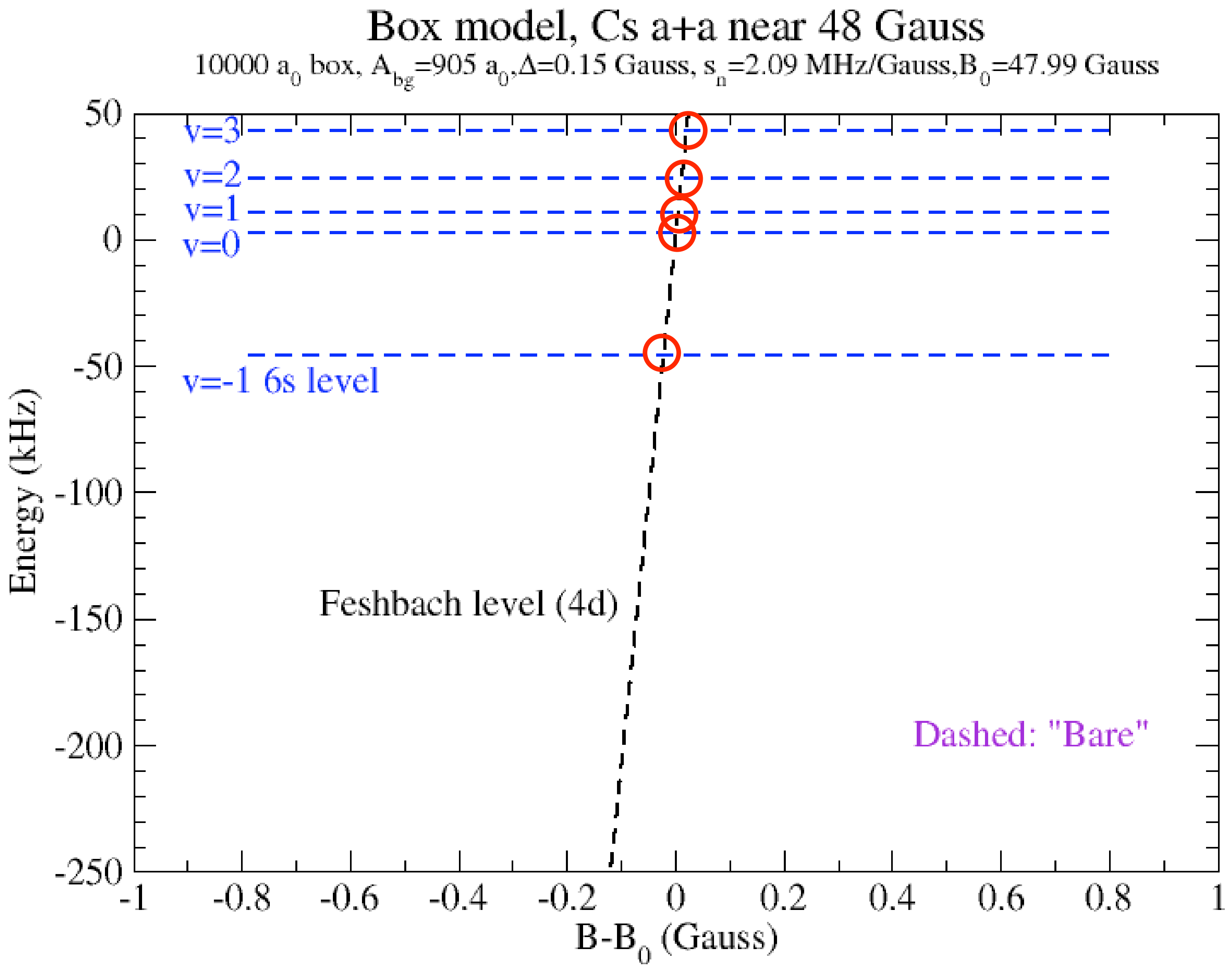
Data: Regal, Ticknor, Bohn, Jin, Nature 424, 47 (2003)



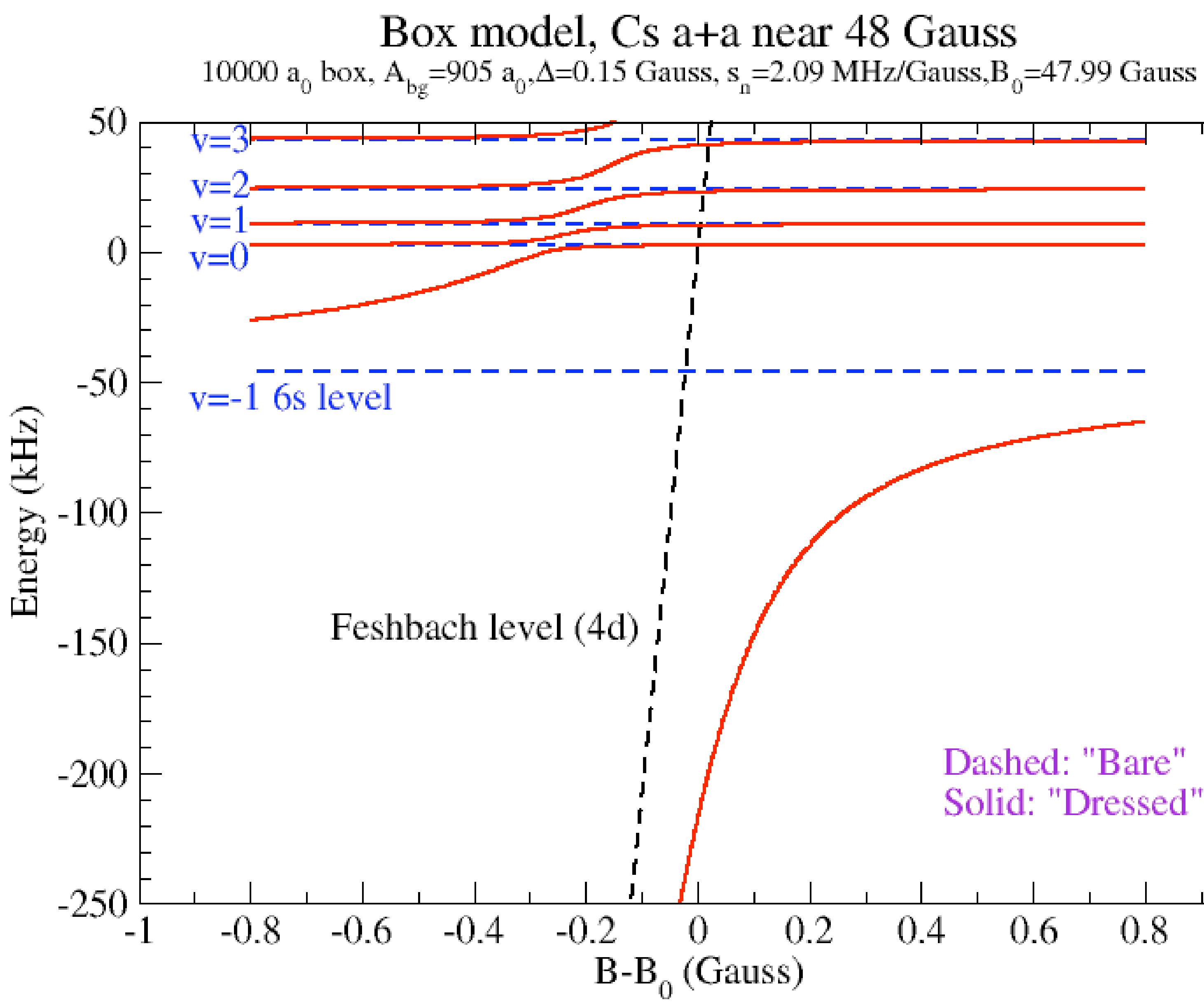
$^6Li_2$ F=1/2, M=1/2 + F=1/2, M=-1/2

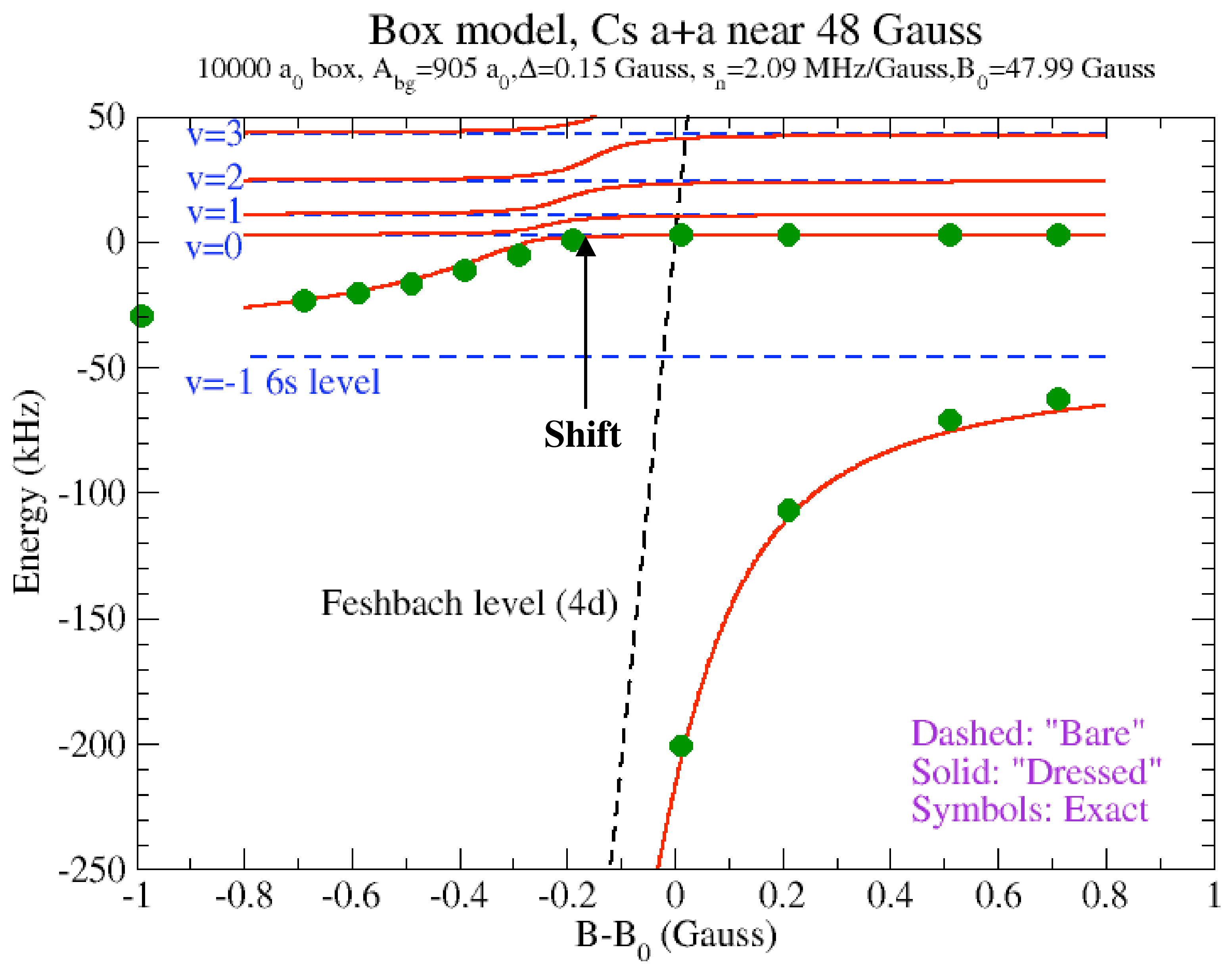
Data: Strecker, Partridge, Hulet, Phys. Rev. Lett. 91, 080406 (2003)



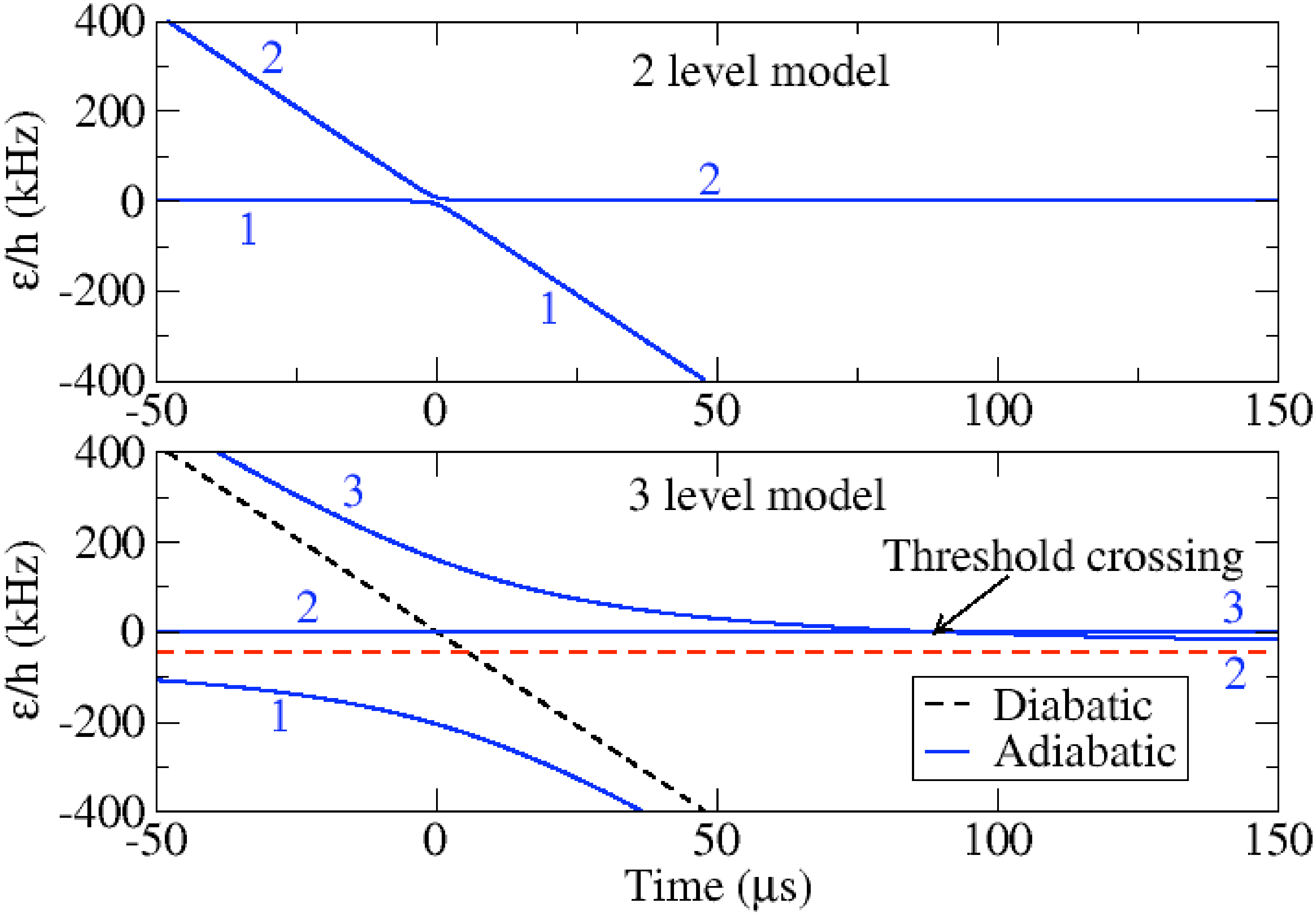


Julienne, Tiesinga, Köhler, cond-mat/0312492; J. Mod. Opt. **51**, 1787(2004)

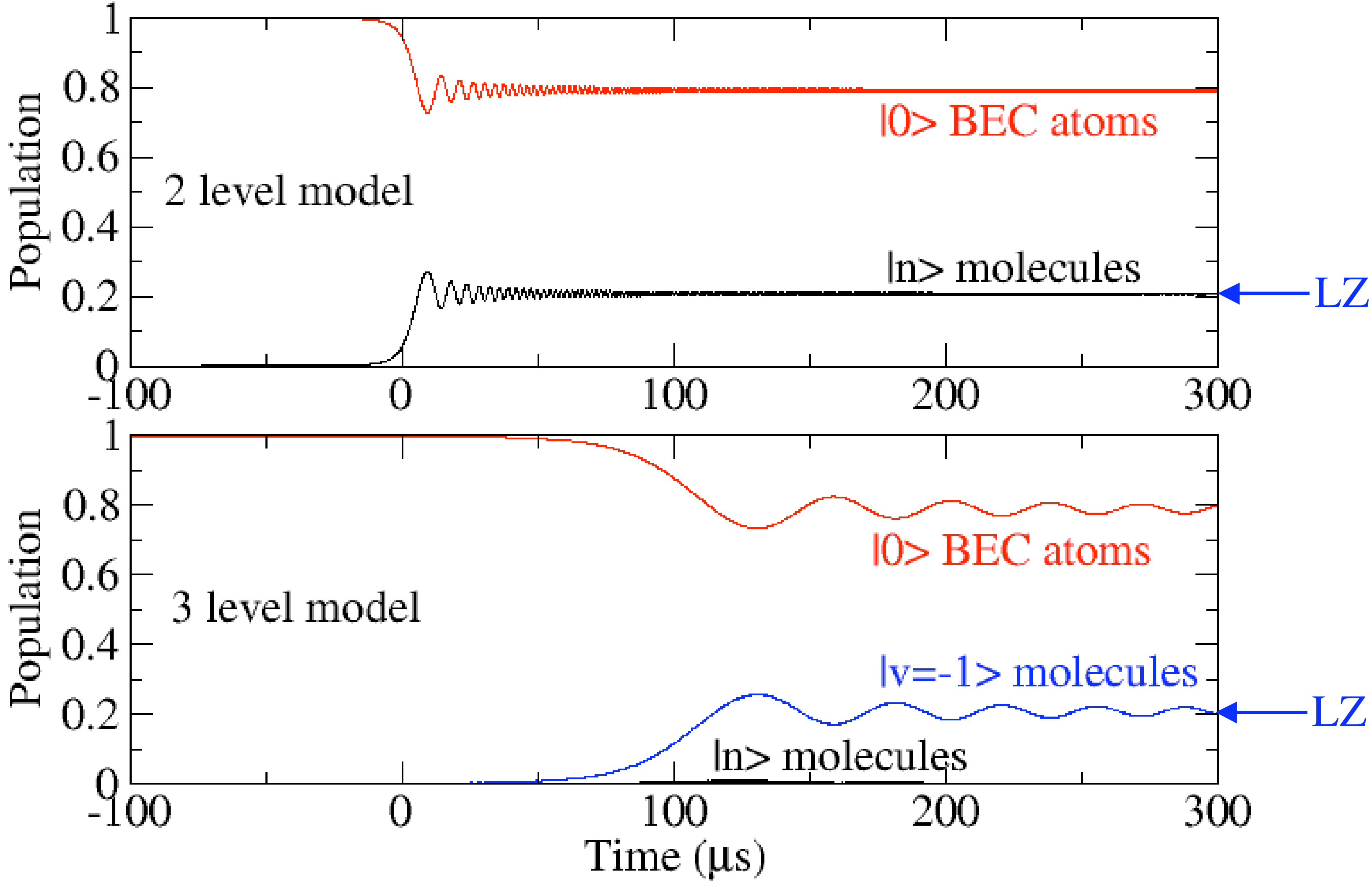

Box model, Cs a+a near 48 Gauss
10000 a0 box, Abg=905 a0,Δ=0.15 Gauss, sn=2.09 MHz/Gauss,B0=47.99 Gauss
v=3
v=2
v=1
v=0
v=-1 6s level
Feshbach level (4d)
Dashed: "Bare"
Solid: "Dressed"
Energy (kHz)
B-B0 (Gauss)
50
0
-50
-100
-150
-200
-250
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1



Box model, Cs a+a near 48 Gauss
10000 a0 box, Abg=905 a0,Δ=0.15 Gauss, sn=2.09 MHz/Gauss,B0=47.99 Gauss
v=3
v=2
v=1
v=0
v=-1 6s level
Shift
Feshbach level (4d)
Dashed: "Bare"
Solid: "Dressed"
Symbols: Exact
Energy (kHz)
B-B0 (Gauss)
50
0
-50
-100
-150
-200
-250
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

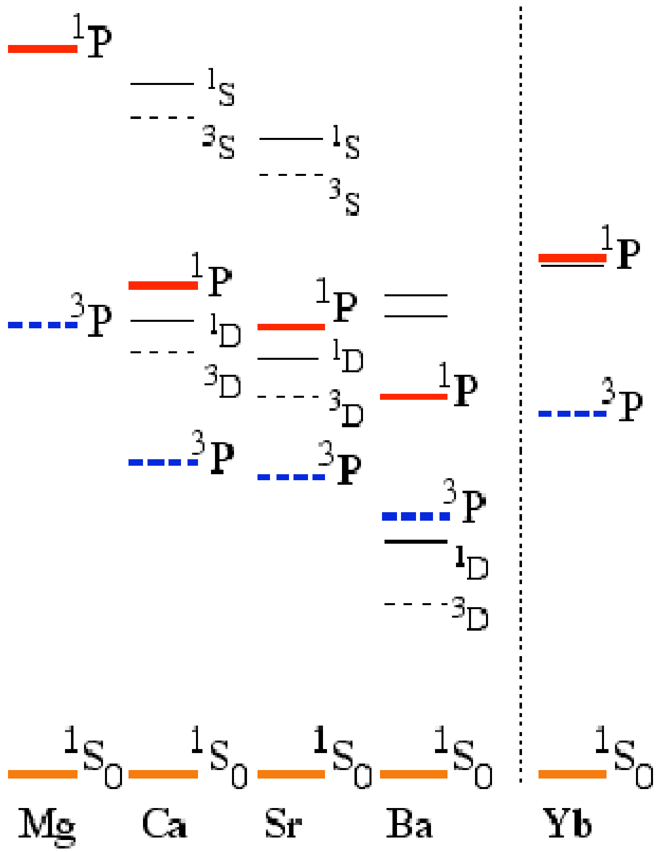


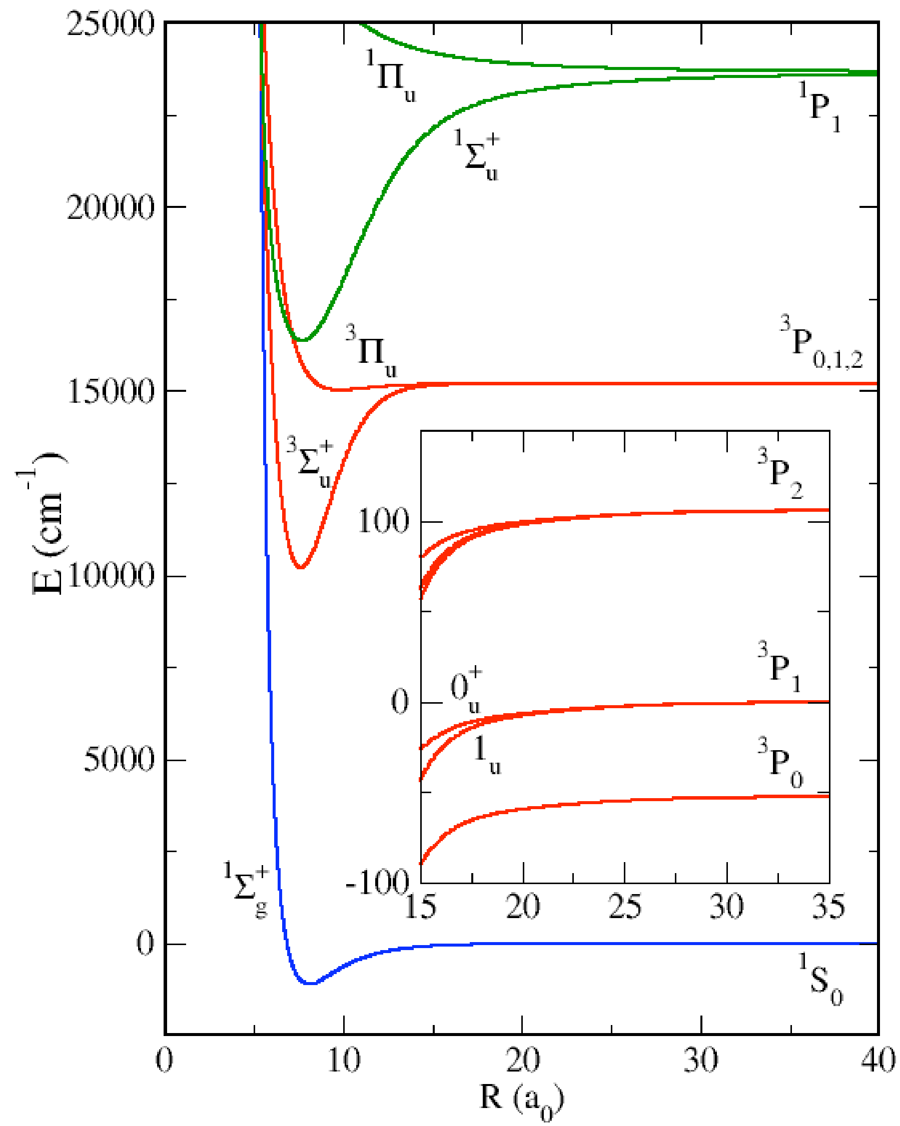
2 level model
3 level model
Threshold crossing
Diabatic
Adiabatic
ε/h (kHz)
Time (μs)



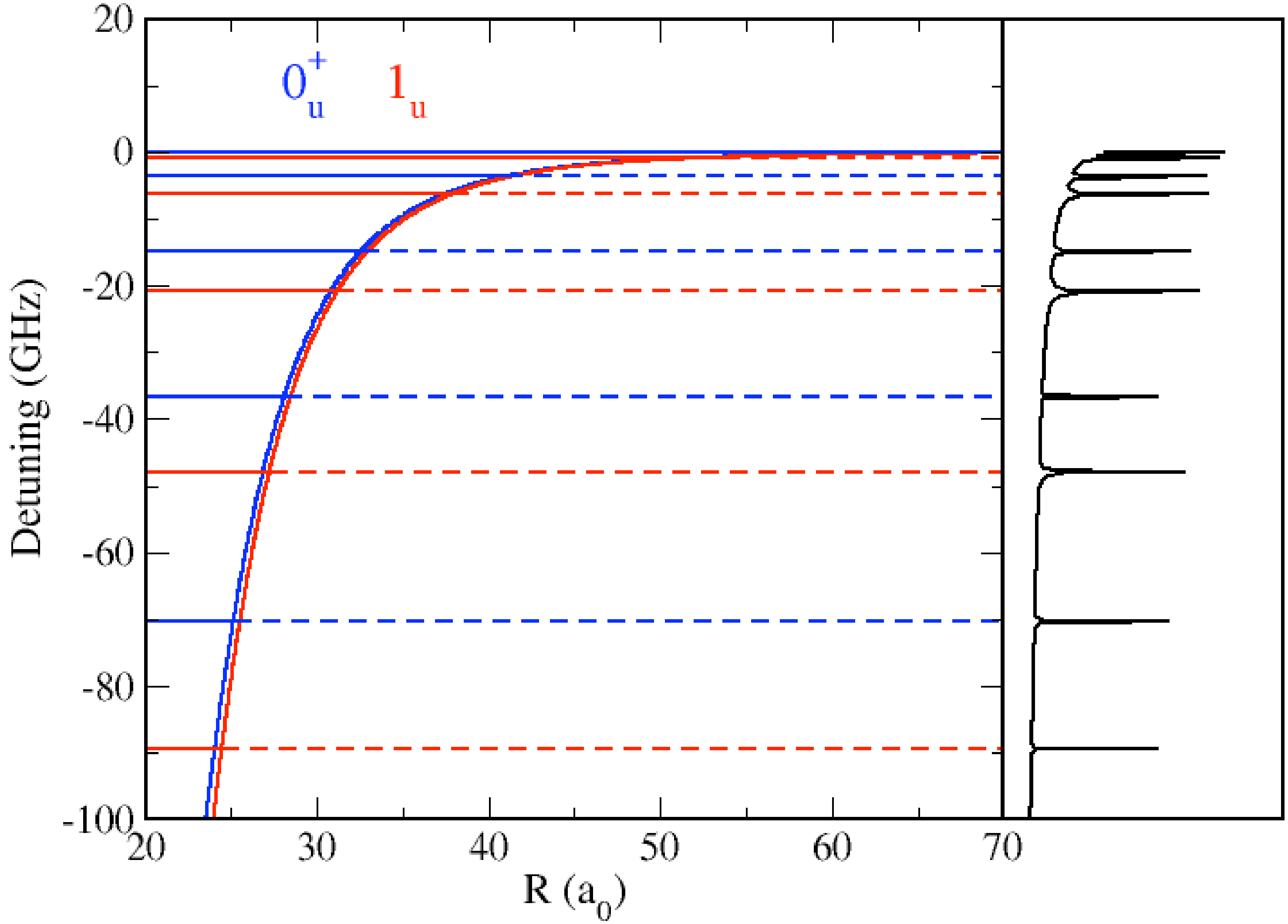
Population
2 level model
|0> BEC atoms
|n> molecules
LZ
3 level model
|0> BEC atoms
|v=-1> molecules
|n> molecules
LZ
Time (μs)

Group II Atoms





Ca photoassociation spectrum



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