The small tunneling amplitude boson-Hubbard dimer

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Abstract
The boson-Hubbard dimer describes a BEC in a double well potential using the two-body approximation, which is robust in the small tunneling regime. Analytical expressions for the energy eigenstates of this model are obtained by applying perturbation theory in the small tunneling amplitude limit. They are compared with the corresponding numerical solutions and the limits of their validity are determined.

The boson-Hubbard dimer Hamiltonian
A BEC in a double well potential with sufficiently distant local minima can be described by the two-site Hamiltonian:

\[ H = -U (a_1^{\dagger} a_2 a_2^{\dagger} a_1 + a_2^{\dagger} a_1 a_1^{\dagger} a_2) + W_0 (a_1^{\dagger} a_1 + a_2^{\dagger} a_2) \]

\( k_1, k_2 \) creation and annihilation operators of bosons at the \( n \) well.
\( \kappa \) tunneling amplitude between the two wells.
\( U \) interaction energy between pairs of atoms that are confined in a particular well.
\( i, j \) (Coulombian part).

Mean-field approximation
Mean-field approximation \( \Rightarrow \) Discrete Nonlinear Schrödinger (DNLs) dimer [2]

DNLs energy spectrum [6]:

\[ E_{\text{DNLs}} = \frac{(\Delta + 1)}{2} K \]

\( K = \sqrt{2 (N - 1) \cos \theta} \)

Mean-field eigenvalues and eigenvectors up to second order [6]:

\[ \Delta \approx \cos \theta \]

\( \omega_2 \approx 2 (N - 1) \cos \theta \)

\[ \nu \approx 2 \cos \theta \]

Completely localized initial state \( \langle \Psi(0) | = (\frac{1}{2}) \]

Energy eigenvalues for small \( k \)
For \( k = 0 \), the \( N \times N \) eigenvalues form degenerate pairs of levels with energies \( \pm E_{\text{DNLs}} \), where the number of bosons \( n \) is positive integer (for even \( N \)) or half-integer (for odd \( N \)).

Energy eigenvectors for small \( k \) are localized and only allow the occupation of one site of the dimer by \( n \) bosons and of the other site by the remaining \( \frac{N - n}{2} \) bosons.

Evolution of the relative number difference
\[ (\Delta \epsilon) = \sum_{n=1}^{N} \left| \langle \psi(t) | n \rangle - \langle \psi(0) | n \rangle \right|^2 \]

Completely localized initial state \( \langle \Psi(t) | = (\frac{1}{2}) \]

Energy eigenvalues for small \( k \)
For small \( k \), the eigenvalues up to second order in \( k \) are [6]:

\[ E_{\text{DNLs}} \approx \frac{1}{2} k^2 + \frac{1}{2} (2K_{\text{DNLs}} - 1) \]

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Completely localized initial state \( \langle \Psi(t) | = (\frac{1}{2}) \]

Coherent spin initial state
For \( n \neq 0 \), the eigenvalues up to second order in \( k \) are [6]:

\[ E_{\text{DNLs}} \approx \frac{1}{2} k^2 + \frac{1}{2} (2K_{\text{DNLs}} - 1) \]

\( k_1, k_2 \) creation and annihilation operators of bosons at the \( n \) well.
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Evolution of the relative number difference
\[ (\Delta \epsilon) = \sum_{n=1}^{N} \left| \langle \psi(t) | n \rangle - \langle \psi(0) | n \rangle \right|^2 \]

Completely localized initial state \( \langle \Psi(t) | = (\frac{1}{2}) \]

References