Irreversible delocalizing transition of a BEC in a 2D optical lattice

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Abstract

A Bose-Einstein condensate trapped in a two-dimensional ($2D$) optical lattice exhibits an abrupt transition manifested by the macroscopic wavefunction changing character from spatially localized to extended. This transition takes place as the interwell potential barrier is adiabatically decreased below a critical value and is irreversible since increasing the interwell barrier back to its initial value does not restore localization.

This is in sharp contrast with the one-dimensional case where a similar delocalization is continuous and reversible. The different behavior reflects the existence of a critical point for the appearance of localized stationary states in two dimensions.

The boson-Hubbard Hamiltonian

Many-body Hamiltonian of a BEC in an optical lattice (ignoring the trapping potential)

$$\mathcal{H} = \int d^3r \left[\hat{\Psi}^\dagger(\vec{r})(-\frac{\hbar^2}{2m}\nabla^2)\hat{\Psi}(\vec{r}) + V_{opt}\hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r}) + \frac{2\pi\hbar^2 a}{m}\hat{\Psi}^\dagger(\vec{r})\hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r})\hat{\Psi}(\vec{r}) \right]$$

Expanding the field operators in the basis of localized on individual lattice sites single boson states, $\phi_i(\vec{r}-\vec{r}_i)$ [1],

$$\hat{\Psi}(\vec{r}) = \sum_i b_i \phi_i(\vec{r}-\vec{r}_i)$$

we derive the boson-Hubbard Hamiltonian (a single band tight-binding approximation taking into account only the lowest vibrational states)

$$\mathcal{H} = -k \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i b_i^\dagger b_i^\dagger b_i b_i$$

$b_i^\dagger$, b_i: creation and annihilation operators of bosons at the i^{th} well.

k: tunneling amplitude between adjacent wells.

$U = \frac{2\pi\hbar^2 a}{m}\int |\phi(\vec{r})|^4 d^3r$: interaction energy between pairs of atoms confined in a particular well

Mean-field approximation

A mean-field approximation leads to the discrete nonlinear Schrödinger (DNLS) equation for the macroscopic wavefunction Ψ_n of the condensate at trap n [2,3]

$$i\hbar\frac{d\Psi_n}{dt} = -k\sum_\delta \Psi_\delta + 2UN|\Psi_n|^2\Psi_n,$$

where we have used that $N-1 \approx N$.

The sum over δ is over the nearest neighbors of lattice site n. In a $2D$ lattice for example, denoting a particular potential well n by the integer pair (n_x, n_y), the first term in the right hand side is

$$-k(\Psi_{n_x+1,n_y} + \Psi_{n_x-1,n_y} + \Psi_{n_x,n_y+1} + \Psi_{n_x,n_y-1})$$

Stationary solutions in $1D$ and $2D$

Stationary solutions of DNLS:

$$\Psi_n = \psi_n e^{-(i/\hbar)\Omega t}$$

where ψ_n is time-independent.

There is a band of extended (Bloch) stationary states and a branch of single-peaked localized states [4]. A bifurcation exists in $2D$ (and in $3D$ as well) in the branch of the localized states, leading to its disappearance below a critical value of the parameter $\frac{2UN}{k}$ [4].

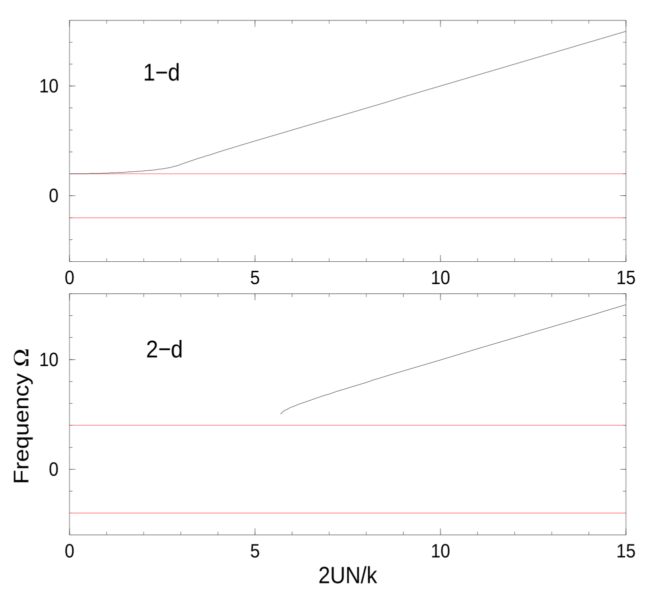

Fig.1: Frequency Ω (in units of $k/\hbar$) of the localized stationary solutions of DNLS versus the dimensionless parameter $\frac{2UN}{k}$ in a $1D$ and $2D$ lattice. Red lines represent the edges of the energies of the extended solutions, which form a band centered symmetrically around zero.

BEC can demonstrate the bifurcation point in $2D$

Apart from Bose-Einstein condensates, DNLS has also found applications

- in interacting electron-lattice models in solid state physics [4,5]. Localized solutions of DNLS correspond to polarons.
- in arrays of coupled nonlinear optical waveguides [6].
- for local intramolecular stretching vibrations in symmetric polyatomic molecules [7]
- as a generic model for studying nonlinear effects (breathers, for example) [8]

BECs, due to their unique and precise manipulation, have been proved suitable systems for demonstrating various well-known properties of solid state physics [9–12]. Here, the current experimental abilities (see Ref. [13], for example) can be used to demonstrate the existence of the bifurcation point in the localized states of DNLS in $2D$, through a striking transition as follows:

A decrease of the intensity of the laser generating the optical lattice $\Rightarrow$ *decreases the interwell barriers* $\Rightarrow$ *increases the tunneling amplitude* k $\Rightarrow$ *decreases the parameter* $\frac{2UN}{k}$ $\Rightarrow$ *an irreversible transition from the localized branch to the extended states can be induced.*

Parameter values

Estimate of the parameters from the experiment of Ref. [13]

$N = 10^4$ atoms of ^{87}Rb

S-wave scattering length: $a = 5.8\ nm$

The optical lattice is generated by a laser with $\lambda = 840\ nm$
Lattice constant: $\frac{\lambda}{2} = 0.42\ \mu m$
Linear extent of the transverse confinement $\rightarrow 0.7\ \mu m$
$\Rightarrow$ Effective volume of the local mode $\phi(\vec{r}) \rightarrow 0.2\ \mu m^3$

The interaction energy U is of the order of $10^{-4}\ neV$

Recoil energy $E_R = \frac{2\pi^2\hbar^2}{m\lambda} \sim 0.01\ neV$

Variation of the height of the barriers: from $\sim E_R$ up to $\sim 50E_R$
(this strongly affects k, but only slightly U [1])
$\Rightarrow$ the ratio $\frac{2UN}{k}$ varies from ~ 1 up to $\sim 10^5$
$\Rightarrow$ k varies from $\sim 10^{-5}\ neV$ to $\sim 1\ neV$

Irreversible transition in $2D$

Variation of the interwell barriers of the optical lattice by crossing the bifurcation point

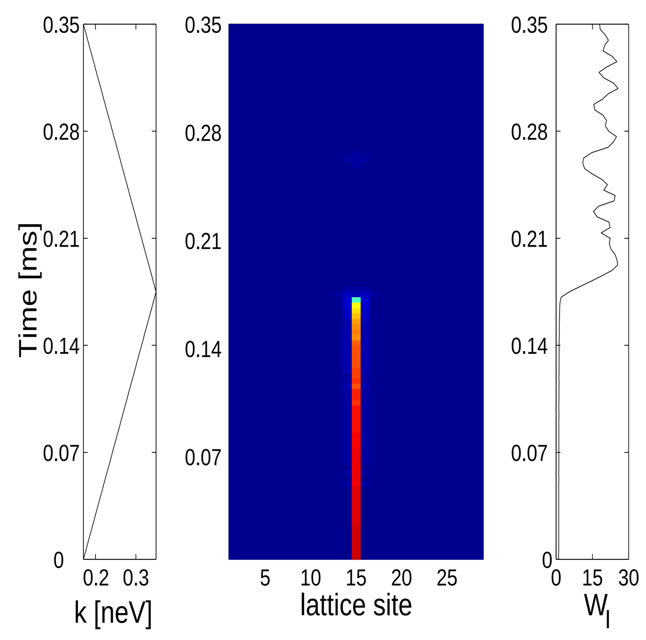

Fig.2: Demonstration of abrupt delocalizing transition in $2D$ as the tunneling amplitude k is linearly changed from $0.17\ neV$ to $0.35\ neV$ and back, during time interval $0.35\ ms$ (left panel). The evolution of the condensate is illustrated by the cross-section $\Psi_{n_x=15,n_y}$ of the wavefunction in the center panel where red color indicates high amplitude (~ 1) and blue color indicates low amplitude ($\ll 1$), and by its linear extent $W_l = Z^{1/D}$, as represented through the participation number, $Z = 1/\sum_n |\Psi_n|^4$, in the right panel. The remaining parameters are $U = 10^{-4}\ neV$ for $N = 10^4$ atoms in a 30×30 optical lattice [14].

Variation of the interwell barriers without crossing the bifurcation point

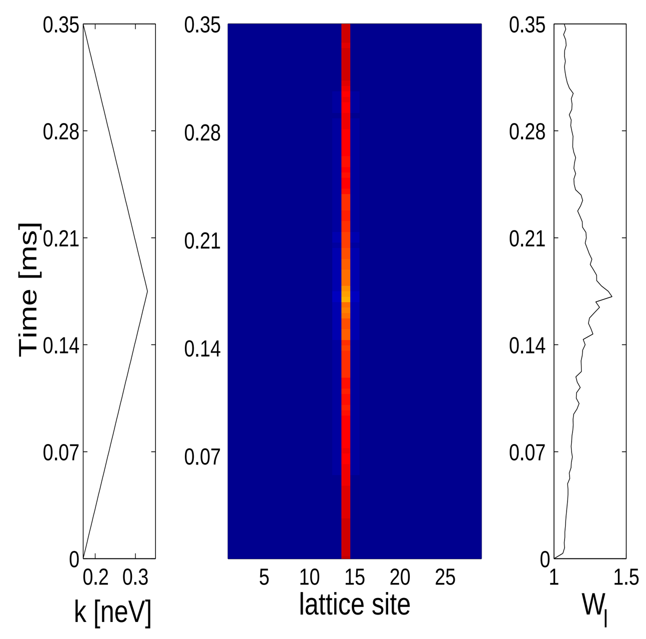

Fig.3: Absence of delocalizing transition in $2D$ as the tunneling amplitude k is linearly changed from $0.17\ neV$ to $0.33\ neV$ and back, during the time interval $0.35\ ms$ (left panel), without the ratio $\frac{2UN}{k}$ crossing the critical point. The center and right panels are analogous to Fig.2, as are the remaining parameters [14].

Absence of irreversible transition in $1D$

Large variation of the interwell barriers in 1D

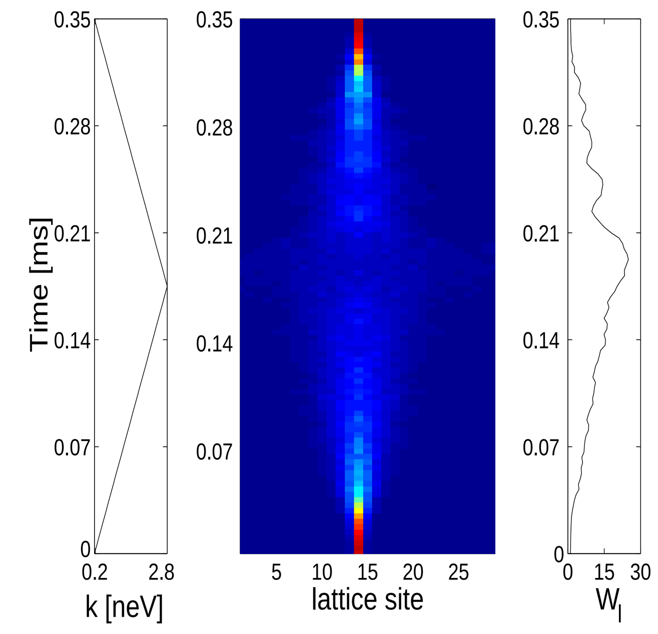

Fig.4: In $1D$ the delocalizing transition is replaced by a continuous delocalization and re-localization as the tunneling amplitude k is linearly changed from $0.2\ neV$ to $2.8\ neV$ and back during time interval $0.35\ ms$ (left panel). The center (right) panel depicts the evolution of the $1D$ wavefunction, (linear extent W_l). U and N are as in Fig. 2 and the $1D$ lattice consists of 30 sites [14].

The time scale of the transition

Relevance of the time scale of system's intrinsic dynamics

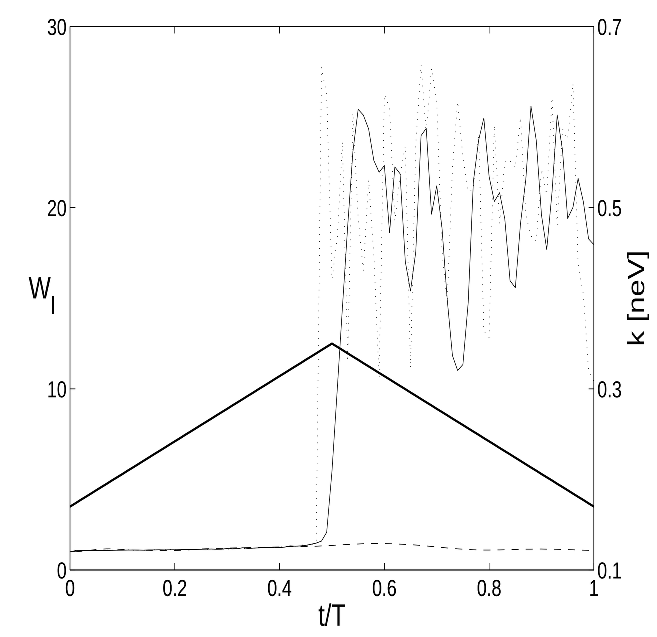

Fig.5: The evolution of the linear extent W_l of a $2D$ condensate as the coupling k is swept from $0.17\ neV$ to $0.35\ neV$ and back (sweeping profile shown by the thick solid line, referring to the right axis) for three different sweeping times T: $3.5\ ms$ (dotted line), $0.35\ ms$ (thin solid line), and $17.5\ \mu s$ (dashed line). The remaining parameters are as in Fig. 2 [14].

Conclusions

- Demonstration of an abrupt, irreversible, delocalizing transition in a $2D$ optical lattice, as the height of the periodic potential varies in a closed loop.
- The disappearance of localized stationary modes below a bifurcation point drives the condensate wavefunction to the branch of extended Bloch states.
- Similar phenomena should appear in $3D$ optical lattices.
- Such a delocalizing transition is absent in $1D$ since the mean-field nonlinearity ensures the existence of a continuous branch of localized solutions.
- The transition is sensitive to the time scale of the variation of the interwell barriers.

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