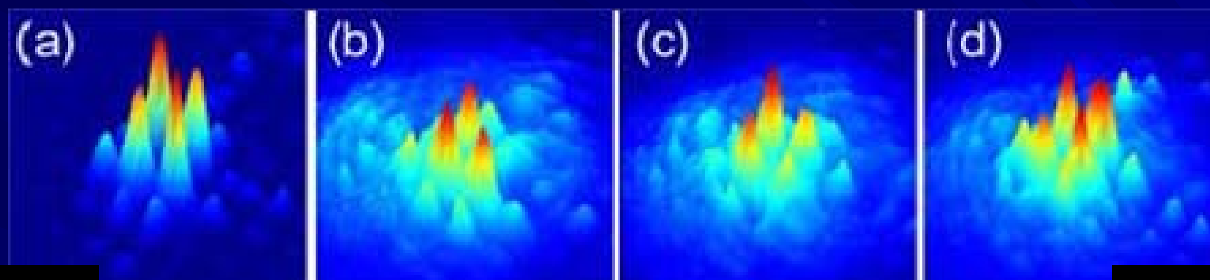
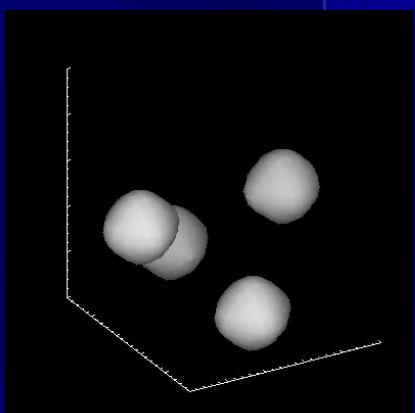
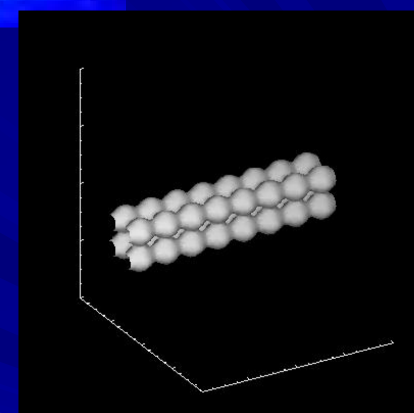
Nonlinear dynamics and solitons in optical lattices







Yuri Kivshar

Nonlinear Physics Centre
Australian National University
Canberra, Australia

Collaborators: “BEC group”

Elena Ostrovskaya, a key researcher

Tristram Alexander, a postdoc

Pearl Louis, a PhD student

Beata Dąbrowska, a PhD student

Our evolution: 1993-2004

Nonlinear Physics Center

2004

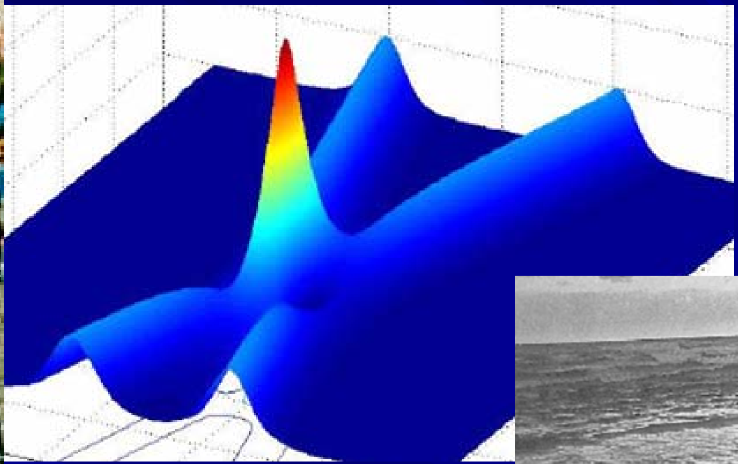
1993

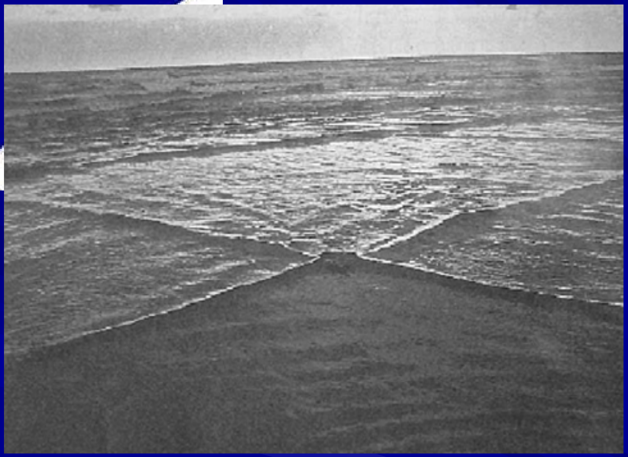
(Only “BEC people” of the Center)

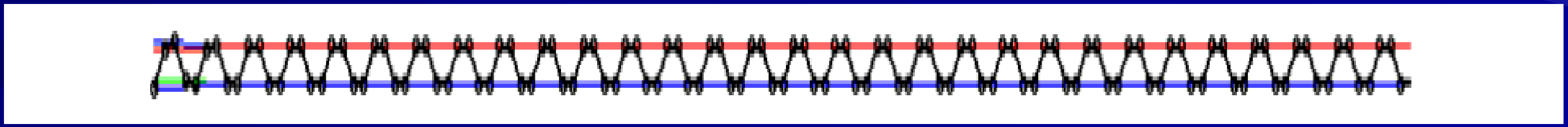
Outline

- Solitons: optics vs. BEC
- Bloch waves and band-gap spectrum
- Gap solitons in 1D lattices
- Mobility of 1D gap solitons
- 2D optical lattices and 2D gap solitons
- 2D discrete vortices
- Experimental observations in optics
- 3D discrete vortices

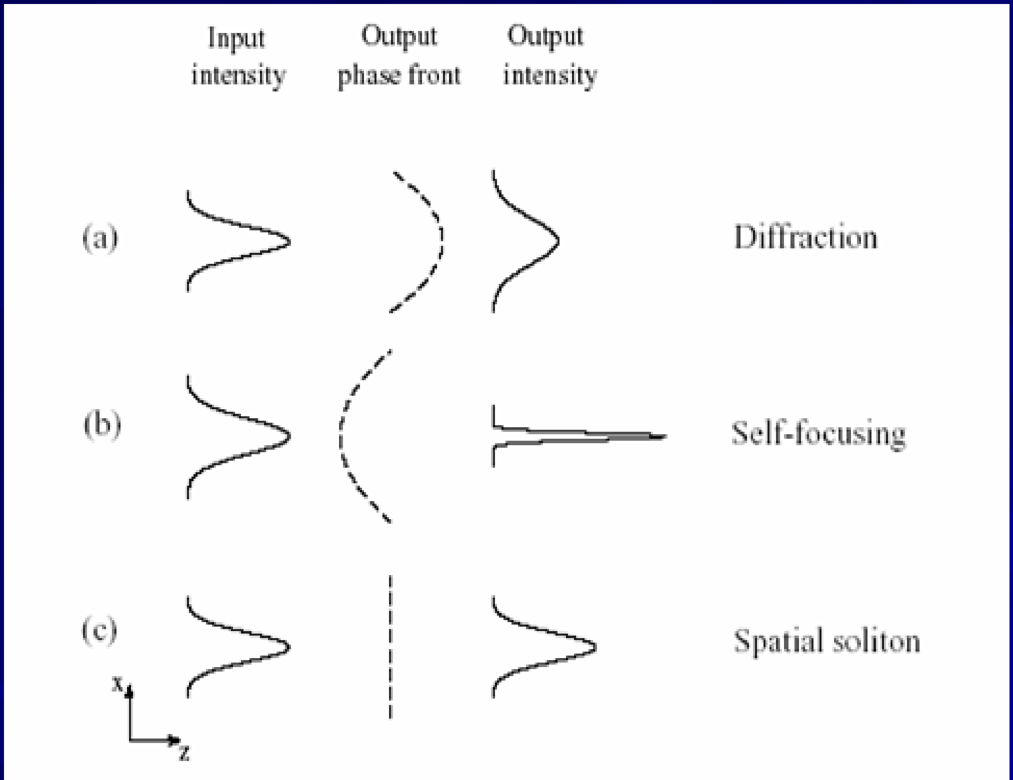
Solitons



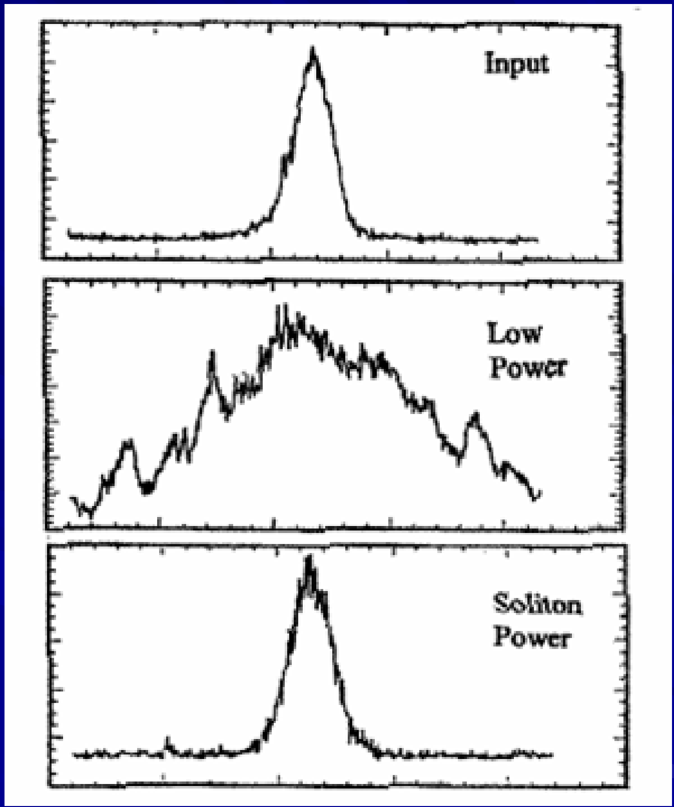




Optics: Self-focusing and solitons



experiment



Nonlinear Kerr medium: $n^2 = n_0^2 + n_2 I$

Matter waves in the mean-field picture

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi+V_{\mathrm{Trap}}(\mathbf{r})\psi+g\,\psi|\psi|^2$$

- Striking similarity with coherent light waves
- “Kerr-type” nonlinearity due to atom-atom interactions
- Main difference: atoms have mass!
- Usually a trapping potential is included

Nonlinear matter waves

- Nonlinearity grows with the number of atoms N (nonlinearity is always there)
- Repulsive: dark solitons and vortices
- Attractive: bright matter-wave solitons
- Can be tuned via Feshbach resonances

  "Focusing": attractive interactions ($g<0$)

  "Defocusing" : repulsive interactions ($g>0$)
- Optical lattices create tunable Bragg gratings

Solitons in nonlinear atom optics

Macroscopic wave function: the mean field theory

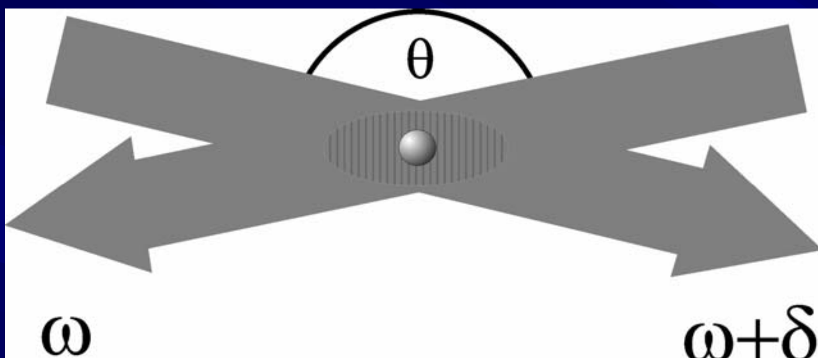
The Gross-Pitaevsky equation vs. the NLS equation

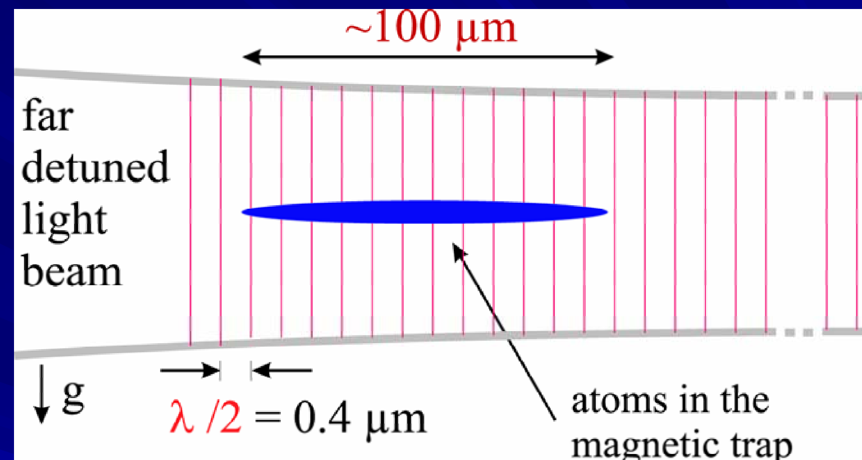
- **Vortices and vortex solitons (1999)**
- **Dark solitons (1999-2000)**
- **Bright solitons (2002)**
- **Matter-wave gap solitons (2004)**

Solitons
in periodic structures

BEC in 1D optical lattices

$$V_{\text{Latt}} = V_0 \sin^2(k_L x + \delta t)$$





$$V_0 \sim I; \qquad k_L \equiv \frac{\pi}{d} = \frac{\omega}{c}\cos\theta; \qquad v = \frac{\delta}{2k_L}$$

$$\lambda_D \equiv \frac{2\pi\hbar}{p_x} >> d \quad \Rightarrow \quad \frac{p_x^2}{2m} \equiv E_k \le E_R \equiv \frac{\hbar^2 k_L^2}{2m}$$

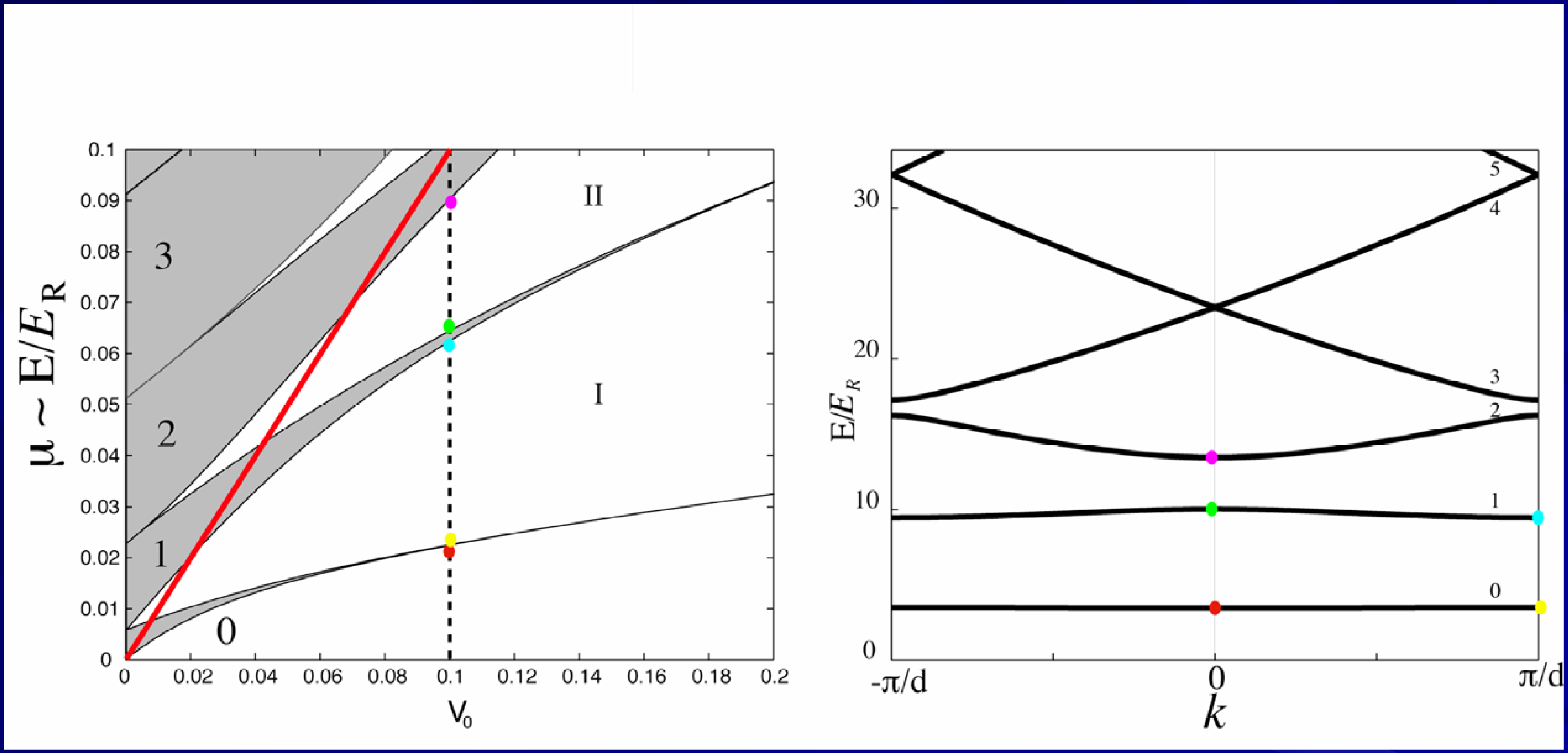
Atomic band-gap structures

BEC in optical lattices -

nonlinear band-gap structures for matter waves

- **Unprecedented control over parameters**
- **Nonlinearity that can be turned off or reversed**
- **Degree of discreteness can be varied**
- **Dimensionality can be changed**
- **Effective dynamical tunability**

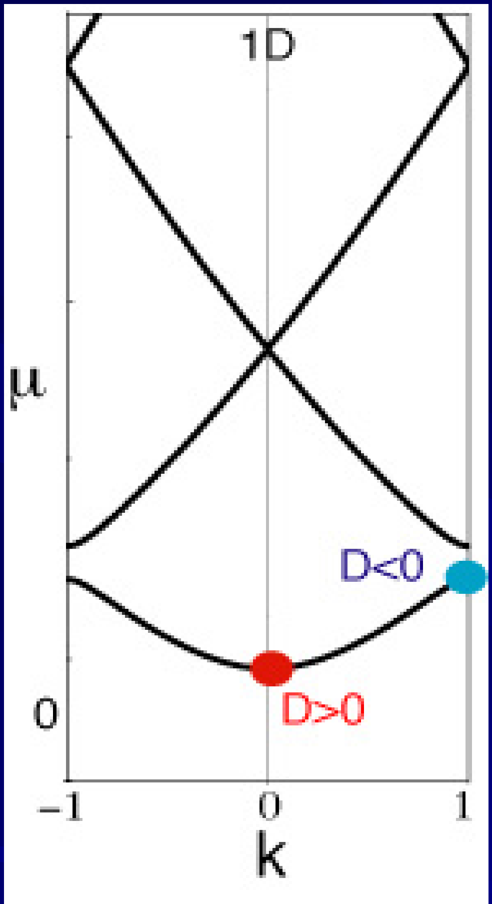
Band-gap spectrum



$$\psi(x,t) = \psi(x)\exp(-i\mu t)$$

$$\frac{1}{2}\frac{d^2\psi}{dx^2} - \{V_0 \sin^2(k_L x) - \mu\}\psi = 0$$

Gap solitons



Dispersion relation in a free space:

$$E(p) = \frac{p^2}{2m}$$

Dispersion for BEC in a lattice is defined by

$$E(k) \equiv \mu(k)$$

$$D = \frac{\partial^2 \mu}{\partial k^2}$$

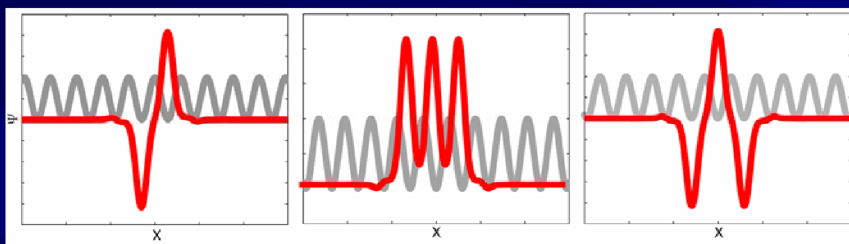
> 0 at the band's centre

< 0 at the band's edge (negative "mass")

Gap solitons - balance between "anomalous" dispersion and positive interaction (repulsion) at the upper band edge

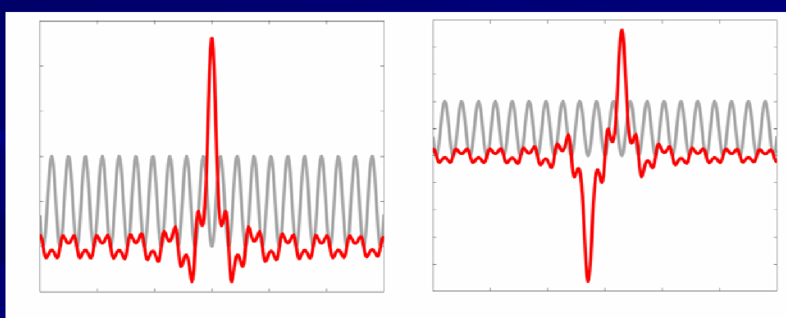
Gap solitons in 1D structures

Repulsive interactions!

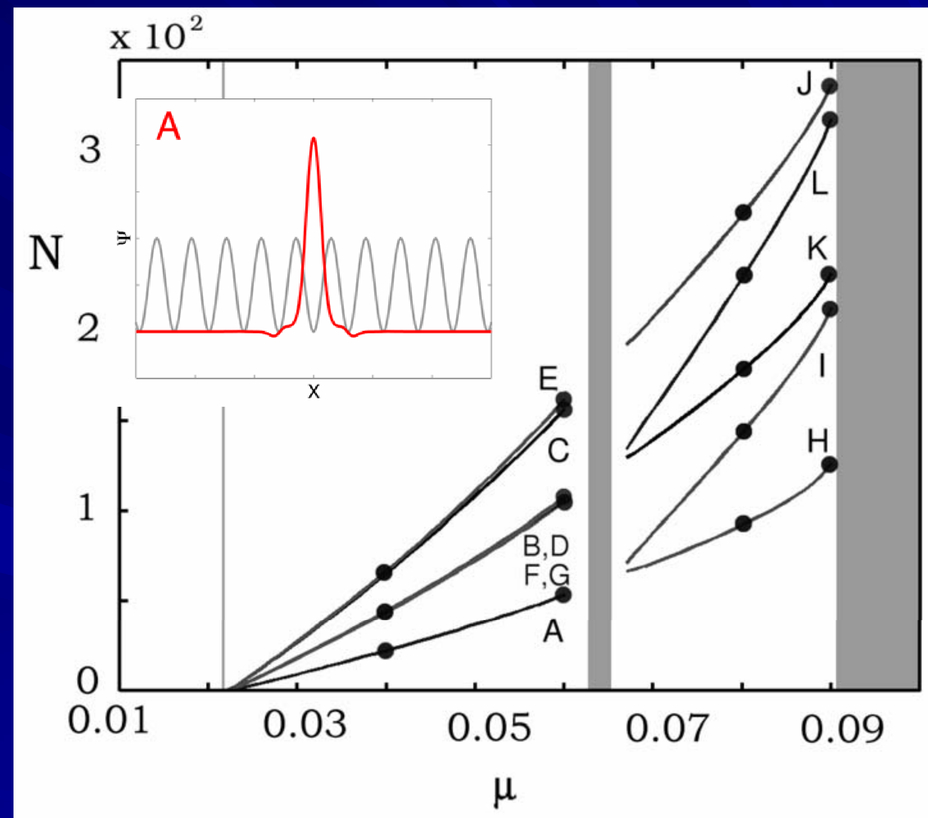


B C E

Inside the band:

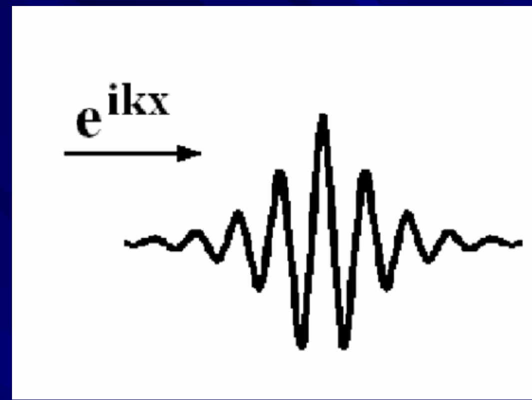


A B



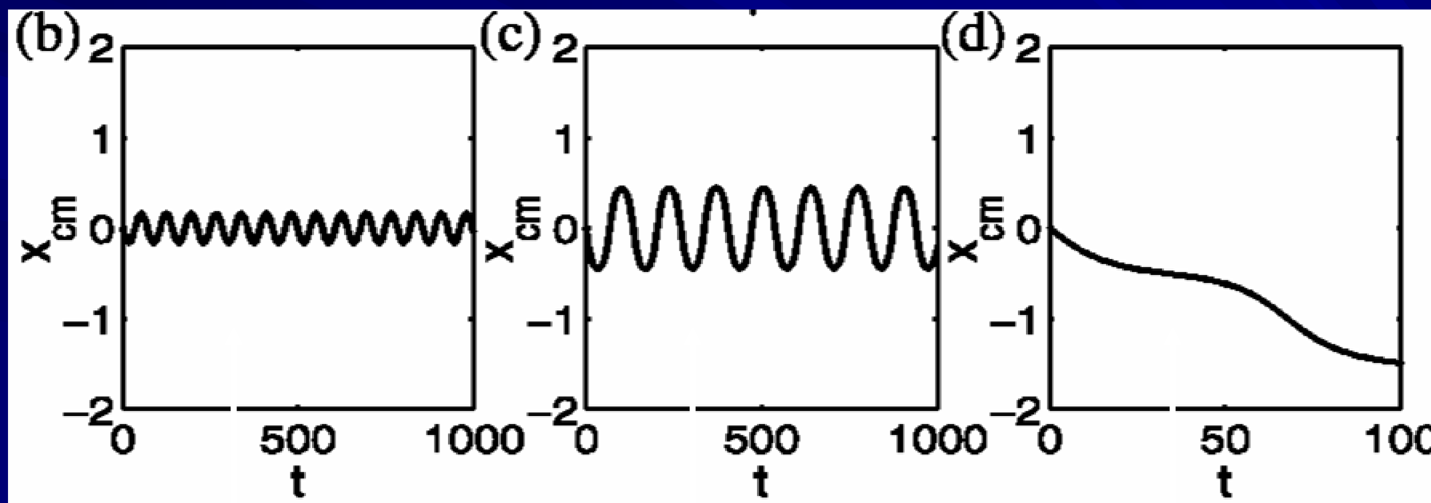
P. Louis et al., Phys. Rev. A **67**, 013602 (2003)

Soliton mobility



μ=7.7
k_{PN}=1.2x10^{-2}

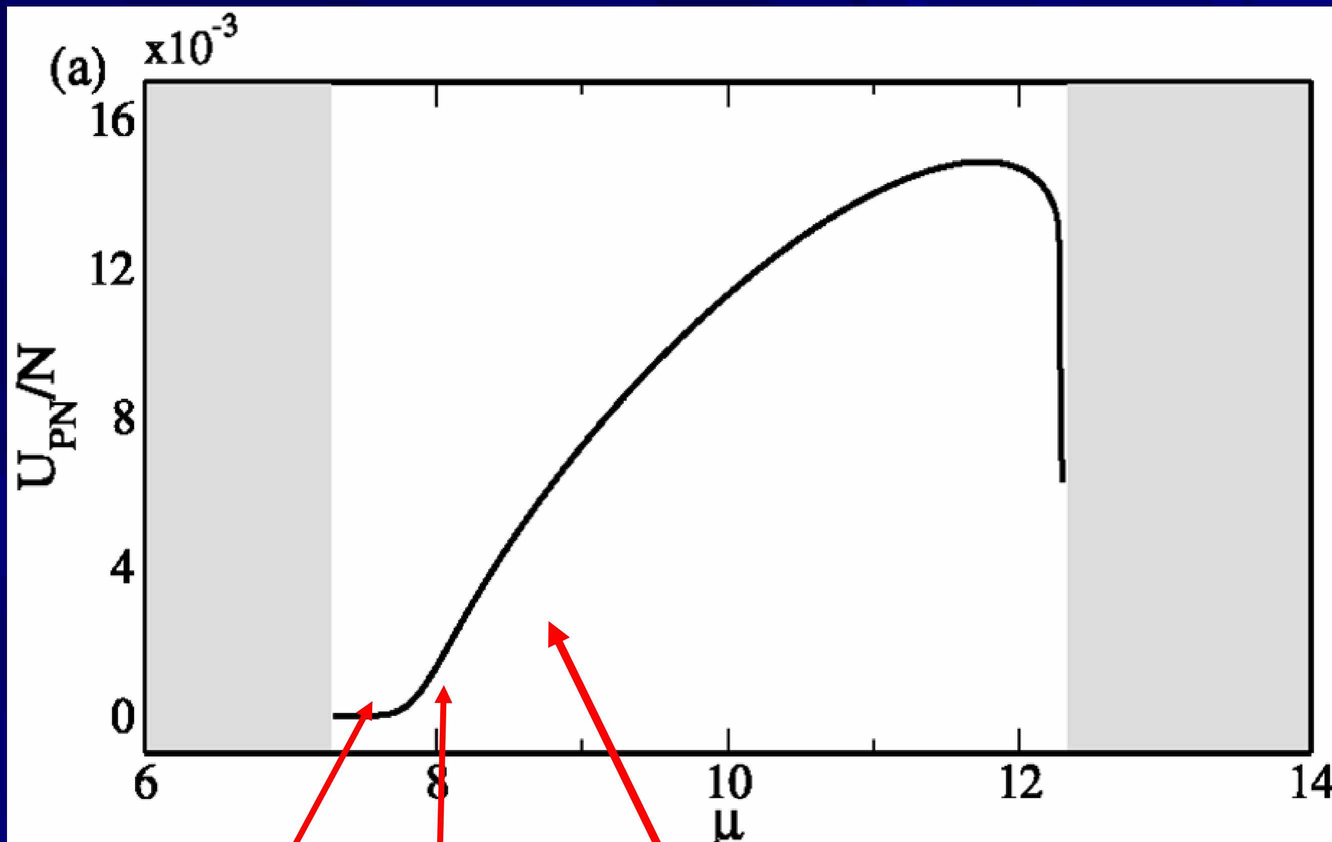
(free motion)



$k<k_{PN}$ $k=k_{PN}$ $k>k_{PN}$

(oscillations within one lattice site)

Peierls-Nabarro potential

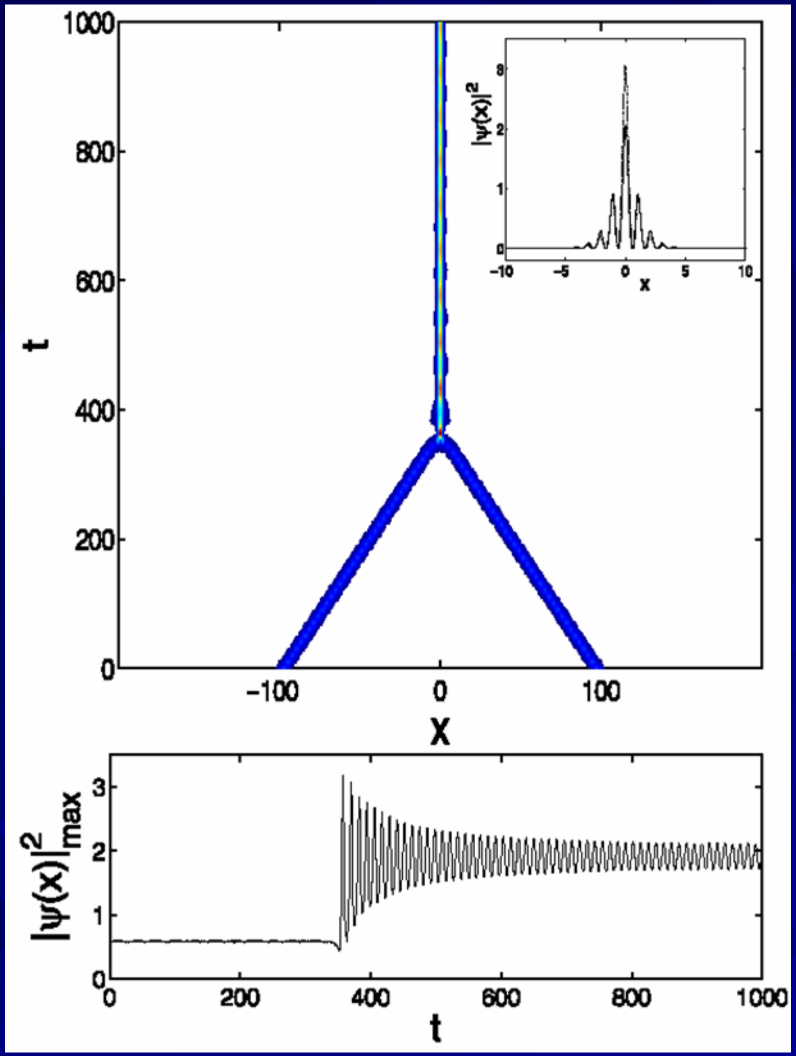


μ<7.5
lattice-free
solitons

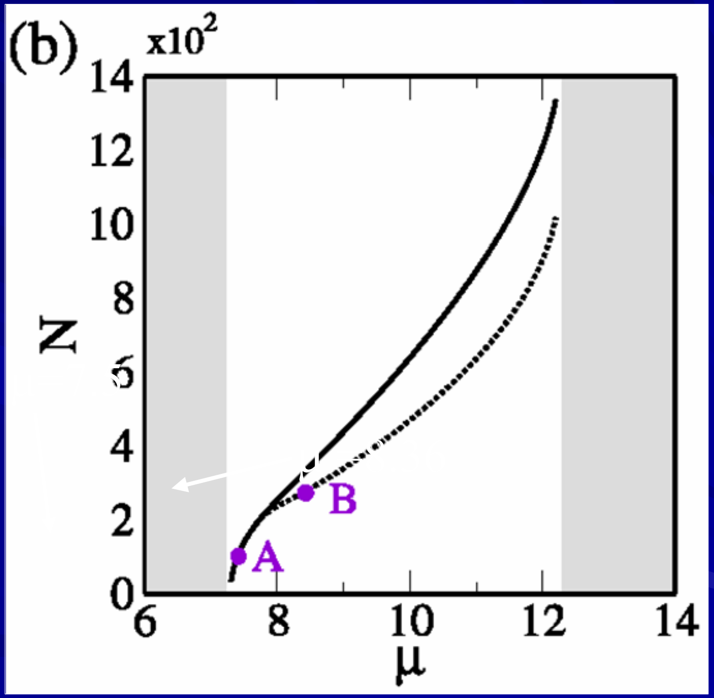
7.5<μ<8.2
discrete NLS
solitons

μ>8.2
centre of mass
oscillations

Soliton fusion

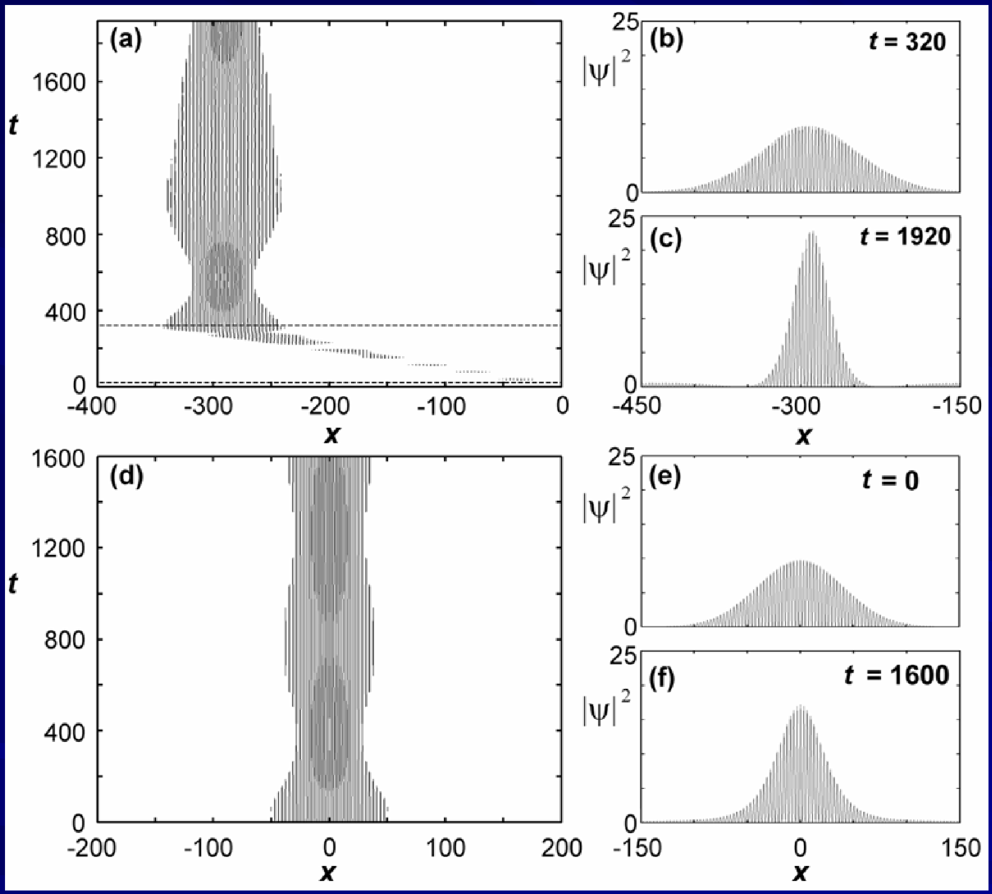


discrete NLS $\mu \geq 7.5$
strongly inelastic scattering



170→270 atoms

Generation of gap solitons



Top half

1. BEC loaded at k=0 by slowly ramping up the lattice.

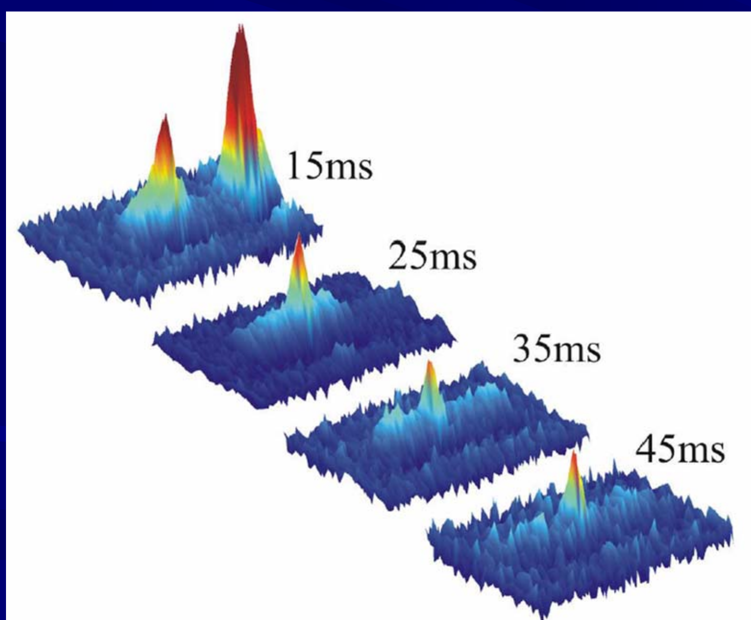
2. Lattice slowly accelerated to $v = v_{recoil}$. Adiabatically moves BEC to the gap edge.

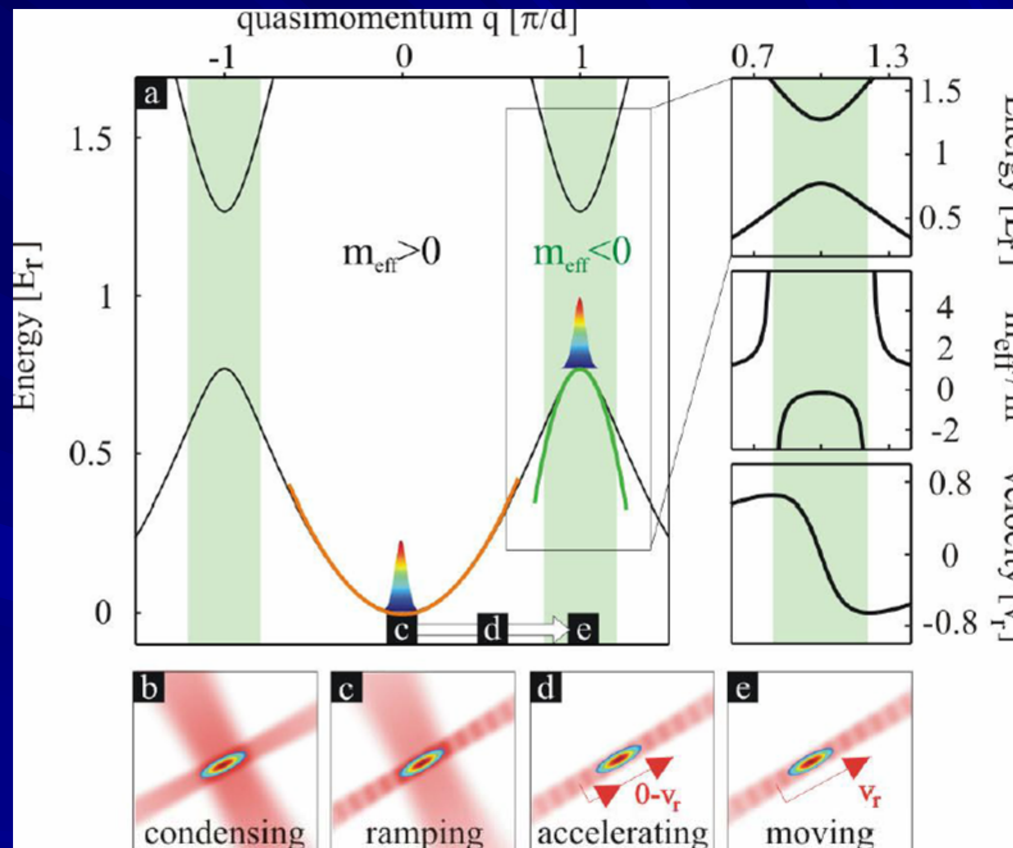
3. Wavepacket evolved at the gap-edge in a lattice moving with velocity $v = v_{recoil}$.

Bottom half

Generates the correct internal structure using interference rather than acceleration.

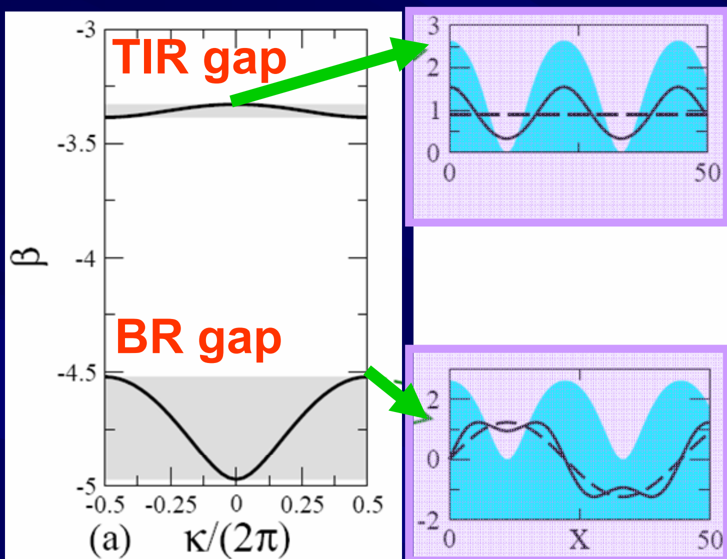
Experiment: Gap Solitons in BEC

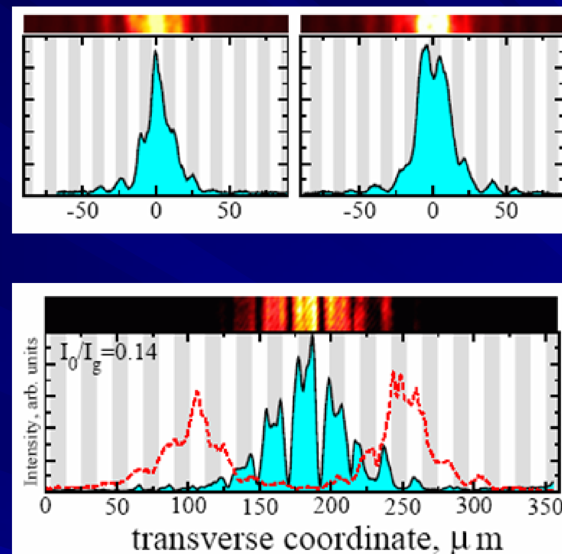




Oberthaler's group, 2004

Spatial gap solitons in optics





Discrete solitons:
Single beam excitation

Gap solitons:
two beams

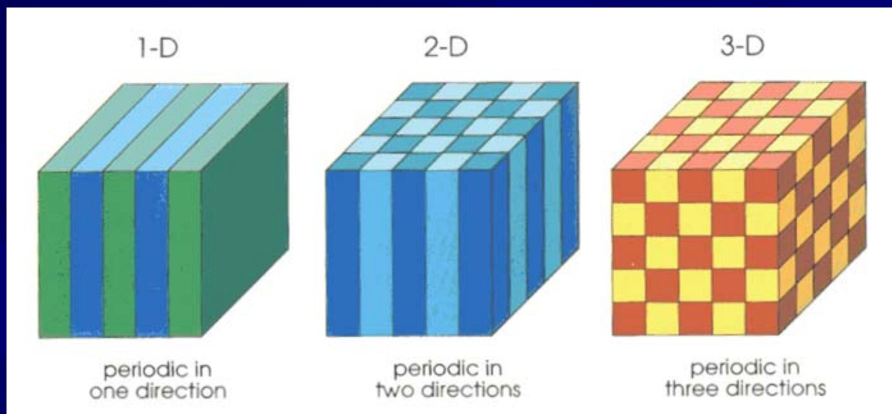
VOLUME 93, NUMBER 8 PHYSICAL REVIEW LETTERS week ending 20 AUGUST 2004

Controlled Generation and Steering of Spatial Gap Solitons

Dragomir Neshev, Andrey A. Sukhorukov, Brendan Hanna, Wieslaw Krolikowski, and Yuri S. Kivshar
Nonlinear Physics Centre and Laser Physics Centre, Centre for Ultra-high bandwidth Devices for Optical Systems (CUDOS), Research School of Physical Sciences and Engineering, Australian National University, Canberra, ACT 0200, Australia

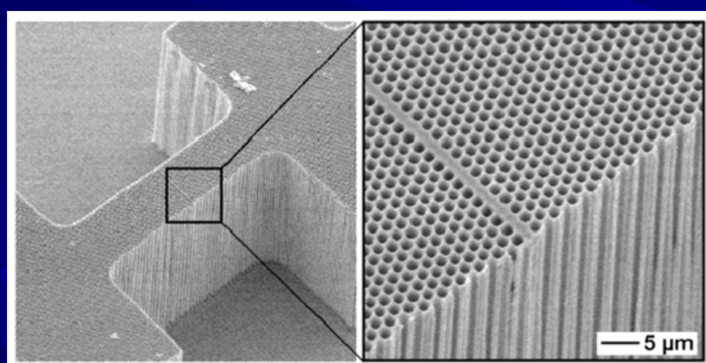
2D optical lattices

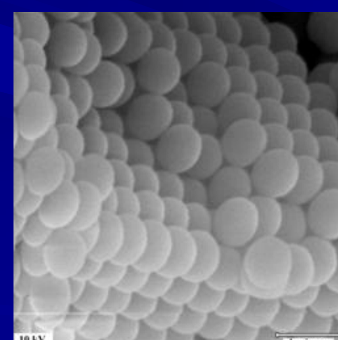
Photonic band-gap structures

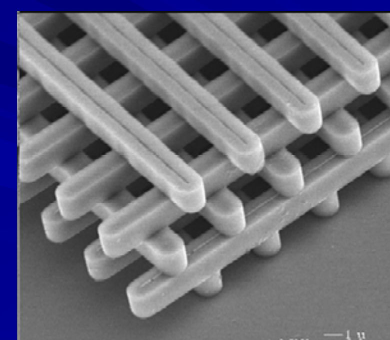


PRL 58 (1987):

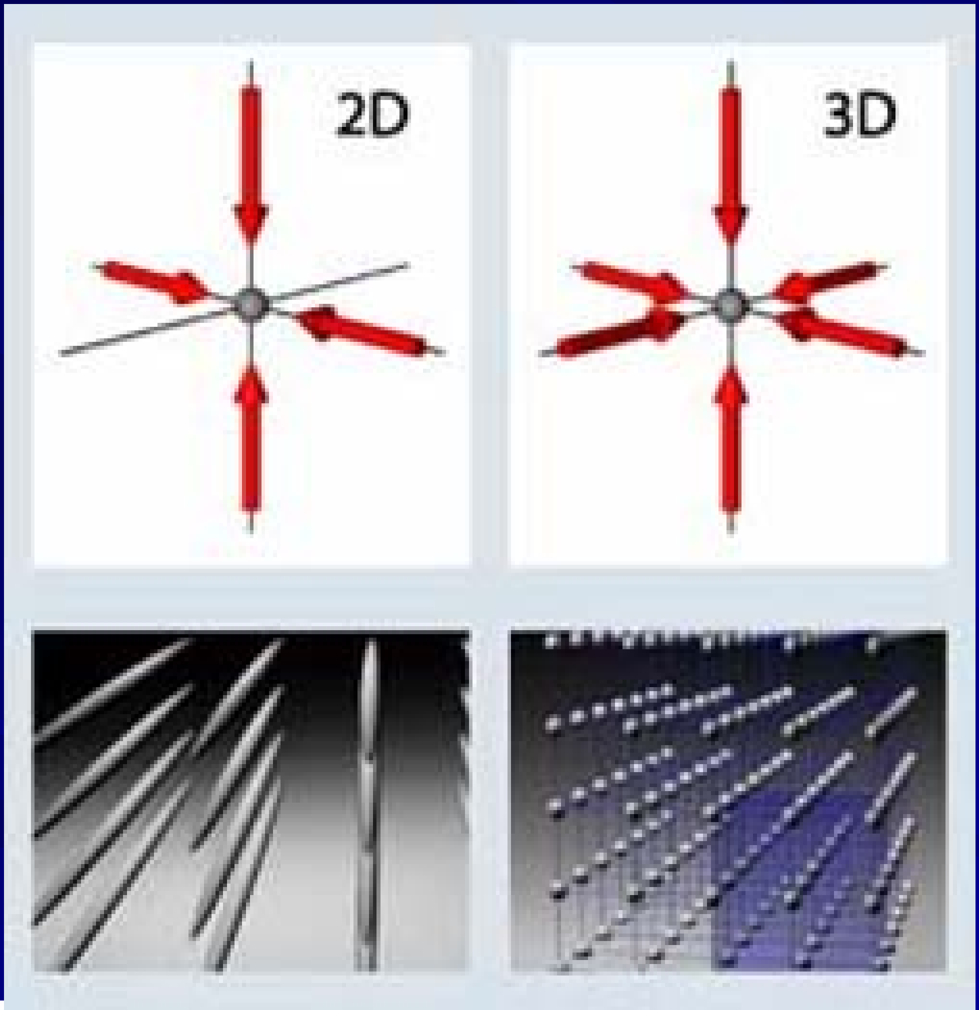
- *S. John*
- *E. Yablonovitch*







Optical lattices in higher dimensions



Munich BEC group

2D: phase coherence
M. Greiner et al.
PRL **87**, 160405 (2001)

3D: coherence break-up
M. Greiner et al.
Nature **415**, 39 (2002)

BEC in a 2D optical lattice

$$i\frac{\partial\psi}{\partial t}+\nabla\psi-V_{\mathrm{Latt}}(\mathbf{r})\psi-\sigma\psi|\psi|^2=0$$

$$V_{\mathrm{Latt}}=V_0\left[\sin^2(k_Lx)+\sin^2(k_Ly)+2\mathbf{e}_1\cdot\mathbf{e}_2\cos\theta\sin(k_Lx)\sin(k_Ly)\right]$$

σ

"Square" lattice and Repulsive nonlinearity $\sigma>0$

Stationary states of noninteracting condensate - Bloch waves:

$$\psi(\mathbf{r},t)=\varphi(\mathbf{r})\exp(-i\mu t);\quad \varphi(\mathbf{r})=u_{\mathbf{k}}(\mathbf{r})\exp(i\mathbf{kr})$$

$$\mathbf{k}\in\ \mathbf{BZ};\qquad u_{\mathbf{k}}(\mathbf{r})=u_{\mathbf{k}}(\mathbf{r}+\mathbf{d})$$

2D band-gap spectrum

$$i\frac{\partial\psi}{\partial t}+\nabla\psi-V_{\mathrm{L}}(\mathbf{r})\psi-\psi|\psi|^2=0$$

$$V_{\mathrm{Latt}}=V_0\left[\sin^2(x)+\sin^2(y)\right]$$

Stationary states:

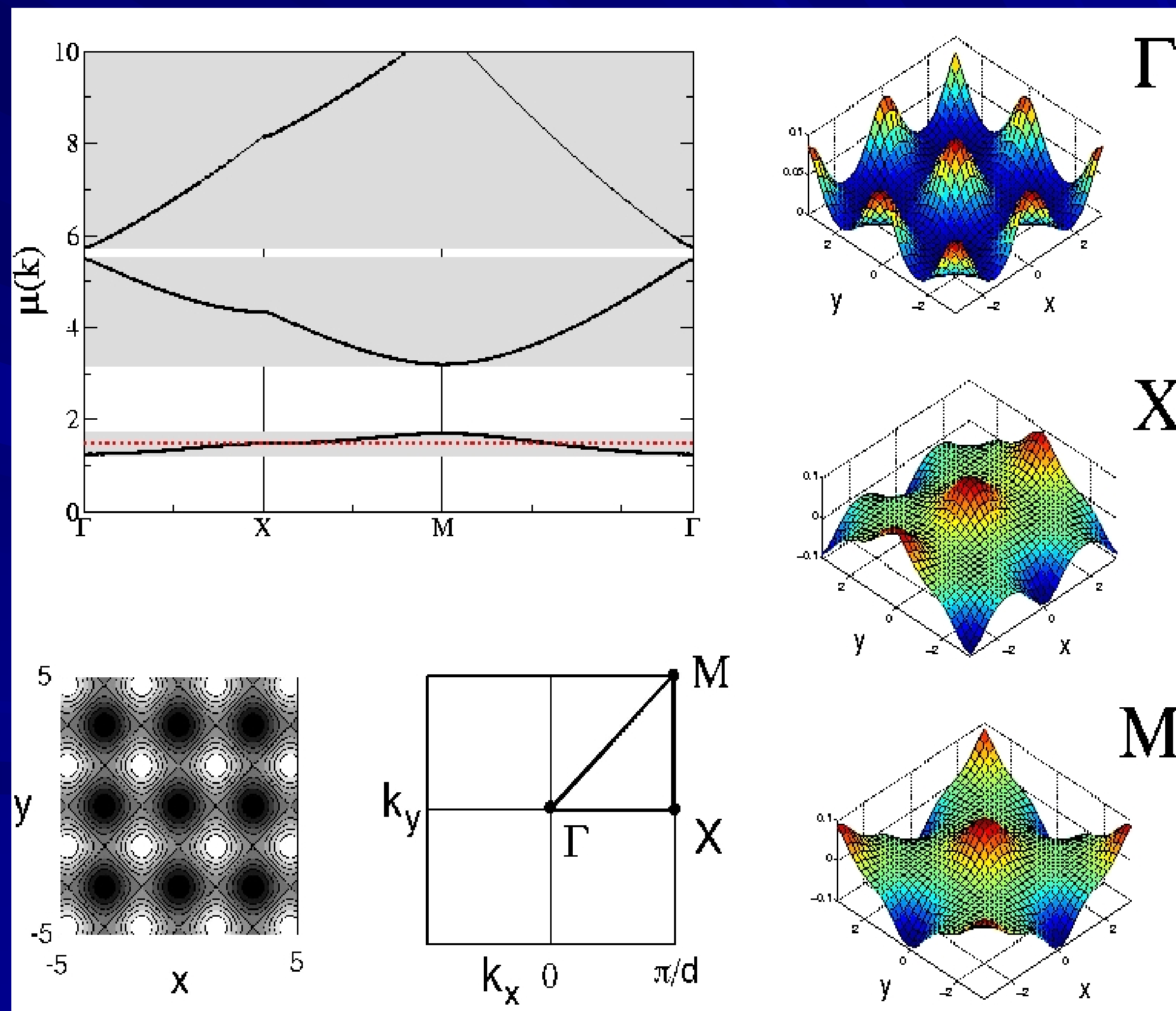
$$\psi(\mathbf{r},t)=\varphi(\mathbf{r})\exp(-i\mu t);$$

Bloch states:

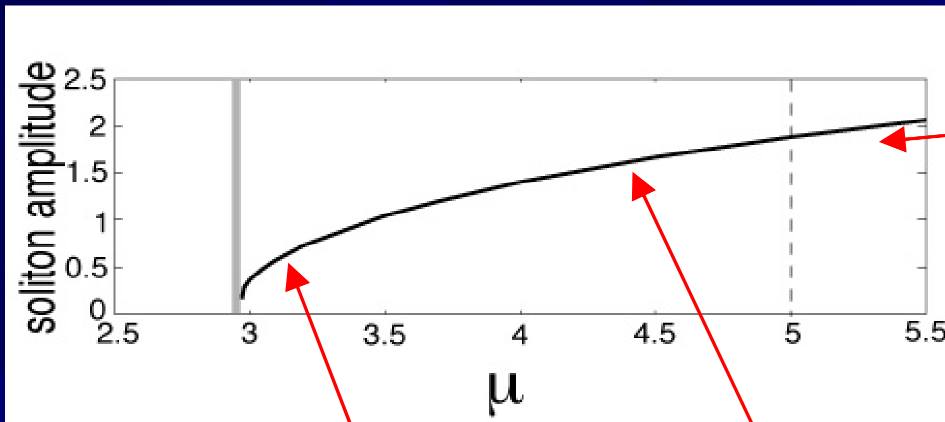
$$\varphi(\mathbf{r})=u_{\mathbf{k}}(\mathbf{r})\exp(i\mathbf{k}\mathbf{r})$$

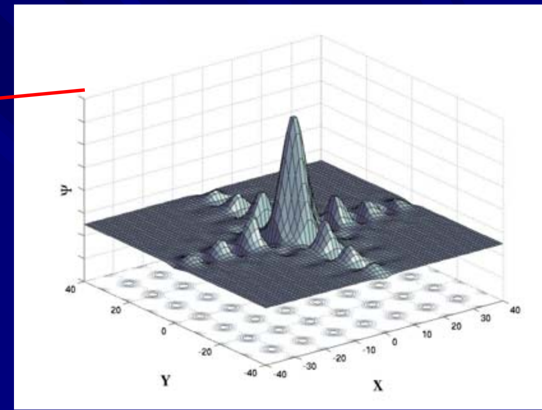
$$\mathbf{k}\in\ \mathbf{BZ};$$

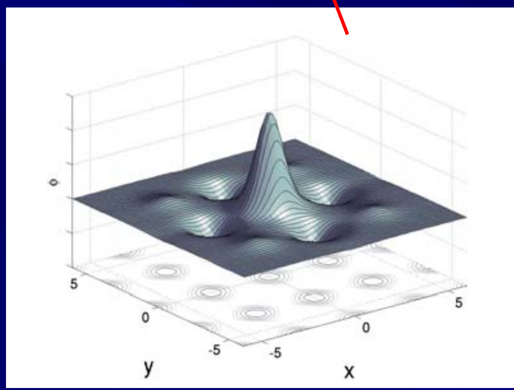
$$u_{\mathbf{k}}(\mathbf{r})=u_{\mathbf{k}}(\mathbf{r}+\mathbf{d})$$

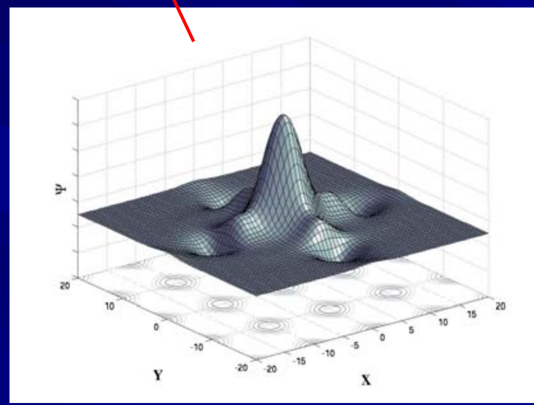


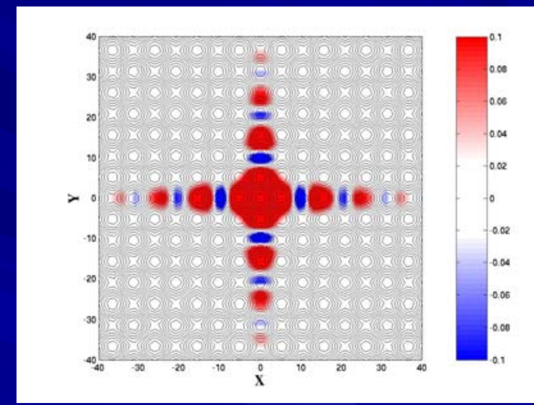
Gap solitons in 2D lattices

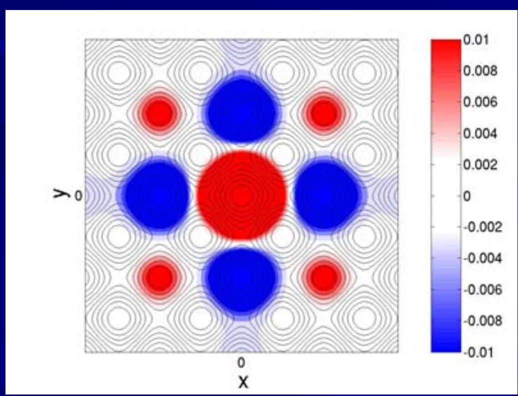


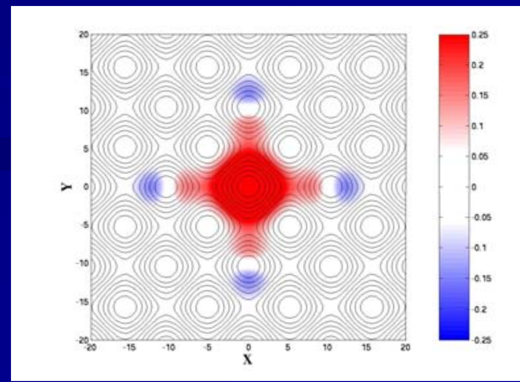










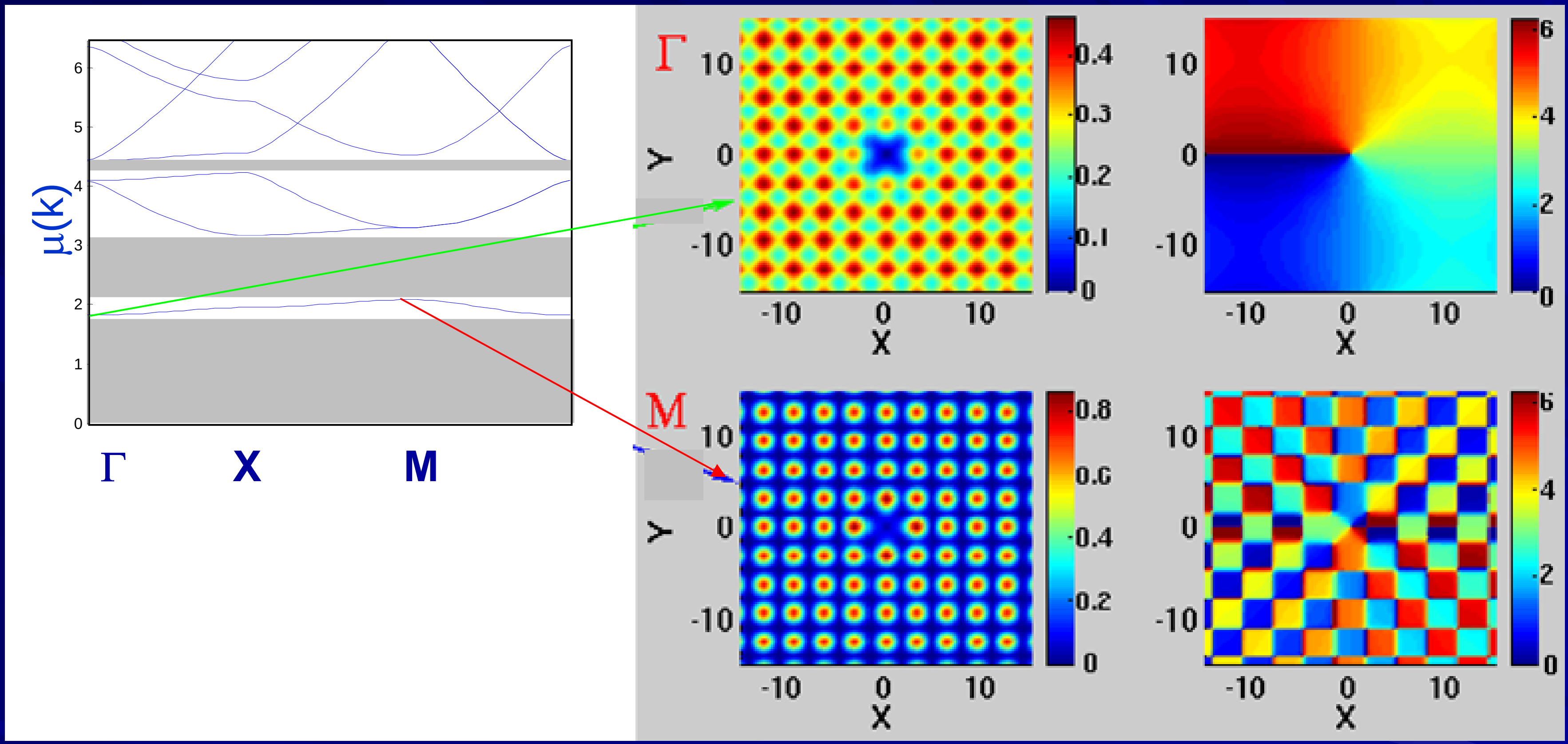


Ostrovskaya et al, PRL <u>90</u>, 160407 (2003)

www.rsphysse.anu.edu.au/nonlinear

2D optical lattices and discrete vortices

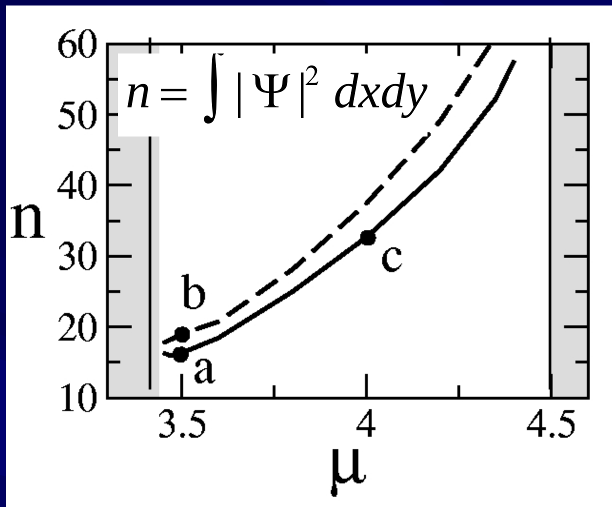
Imprinting a phase: BEC vortex in-band

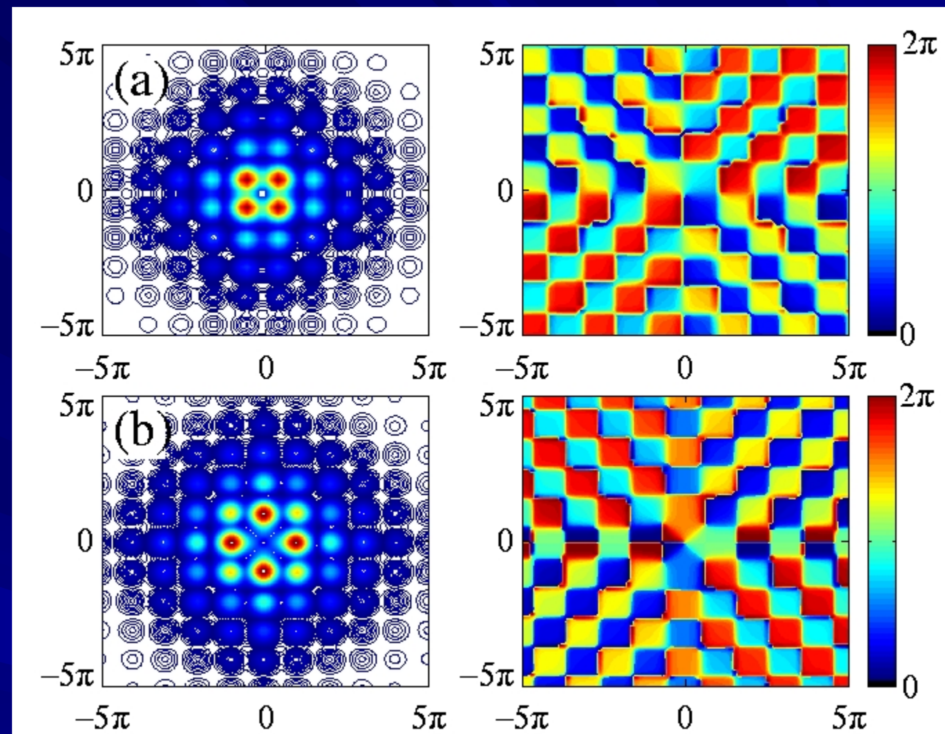


Bloch vortex at k=0: $\psi(r,\theta) \sim A\exp(im\theta)\exp(-a^2r^2)u_{1,\mathbf{k}_\Gamma}(r,\theta)$

Bloch vortex at k=k$_M$: $\psi(r,\theta) \sim A\exp(im\theta)\exp(-a^2r^2)u_{1,\mathbf{k}_M}(r,\theta)$

Stationary in-gap states: gap vortices

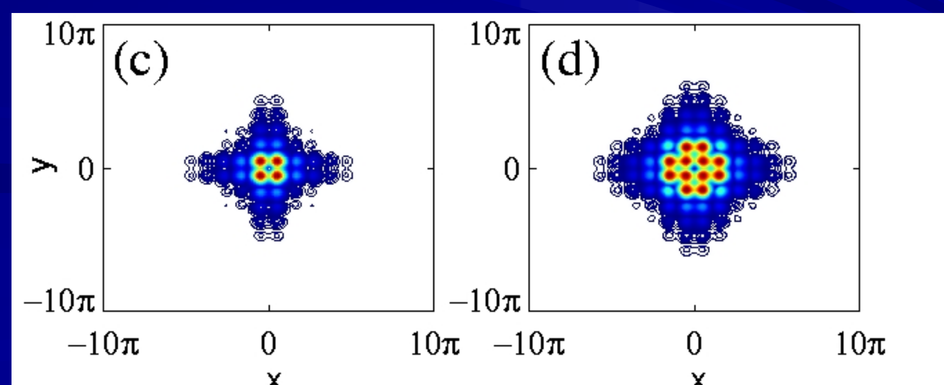




two symmetry types:

(a,c) off-site vortex

(b) on-site vortex

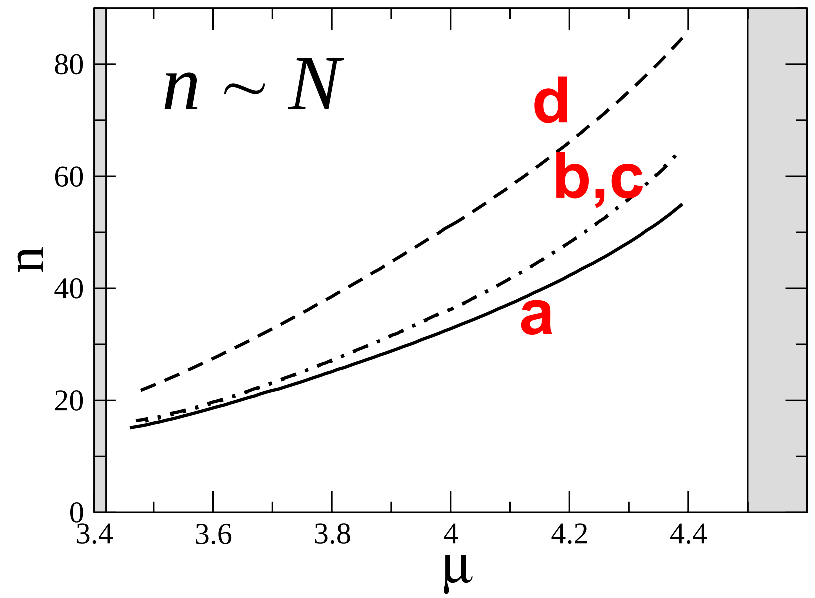


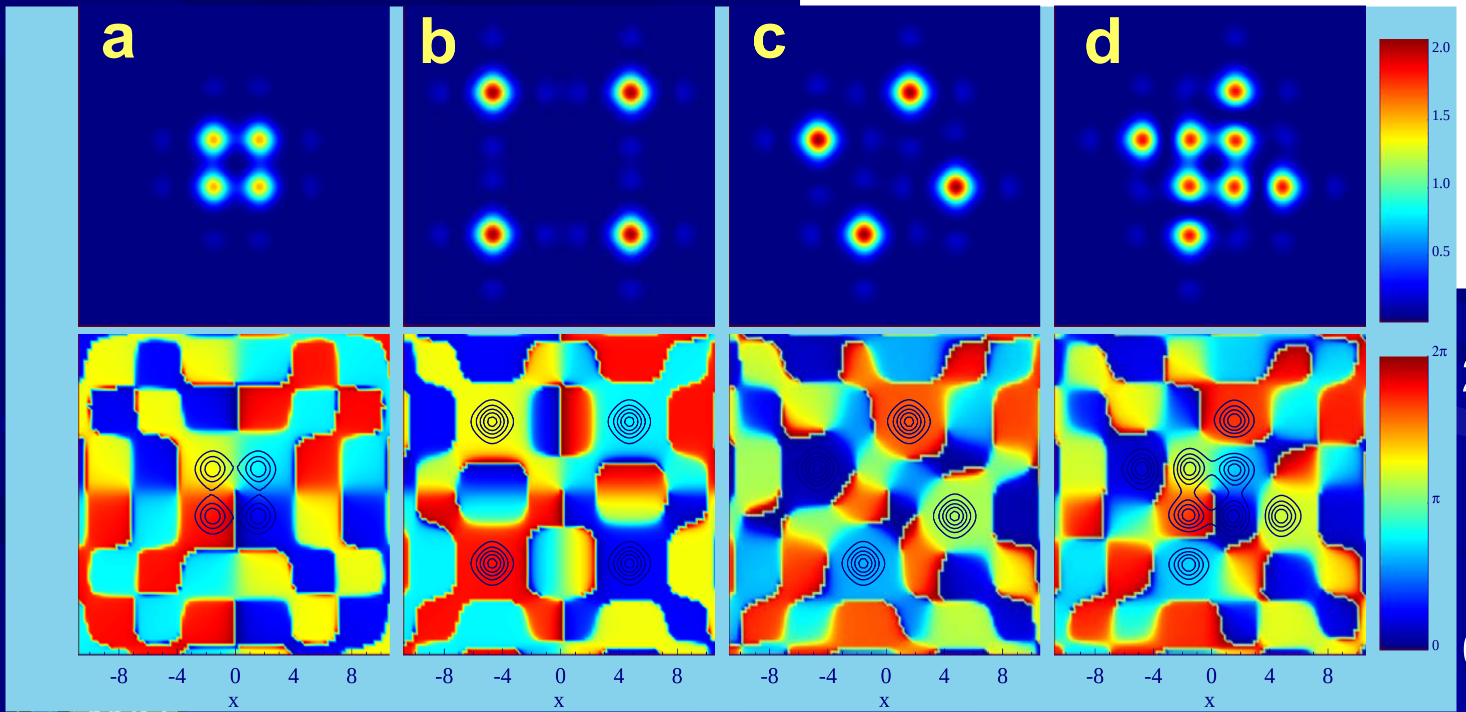
and different widths:

(d) wide off-site vortex

Off-site vortices

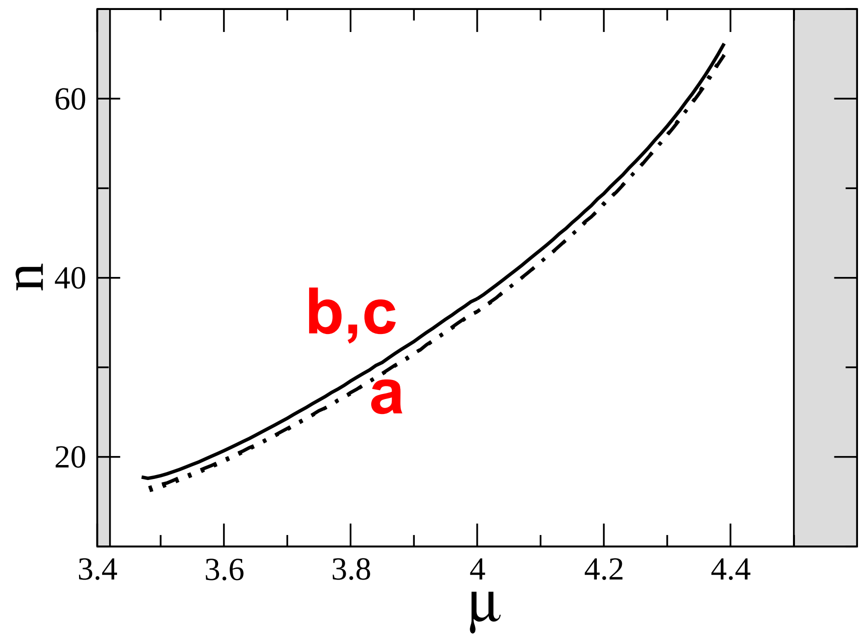
- Off-site vortices of different rad
- Almost degenerate in atom number

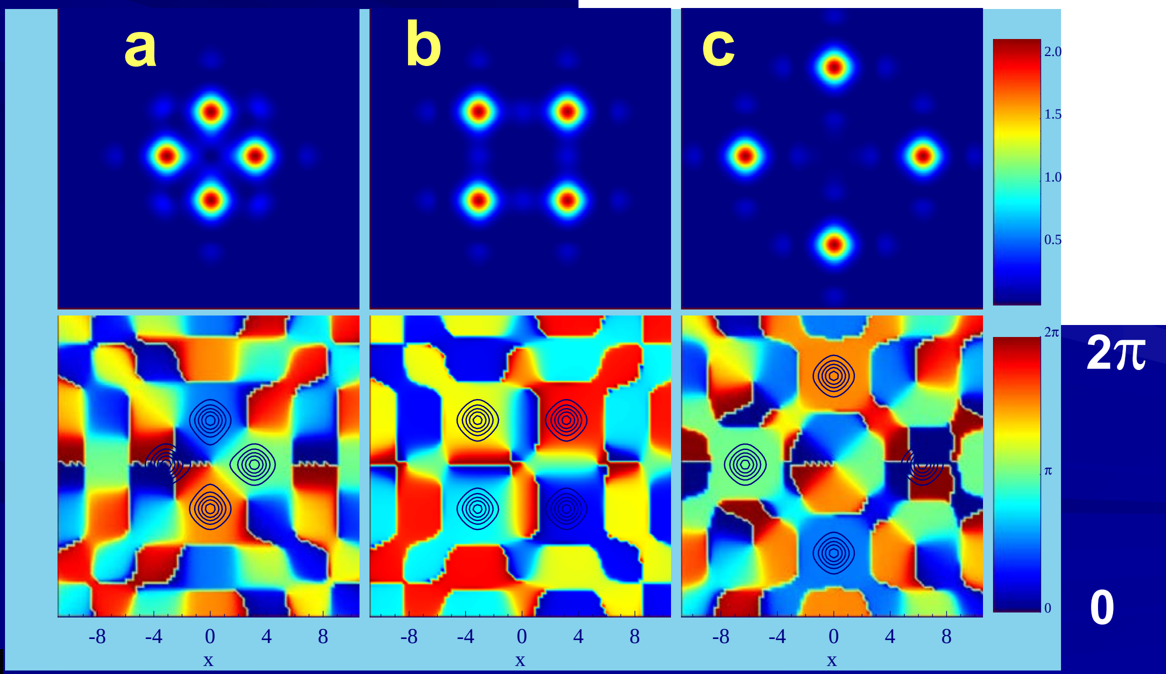




On-site vortices

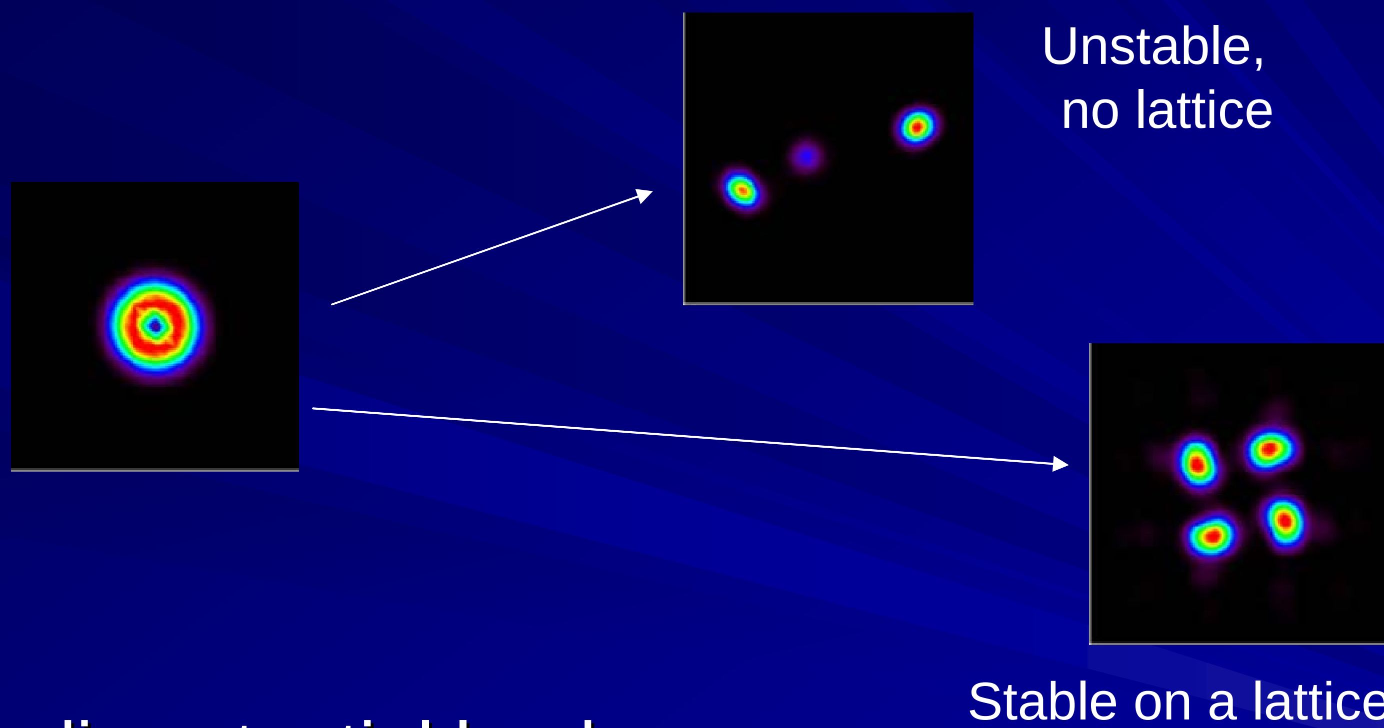
- On-site vortex families are even closer in atom numbers





Discrete vortices in nonlinear optics

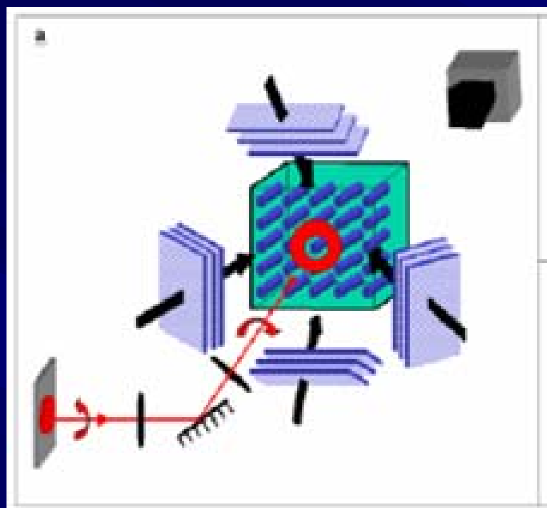
Why use a lattice?



Periodic potential leads to vortex stability

Discrete optical vortex: observation

Optically induced 2D grating in a photorefractive crystal



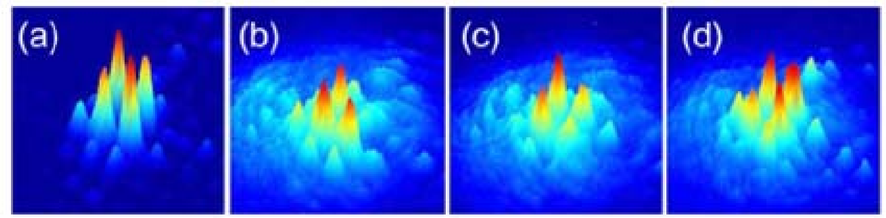
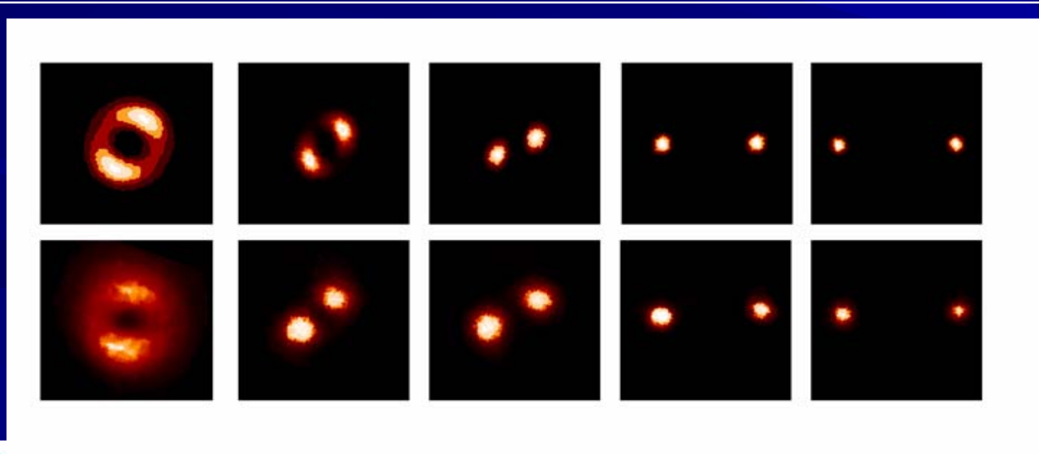


FIG. 5 (color online). Phase structure measurements. (a) Discrete vortex soliton for a lattice period of 20 μm. (b)–(d) Interferograms of the vortex soliton with a weak broad coherent beam, whose relative phase is changed in steps of $\pi/2$.

Neshev et al & Fleischer et al, PRL 92 (2004)



Without a lattice vortex rings decay!

Theory + experiment
Tikhonenko et al, 1995

Energy balance condition

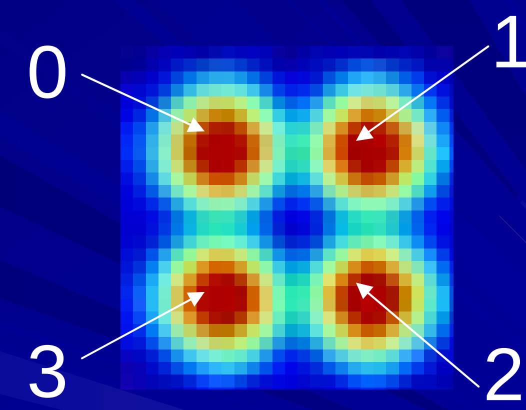
- We examine the existence properties of discrete vortices by solving the energy balance conditions for coupled modes:

N: number of lobes

Inter-lobe coupling strength

$$\sum_{m=0}^{N-1} f_{nm} \sin(\varphi_m - \varphi_n) = 0.$$

Phase of lobe

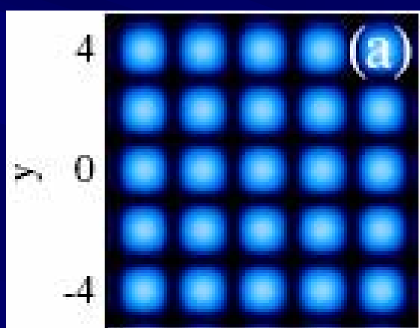


- Energy into and out of a lobe must be equal

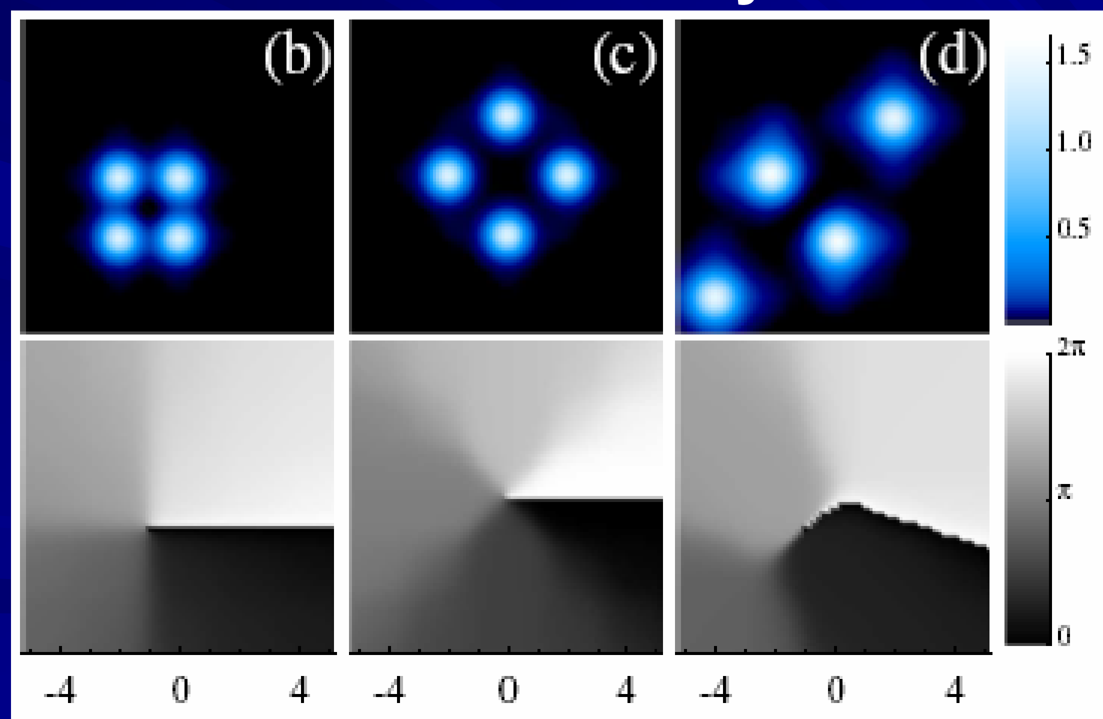
Vortex lattice solitons

Different types of the vortex symmetries

Off-site **On-site** **asymmetric**



lattice



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Two experimental observations

VOLUME 92, NUMBER 12 PHYSICAL REVIEW LETTERS week ending 26 MARCH 2004

Observation of Discrete Vortex Solitons in Optically Induced Photonic Lattices

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We report on the first experimental observation of *discrete vortex solitons* in two-dimensional optically induced photonic lattices. We demonstrate strong stabilization of an optical vortex by the lattice in a self-focusing nonlinear medium and study the generation of the discrete vortices from a broad class of singular beams.

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Observation of Vortex-Ring "Discrete" Solitons in 2D Photonic Lattices

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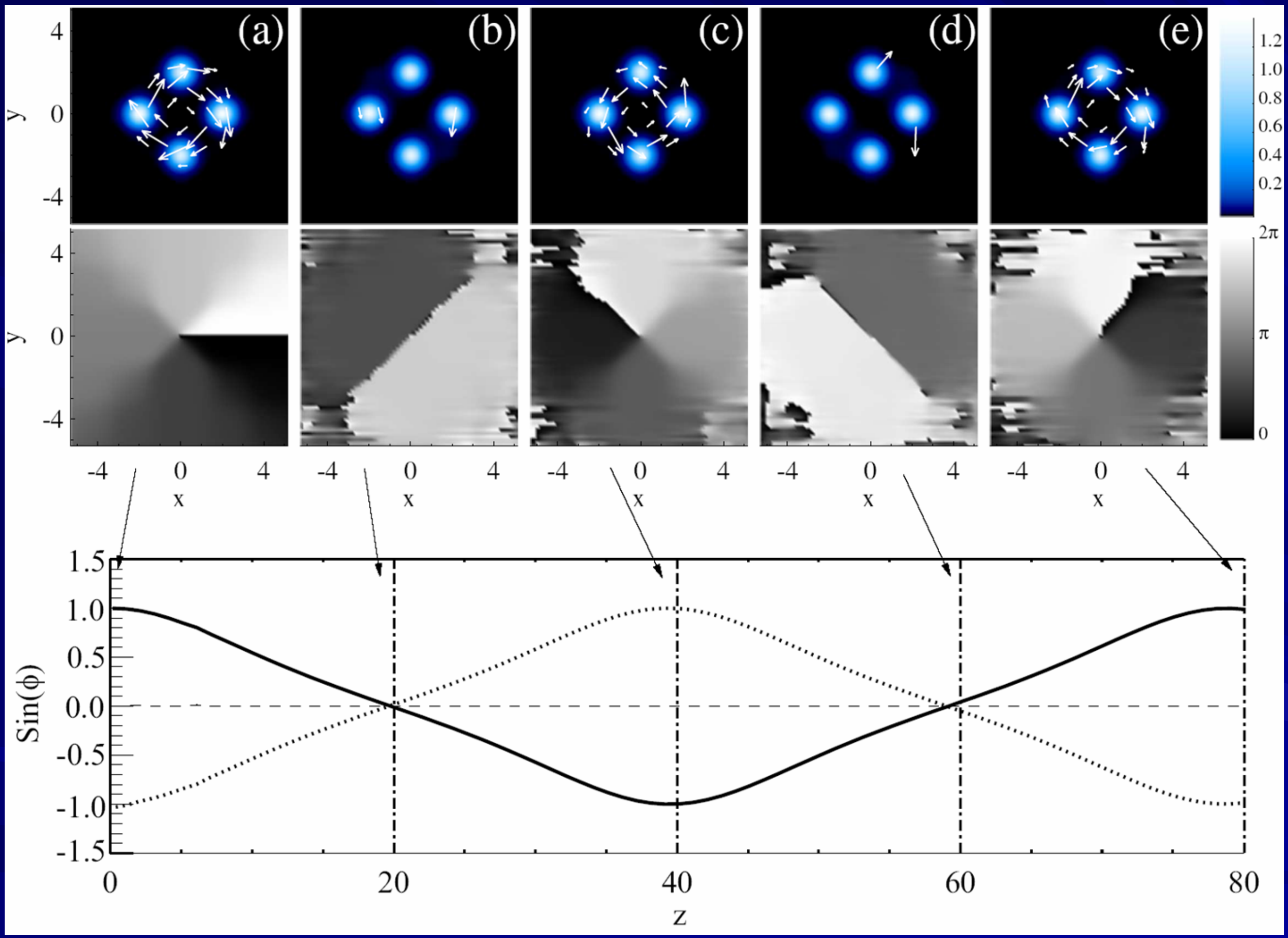
We present the experimental observation of both on-site and off-site vortex-ring solitons of unity topological charge in a nonlinear photonic lattice, along with a theoretical study of their propagation dynamics and stability.

Charge flipping of on-site vortex



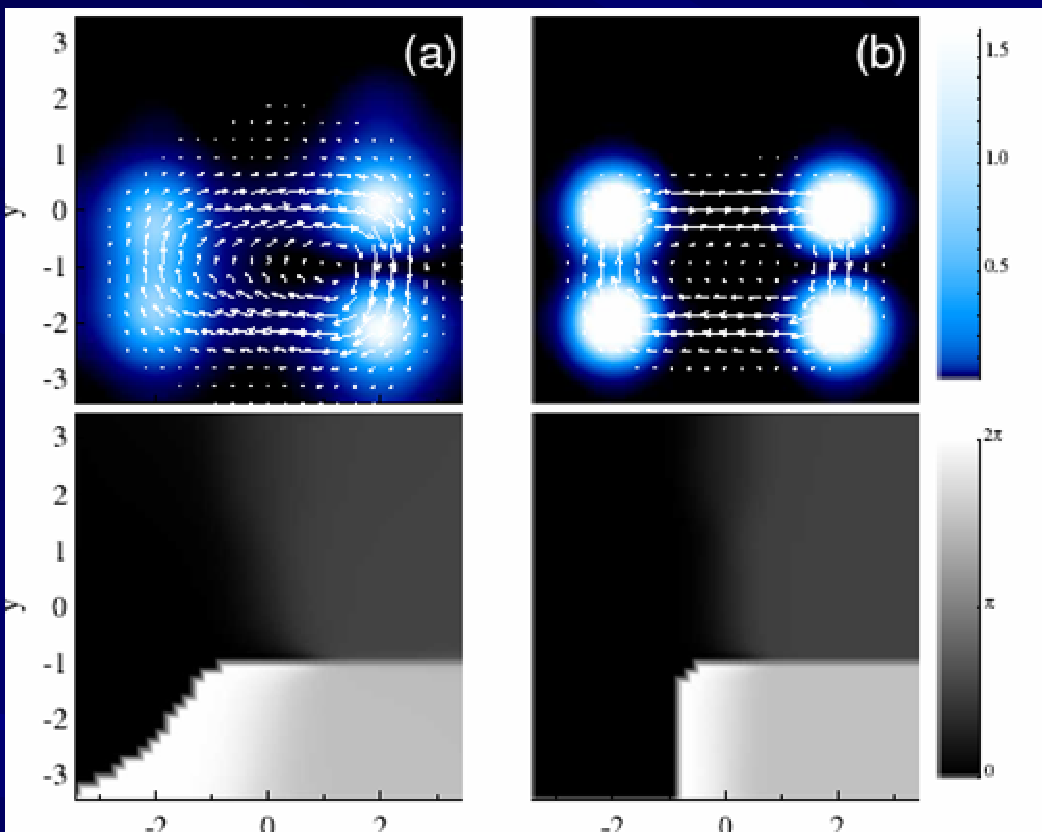
A small, but finite, perturbation leads to a charge reversal and a new effect:

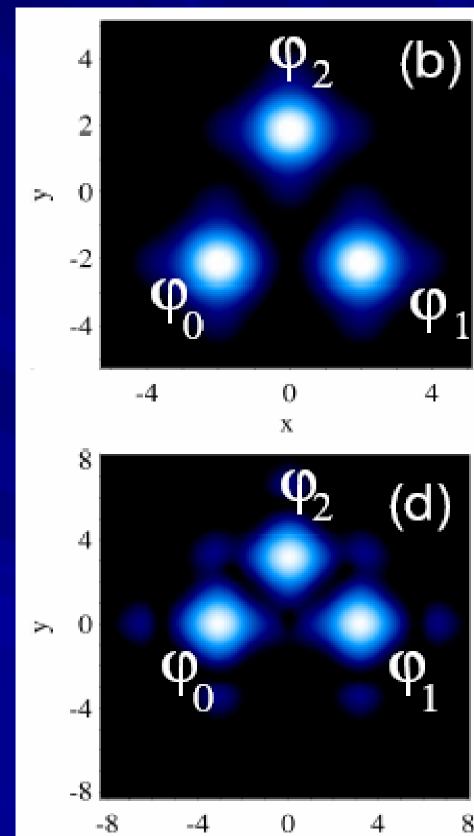
periodic flipping of the vortex charge

Alexander, Sukhorukov, & Kivshar, PRL **93,** 063901 (2004)

Asymmetric vortices

Nonlinearity allows for the symmetry breaking





semi-infinite

in-gap

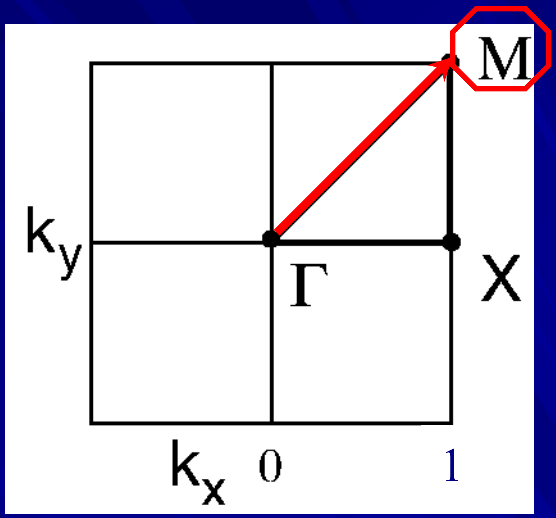
- Symmetric rectangular vortex does not exist
- Diagonal coupling - weaker than nearest-neighbor coupling

Alexander, Sukhorukov, & Kivshar, PRL <u>93</u> 063901 (2004)

How to generate a gap vortex?

Gap-state preparation requirements:

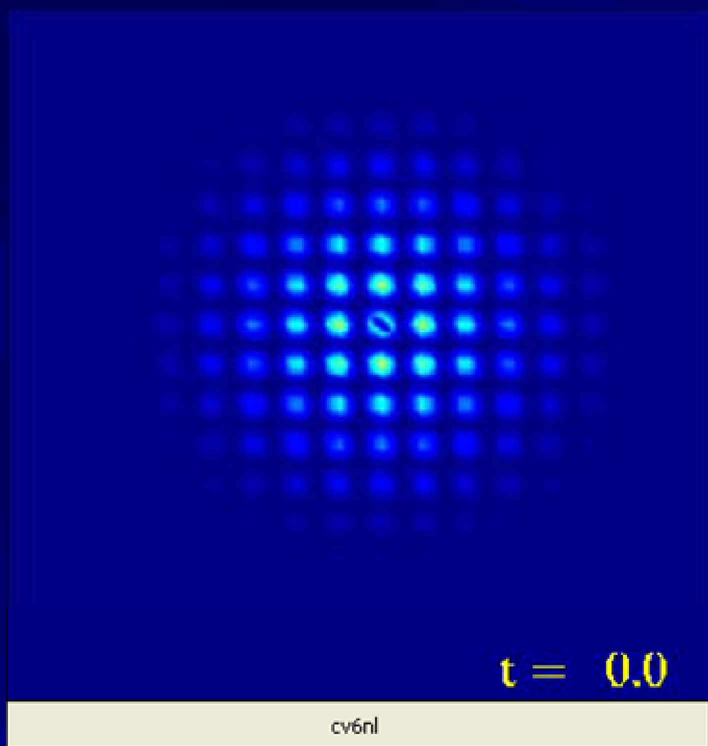
- Negative effective diffraction regime $D_{x,y} < 0$
- Adiabatic drive to the Brillouin zone's edge $t >> 1/\Delta\mu_{gap}$
- Phase ramp imprint



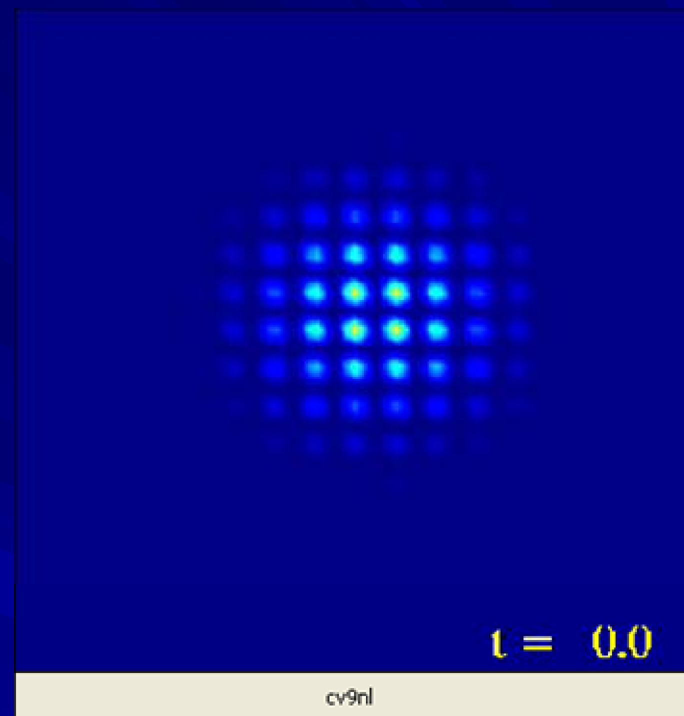
Simulation procedure:

- Start with a broad BEC wavepacket at the correct band edge (with a nontrivial Bloch-wave phase)
- Imprint a charge one vortex phase
- Let evolve in time

Generation dynamics



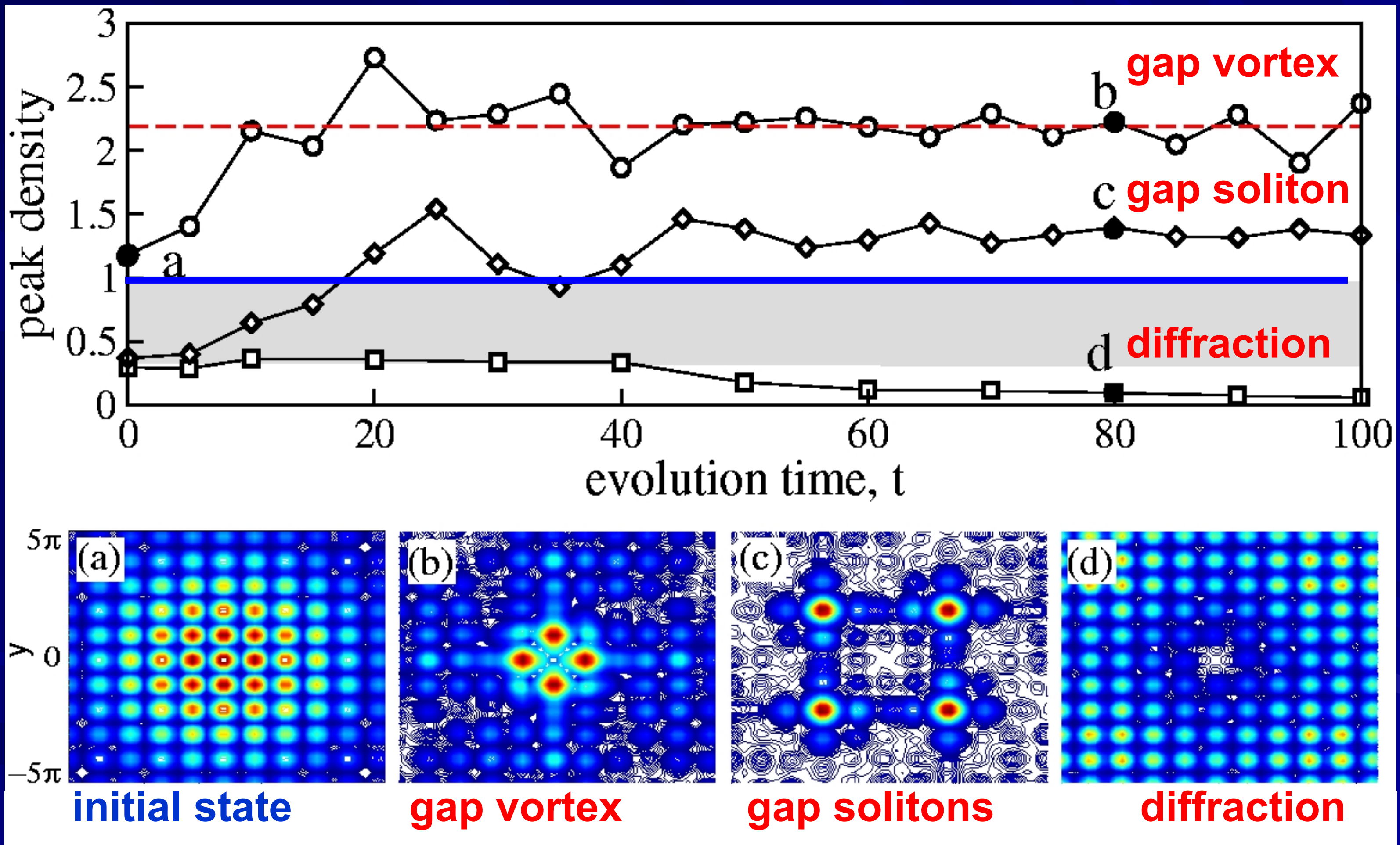
Discrete diffraction



Gap vortex

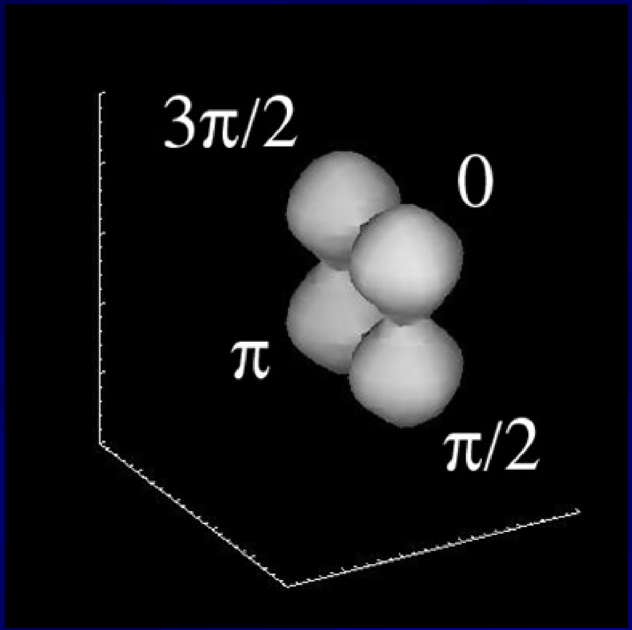
Possible outcomes

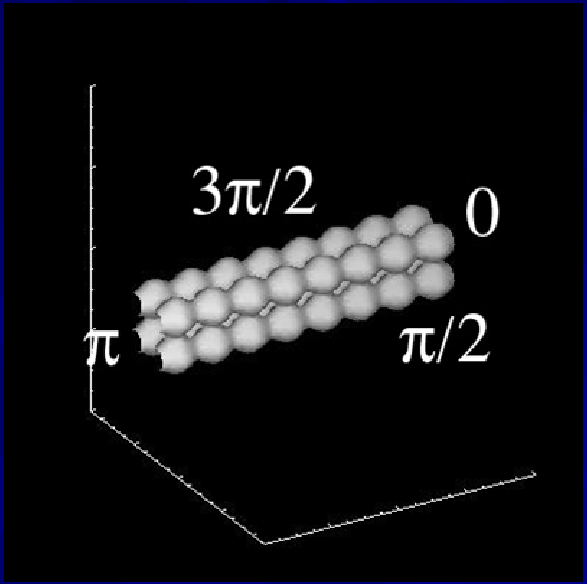
- Necessary: **atom number** above vortex state threshold
- Sufficient: **peak density** above threshold

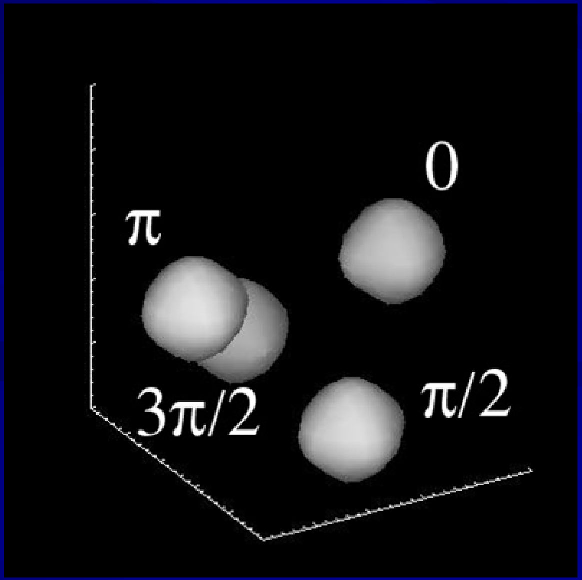


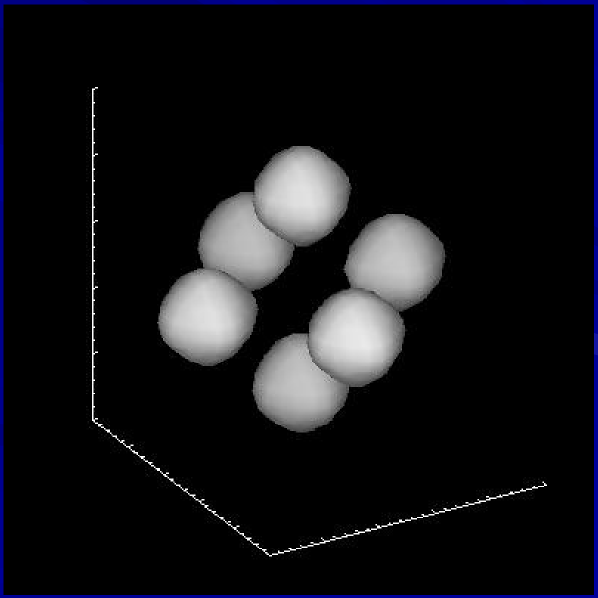
3D discrete vortices

3D discrete vortices

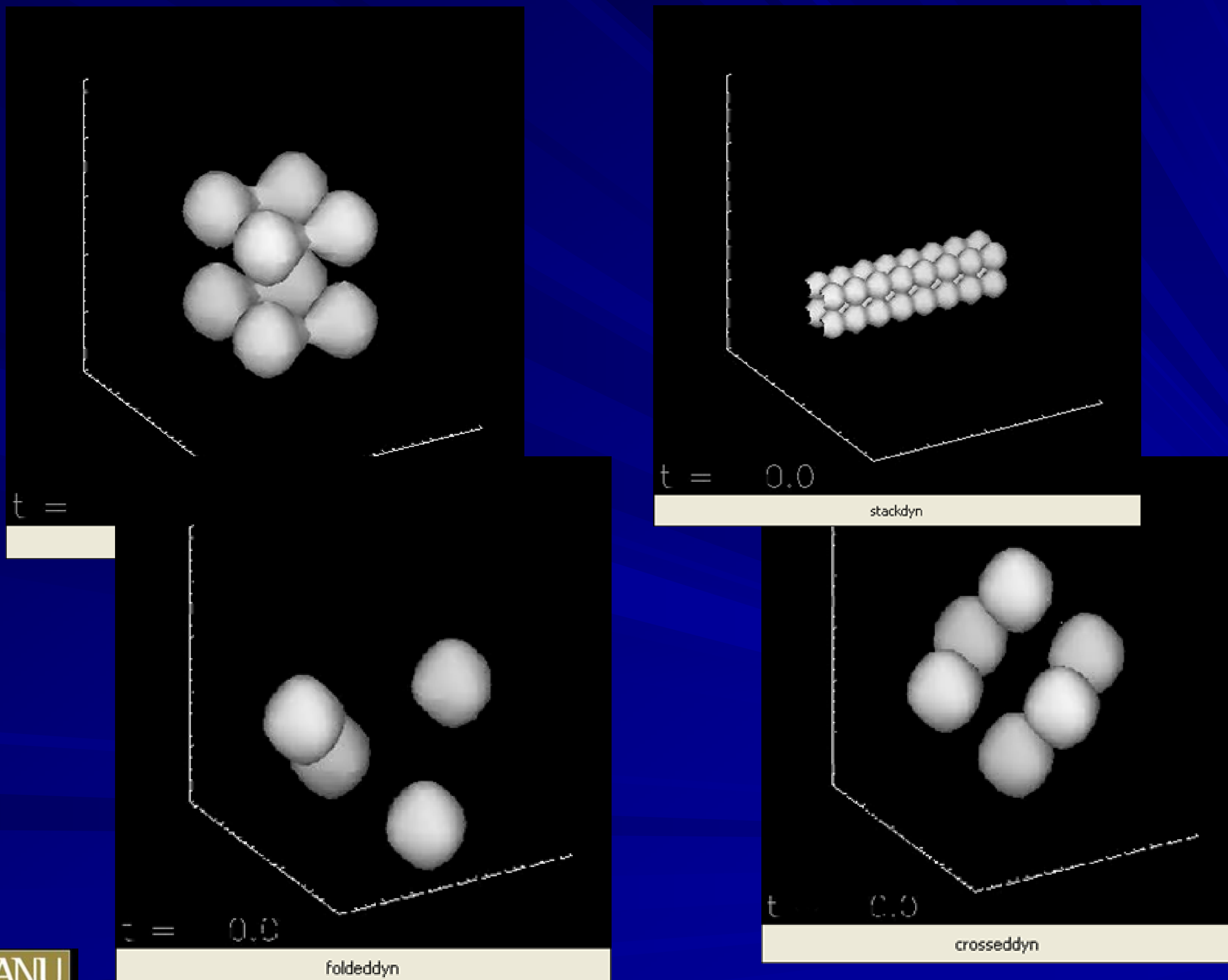








Dynamics of 3D discrete vortices

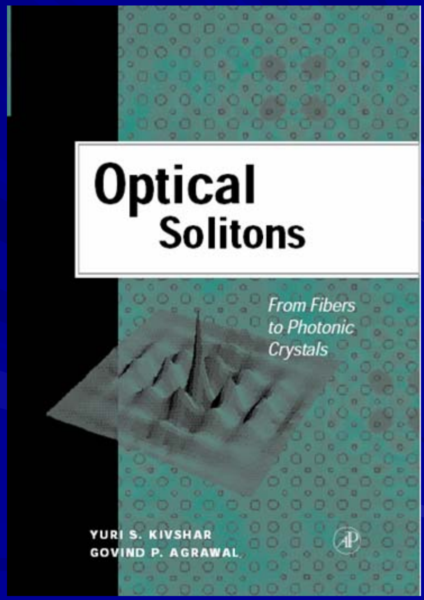


Conclusions

•**Nonlinear optics provides a solid background for the physics of nonlinear matter waves and solitons**

•**Much to learn from the field of spatial optical solitons**
•**Novel features: a strong trap; 2D vs. 3D; strong repulsion**

Last 2-3 years show a strong influence
of both the fields on each other



Novel directions for matter waves in lattices
Attractive condensate in a lattice (1D vs 2D & 3D)
>In-gap nonlinear effects (e.g. gap vortex)
>Atomic bandgap engineering - BEC in superlattices
>Spinor, multi-component, and atom-molecular BEC