Interaction of Impurity Atoms in Bose-Einstein Condensates

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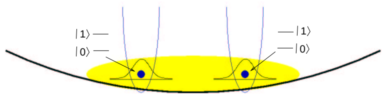
Abstract

The interaction of two spatially separated impurity atoms through phonon exchange in a Bose-Einstein condensate is studied within a Bogoliubov approach. The impurity atoms are held by deep and narrow trap potentials and experience level shifts which consist of a mean-field part and vacuum contributions from the Bogoliubov-phonons. In addition there is a conditional energy shift resulting from the exchange of phonons between the impurity atoms. Potential applications to quantum information processing are discussed.

Effective interaction of impurity atoms in a BEC



- Impurity atoms in sufficiently deep traps ⇒ Only ground state ϕ_0 of traps occupied.
- Interaction with condensate is given by

$$\hat{H}_{\rm int} = \sum_{\alpha,\beta} |\alpha,\beta\rangle\langle\alpha,\beta| \left(\frac{\kappa_\alpha}{2}\int \mathrm{d}\mathbf{r}\, |\phi_0(\mathbf{r}-\mathbf{r}_1)|^2 \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}) + \frac{\kappa_\beta}{2}\int \mathrm{d}\mathbf{r}\, |\phi_0(\mathbf{r}-\mathbf{r}_2)|^2 \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\right),$$

with state dependent coupling constants $\kappa_0 = 0$, $\kappa_1 = \kappa$.

- Dynamics of the condensate is calculated by standard Bogoliubov approach [1]

$$\hat{\psi}(\mathbf{r},t) = \psi_0(\mathbf{r}) + \sum_j{}' u_j(\mathbf{r})\hat{b}_j - v_j^*(\mathbf{r})\hat{b}_j^\dagger$$

with ψ_0 being the condensate wave function.

- Reduced statistical operator matrix elements $\tilde{\varrho}_{\alpha\beta,\gamma\delta} = \langle\alpha\beta,t|\,\tilde{\varrho}\,|\gamma\delta,t\rangle$ for the two impurity atoms are calculated in the usual Born approximation

$$\partial_t\tilde{\varrho}_{10,00} = -\frac{\kappa^2}{4\hbar^2}\int_{t_0}^t \mathrm{d}t'\, \tilde{\varrho}_{10,00}(t')\left\langle \tilde{B}_1(t)\tilde{B}_1(t')\right\rangle \quad (1)$$

$$\partial_t\tilde{\varrho}_{01,00} = -\frac{\kappa^2}{4\hbar^2}\int_{t_0}^t \mathrm{d}t'\, \tilde{\varrho}_{01,00}(t')\left\langle \tilde{B}_2(t)\tilde{B}_2(t')\right\rangle \quad (2)$$

$$\partial_t\tilde{\varrho}_{11,00} = -\frac{\kappa^2}{4\hbar^2}\int_{t_0}^t \mathrm{d}t'\, \tilde{\varrho}_{11,00}(t')\Big\{\left\langle \tilde{B}_1(t)\tilde{B}_1(t')\right\rangle + \left\langle \tilde{B}_1(t)\tilde{B}_2(t')\right\rangle + \text{ terms with } 1 \leftrightarrow 2\Big\}. \quad (3)$$

- Correlation functions are given by

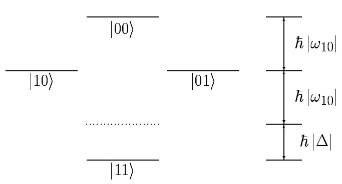
$$\left\langle \tilde{B}_l(t)\tilde{B}_{l'}(t')\right\rangle = \sum_j{}' \mathrm{e}^{-\frac{i}{\hbar}E_j(t-t')} S_j(l,l'),$$

where the E_j's are the Bogoliubov energies and

$$S_j(l,l') = \int \mathrm{d}\mathbf{r}\, |\phi_0(\mathbf{r}-\mathbf{r}_l)|^2\, \psi_0(\mathbf{r})(u_j(\mathbf{r}) - v_j(\mathbf{r})) \times \int \mathrm{d}\mathbf{r}'\, |\phi_0(\mathbf{r}'-\mathbf{r}_{l'})|^2\, \psi_0(\mathbf{r}')(u_j^*(\mathbf{r}') - v_j^*(\mathbf{r}')).$$

- Eqs. (1)-(3) correspond to effective coarse grained Hamiltonian for (symmetric located) impurity atoms in interaction picture. It reads

$$\tilde{H}_{\rm eff} = |10\rangle\langle 10|\,\hbar\omega_{10} + |01\rangle\langle 01|\,\hbar\omega_{10} + |11\rangle\langle 11|\,\hbar(2\omega_{10}+\Delta).$$

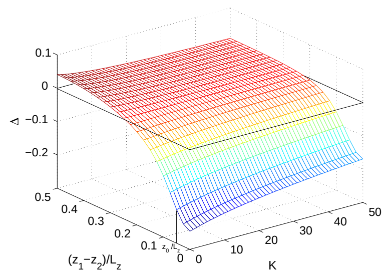


Homogeneous condensate

- Assume a condensate in a box with periodic boundary conditions, impurities located on the z-axis.
- Energy shift

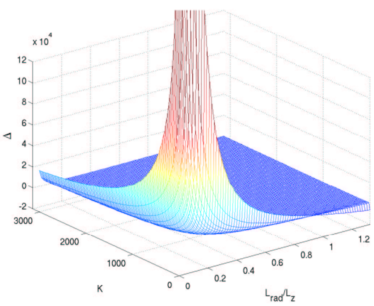
$$\Delta = -\frac{\kappa^2 N_0}{2\hbar V^2}\sum_{\mathbf{k}}{}' \frac{\varepsilon^0_{\mathbf{k}}}{E^2_{\mathbf{k}}}\cos(\mathbf{k}\cdot\Delta\mathbf{r})\exp\left(-\frac{z_0^2k^2}{2}\right)$$

with $E_{\mathbf{k}} = \sqrt{\varepsilon^0_{\mathbf{k}}(\varepsilon^0_{\mathbf{k}}+2gn_0)}$, $\varepsilon^0_{\mathbf{k}} = \frac{\hbar^2\mathbf{k}^2}{2m_{\rm B}}$, N_0 the number of condensed atoms, z_0 the width of the impurity traps and $\Delta\mathbf{r}$ the distance between the impurities.



Here, $K = gN_0 2m_{\rm B}L_z^2/\hbar^2 V$.

- Influence of the ratio $L_{\rm rad}/L_z$, where $L_{\rm rad} = L_x = L_y$:



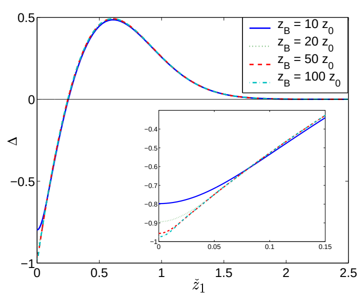
⇒ 1D geometry preferred.

Ideal, 1D trapped condensate

- Energy shift

$$\Delta = -\frac{\kappa^2_{\rm 1D}}{2\hbar}\sum_{\nu=1}^{\infty}\frac{1}{\hbar\nu\omega_{\rm B}}\frac{N_0\exp\left(-\check{z}_1^2-\check{z}_2^2\right)}{\pi(z_0^2+z_{\rm B}^2)}\frac{\check{z}^\nu}{2^\nu\nu!}H_\nu(\check{z}_1)\,H_\nu(\check{z}_2)$$

where $\check{z} = z_{\rm B}^2/(z_{\rm B}^2+z_0^2)$, $\check{z}_l = z_l/\sqrt{z_{\rm B}^2+z_0^2}$, $z_{\rm B}$ the width of the condensate trap, z_0 the width of the impurity traps. Furthermore $\kappa_{\rm 1D} = \kappa/(2\pi a_\perp^2)$ with the radial confinement $a_\perp^2 = \hbar/m_{\rm B}\omega_\perp$. H_ν are Hermite polynomials.



In this figure $\check{z}_1 = -\check{z}_2$.

- Influence of the width of the impurity traps is very weak.

Interacting, 1D trapped condensate

- Condensate dynamics is solved within the Thomas-Fermi approximation [1].
- The solutions of the Bogoliubov-de Gennes equation are given by [2]

$$f_j^{\pm}(z) = \sqrt{\frac{2j+1}{2R_{\rm TF}}}\left[\frac{2\mu}{E_j}\left(1-\frac{z^2}{R_{\rm TF}^2}\right)\right]^{\pm\frac{1}{2}} P_j\left(\frac{z}{R_{\rm TF}}\right)$$

with $f_j^{\pm} = u_j \pm v_j$.

- The spatial dependence of the energy shift disappears and one gets

$$\Delta = \frac{\kappa^2_{\rm 1D}}{8\hbar R_{\rm TF}g_{\rm 1D}}$$

- Conditions

$$\frac{z_1-z_2}{R_{\rm TF}} \gg \pi\sqrt{\frac{\zeta}{2}}$$

$$\frac{\delta r}{R_{\rm TF}} \gg \max\left\{\sqrt{\zeta}, \frac{z_0}{R_{\rm TF}}\right\},$$

with $R_{\rm TF}$ the Thomas-Fermi radius, δr the distance of the impurities from the edge of the condensate, and $\zeta = \hbar\omega_{\rm B}/2\mu$.

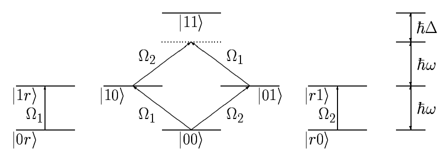
- Furthermore, one is restricted to (coarse graining approximation)

$$\Delta \ll \min\left\{\omega_{\rm B}, \frac{3N_0\kappa^2_{\rm 1D}}{16\hbar^2 R^2_{\rm TF}\omega_{\rm B}}\right\}. \quad (4)$$

- For a tight transverse confinement $\omega_\perp = 2\pi\times 10^4\mathrm{Hz}$, a scattering length $a_{\rm i} = 200\mathrm{nm}$ between impurities and condensate and $a = 5\mathrm{nm}$ within the condensate one finds a conditional frequency shift of $2\pi\times 10^3\mathrm{Hz}$.

Implementation of a quantum phase gate

- A quantum phase gate can be implemented as follows



	Ω_1 π-pulse		Ω_2 2π-pulse		Ω_1 π-pulse	
$\lvert 00\rangle$	→	$i\lvert 10\rangle$	→	$i\lvert 10\rangle$	→	$-\lvert 00\rangle$
$\lvert r0\rangle$	→	$\lvert r0\rangle$	→	$-\lvert r0\rangle$	→	$-\lvert r0\rangle$
$\lvert 0r\rangle$	→	$i\lvert 1r\rangle$	→	$i\lvert 1r\rangle$	→	$-\lvert 0r\rangle$
$\lvert rr\rangle$	→	$\lvert rr\rangle$	→	$\lvert rr\rangle$	→	$\lvert rr\rangle$

- $|r\rangle$ and $|0\rangle$ are the logical states.
- Gate time has to be long compared to the inverse conditional frequency shift Δ^{-1}.
- Because of (4) the major restriction is the trap frequency in weak (longitudinal) direction $\omega_{\rm B}$.

References

[1] P. Öhberg *et al.*, "Low-energy elementary excitations of a trapped Bose-condensed gas", Phys. Rev. A, **56**, R3346 (1997).

[2] D. S. Petrov, G. V. Shlyapnikov, J. T. M. Walraven, "Regimes of quantum degeneracy in trapped 1D gases", Phys. Rev. Lett., **85**, 3745 (2000).

[3] A. Klein, M. Fleischhauer, "Interaction of impurity atoms in Bose-Einstein condensates", cond-mat/0407809.