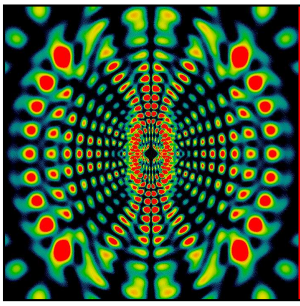


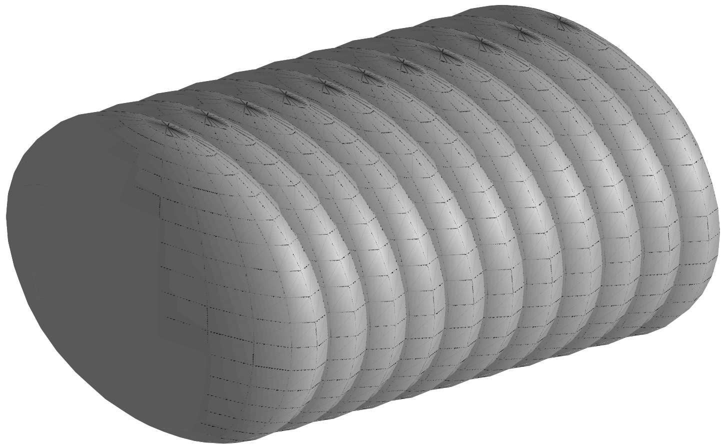
Nonlinear Dynamics in Quantum Systems

BEC of cold atoms loaded into the optical lattices: Quantum chaos approach

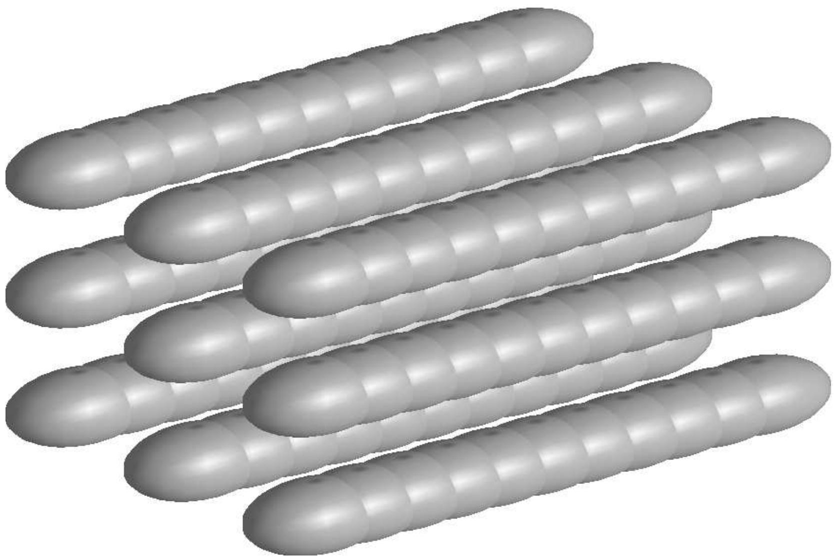
Andrey R. Kolovsky

Workshop "Mesoscopic phenomena in ultracold matter", 11-15 October 2004, Dresden

Optical lattices



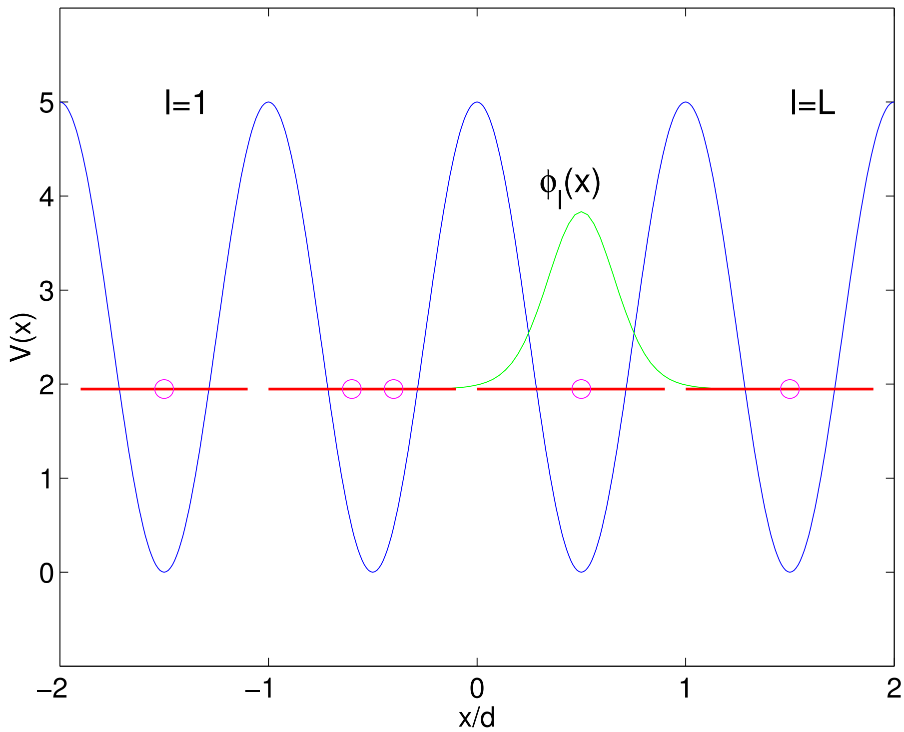
Quasi one-dimensional lattice.



Array of truly 1D lattices.

Bose-Hubbard model

$$\widehat{H} = -\frac{J}{2}\left(\sum_{l} \widehat{a}^{\dagger}_{l+1}\widehat{a}_{l} + h.c.\right) + \frac{W}{2}\sum_{l} \widehat{n}_{l}(\widehat{n}_{l} - 1) \qquad (1)$$



Parameters:

J – the hopping matrix element;

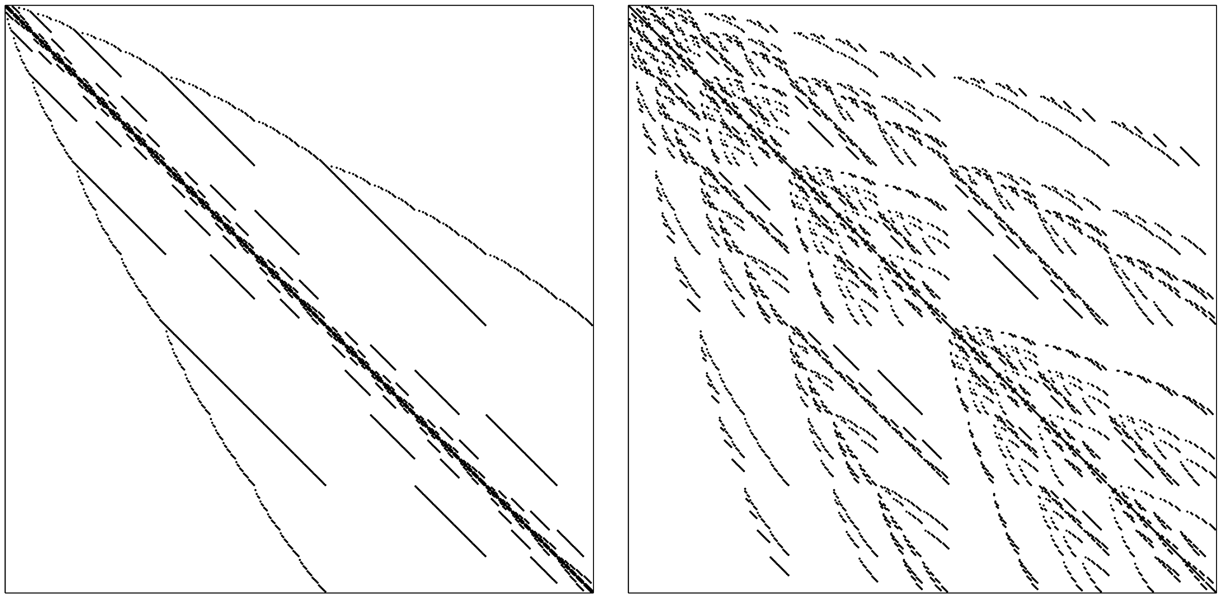
W – the interaction constant;

N – total number of the atoms;

L – the lattice size;

$\mathcal{N}$ – dimension of the Hilbert space.

Wannier *vs.* Bloch basis sets



Matrix of the Hamiltonian in the Wannier and Bloch basis (periodic boundary conditions).

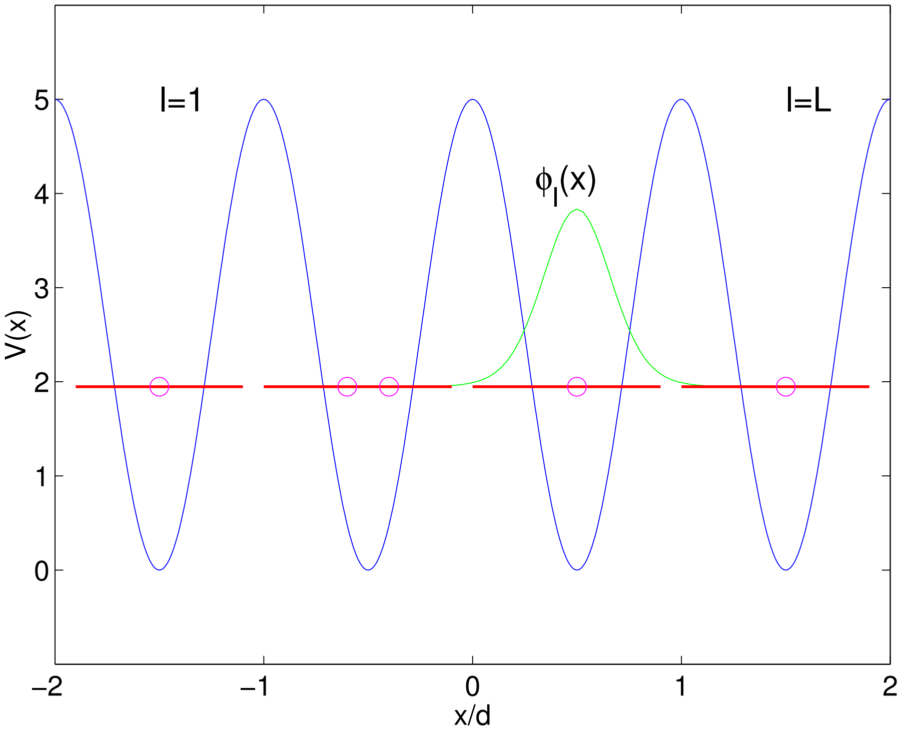
Canonical transformation: $\hat{b}_k = (1/\sqrt{L}) \sum_l \exp(i2\pi kl/L)\hat{a}_l$

$$\hat{H} = -J \sum_k \cos\left(\frac{2\pi k}{L}\right) \hat{b}_k^\dagger \hat{b}_k \tag{2}$$

$$+\frac{W}{2L} \sum_{k_1,k_2,k_3,k_4} \hat{b}_{k_1}^\dagger \hat{b}_{k_2} \hat{b}_{k_3}^\dagger \hat{b}_{k_4} \tilde{\delta}(k_1 - k_2 + k_3 - k_4)$$

Fermi-Hubbard model

$$\widehat{H} = -\frac{J}{2}\sum_{s=\uparrow,\downarrow}\left(\sum_{l}\widehat{c}^{\dagger}_{l+1,s}\widehat{c}_{l,s} + h.c.\right) + W\sum_{l}\widehat{n}_{l,\uparrow}\widehat{n}_{l,\downarrow}\ ,$$

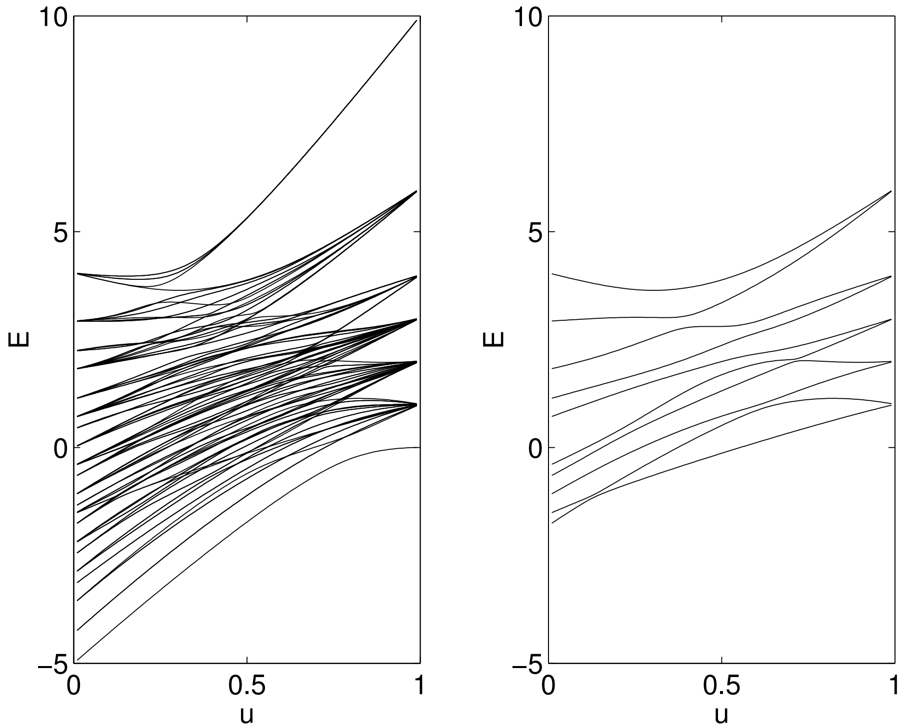


Dimension of the Hilbert space:

$$\mathcal{N} = \mathcal{N}_{\uparrow}\mathcal{N}_{\downarrow}\ ,\quad \mathcal{N}_s = \frac{(L - N_s + 1)!}{N_s!}\ \text{– Fermi system}$$

$$\mathcal{N} = \frac{(N + L - 1)!}{N!(L - 1)!}\ \text{– Bose system}$$

Spectrum of Bose-Hubbard model



The left panel shows the complete spectrum, the right panel singles out the 'odd $\kappa = 0$' symmetry component ($W = u$, $J = 1 - u$, $N = L = 5$).

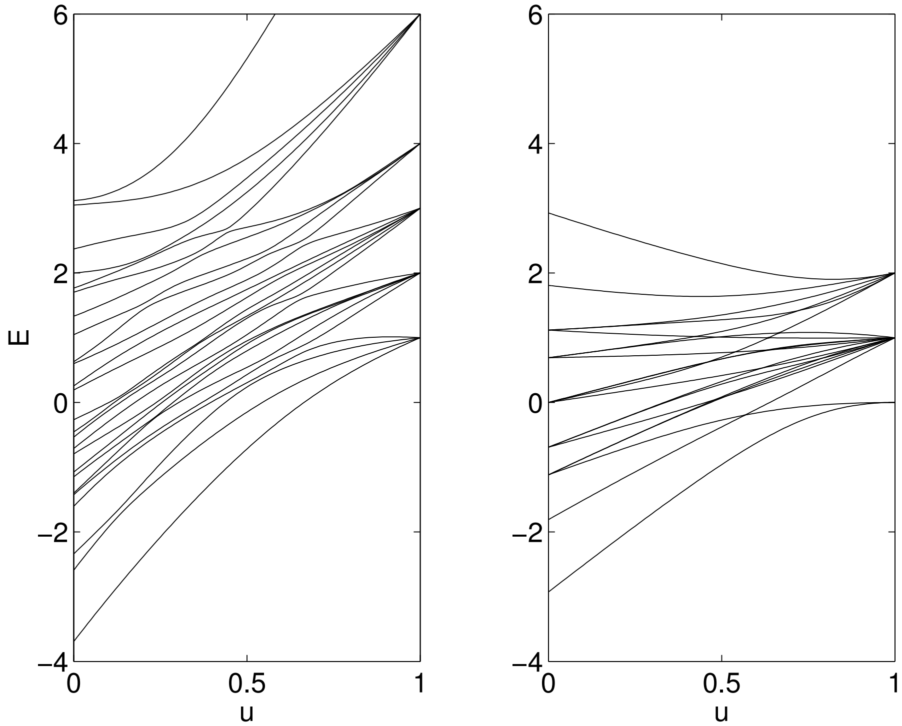
$$E = \frac{W}{2} \sum_{l=1}^{L} n_l \, , \quad \text{for } u = 1$$

$$E = -J \sum_{k=1}^{L} \cos\left(\frac{2\pi k}{L}\right) n_k \, , \quad \text{for } u = 0$$

$$H = \bigoplus_{k=1}^{L} H^{(\kappa)} \, , \quad \kappa = \frac{2\pi k}{L}$$

Bose-Hubbard *vs.* Fermi-Hubbard models

Intermediate conclusion: Bose-Hubbard system is non-integrable. (Note that 1D Hubbard model for fermions is completely integrable!)



The energy levels of BH-model (left panel) and FH-model (right panel) as the functions of the parameter u ($J = 1 - u$, $W = u$) for $N = 5$ ($N_\uparrow = 3$, $N_\downarrow = 2$ for FH-model)and $L = 5$. Only the levels of $\kappa = 2\pi/L$ symmetry are shown.

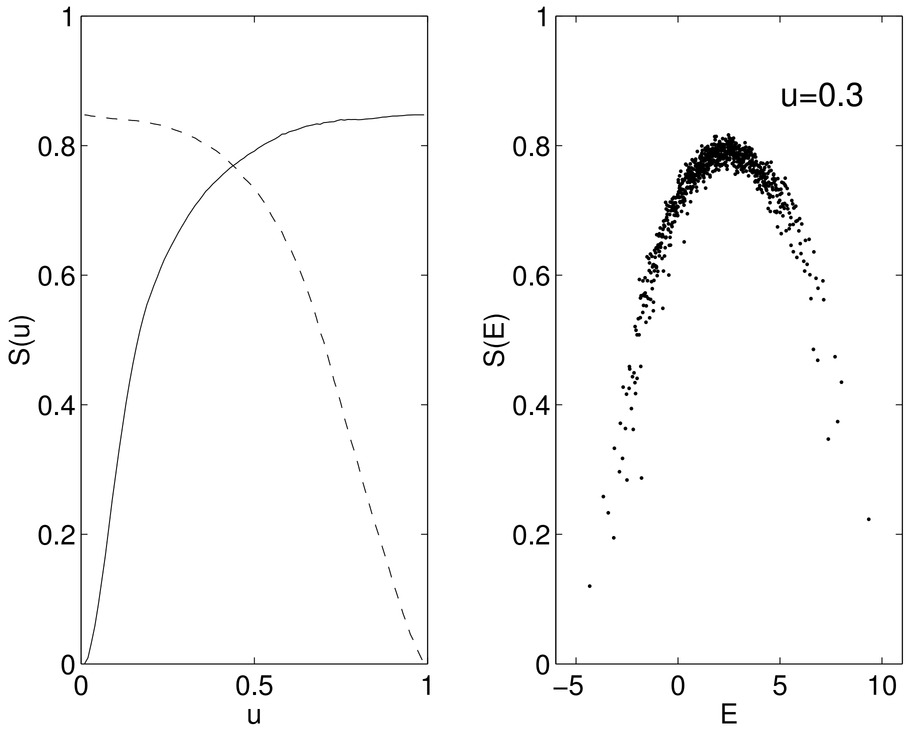
Is it a chaotic system?

Criterion of chaos

Delocalization of the eigenstates:

$$S(u) = \left\langle -\frac{1}{\log \mathcal{N}} \sum_{j=1}^{\mathcal{N}} |c_j|^2 \log(|c_j|^2) \right\rangle ,$$

where c_j are the expansion coefficients of an arbitrary eigenstate of the system in a given basis.



Left panel: Mean entropy $S(u)$ in the ν-basis (solid line) and in the μ-basis (dashed line), as a function of the interaction parameter u ($N = L = 8$). Right panel: Entropy $S(E) = \min[S^{(\nu)}(E), S^{(\mu)}(E)]$ of the individual eigenstates (dots), for $u = 0.3$.

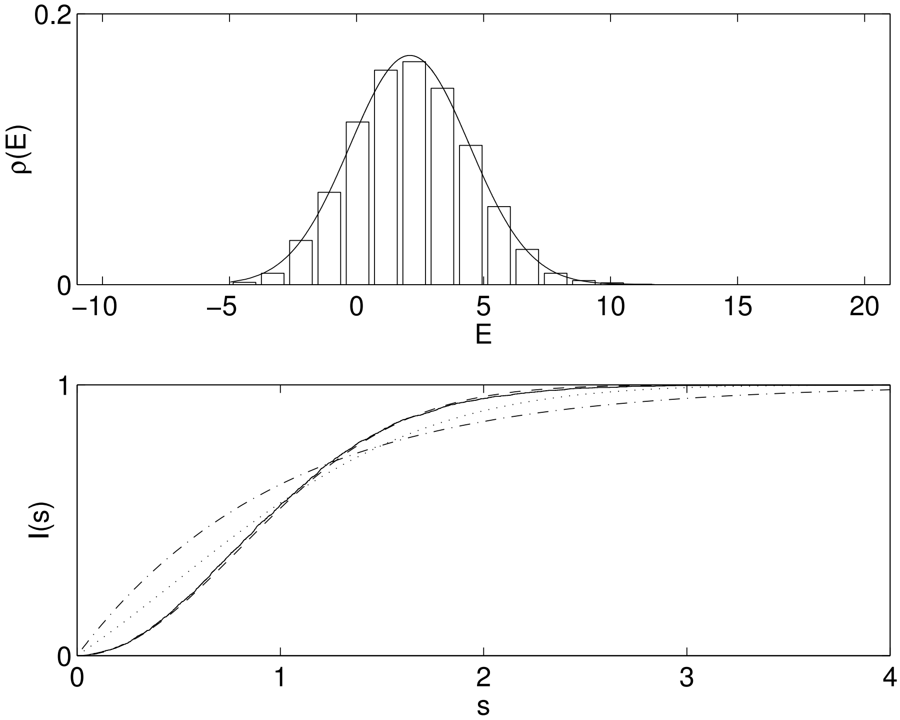
[2] A. R. Kolovsky and A. Buchleitner, e-print cond-mat/0403213

Statistics of energy spectrum

Wigner-Dyson statistics for level spacing distribution,

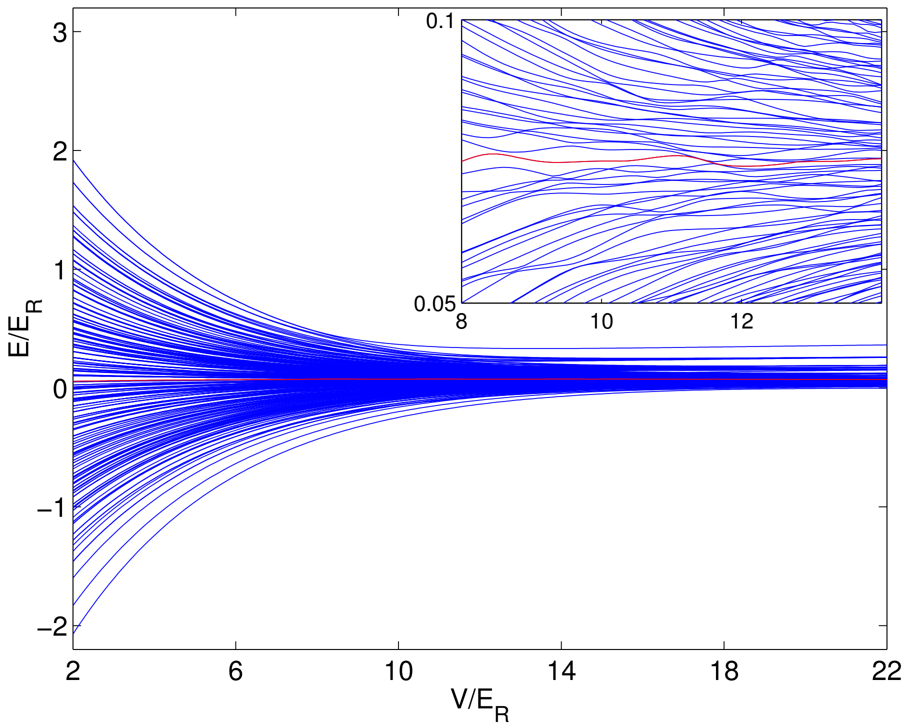
$$P(s) = \frac{\pi^2}{6} s \exp\left(-\frac{\pi s^2}{4}\right) ,$$

$$I(s) = \int_0^s P(s')ds' = 1 - \exp\left(-\frac{\pi s^2}{4}\right) .$$



Density of states and level spacing statistics for $u = 0.3$, $N = L = 9$, and $\kappa = 0$.

Probing chaos



Energy levels of the system of cold atoms in optical lattice as a function of the optical potential depth ($L = N = 7$, only the levels associated with ground Bloch band are shown).

Stages of the suggested experiment:

1. Prepare the system in the (ground) super-fluid state;

2. Excite it by applying a strong static force for 1/4 of Bloch period;

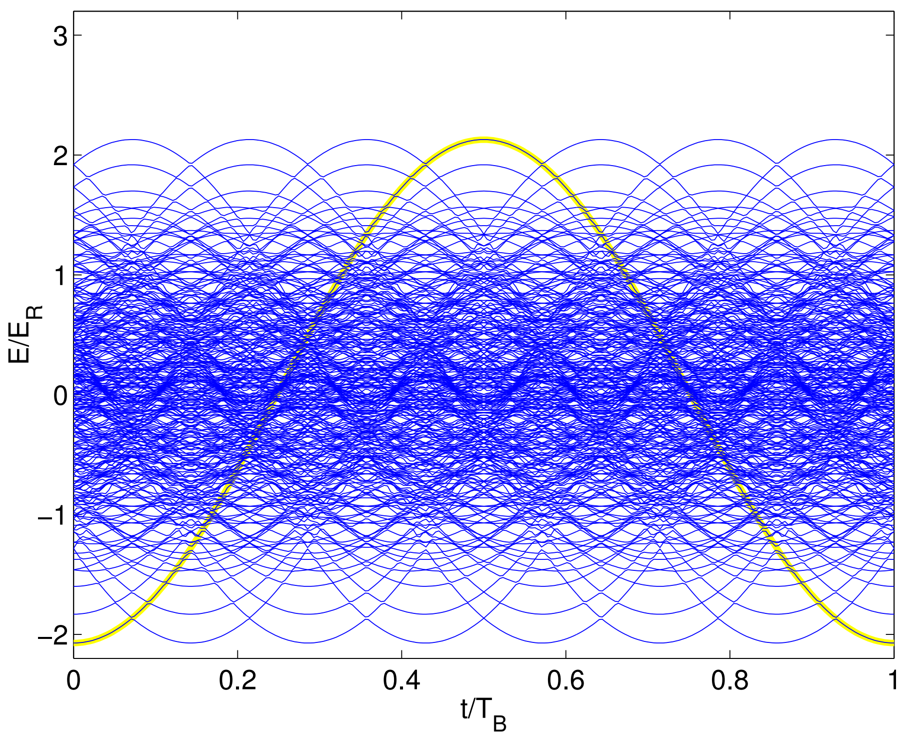
3. Perform an adiabatic passage $V_0 \to V_{max} \to V_0$;

4. Measure the atomic momentum distribution.

[1] M. Greiner, O. Mandel, T. Esslinger, T. W.Hänsch, and I. Bloch, Nature **415**, 39 (2002).

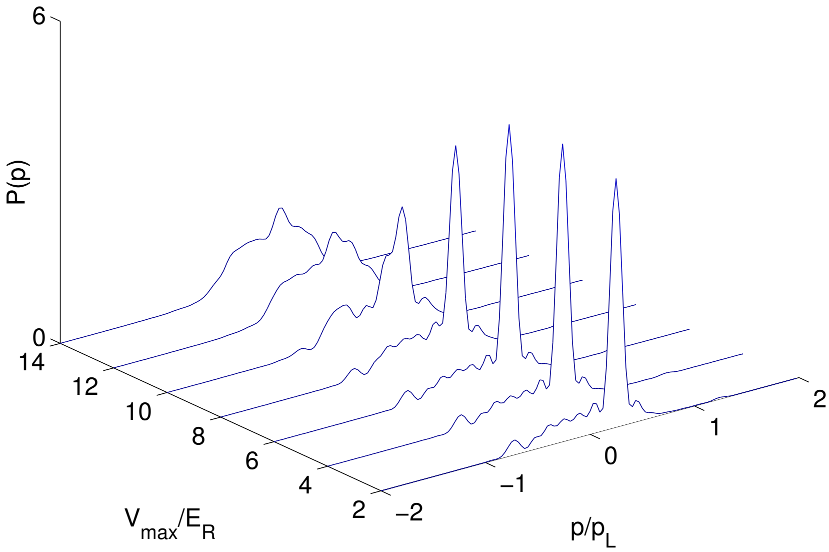
Bloch oscillation

$$\widehat{H}(t) = -\frac{J}{2}\left(\sum_l e^{i\omega_B t}\widehat{a}^\dagger_{l+1}\widehat{a}_l + h.c.\right) + \frac{W}{2}\sum_l \widehat{n}_l(\widehat{n}_l - 1)$$

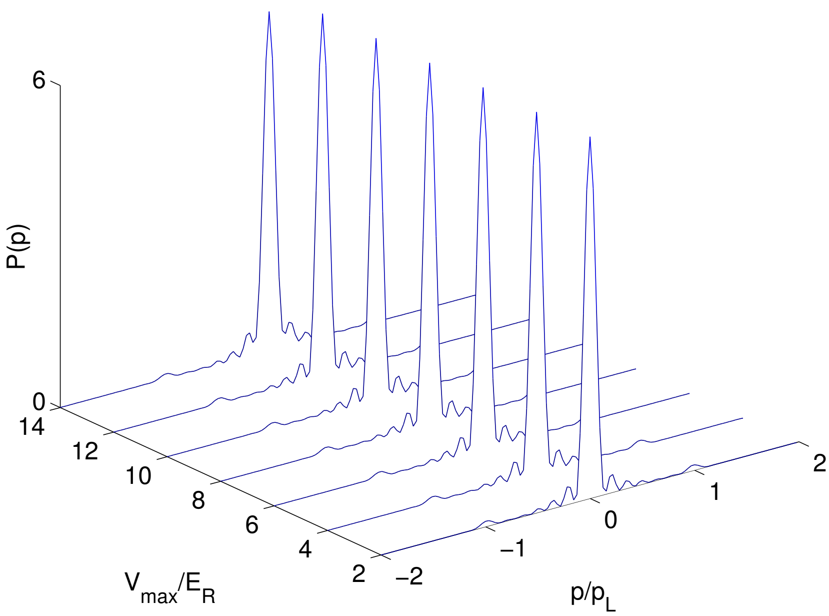


Instantaneous spectrum of the above Hamiltonian ($L = N = 7$).

Momentum distribution



Atomic momentum distribution at the end of an adiabatic passage $V_0 \to V_{max} \to V_0$, $V_0 = 2$.



Conclusions

1. Bose-Hubbard model or, equivalently, the system of cold bosonic atoms in the optical lattices is (generally) a quantum chaotic system.

2. This mesoscopic phenomenon can be well tested in the laboratory experiments by performing an adiabatic passage from the shallow to deep lattice (and back).

3. For the other manifestations of chaos in the system see poster No.