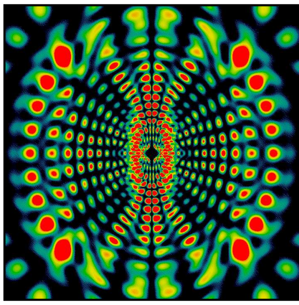


Nonlinear Dynamics in Quantum Systems

Dynamical instability, Chaos and Bloch oscillations of BEC in the titted optical lattices

Andrey R. Kolovsky

11-15 October 2004, Dresden

Introduction

[1] F. Bloch, Z. Phys **52**, 555 (1928).

[2] C. Zener, Proc. R. Soc. A **145**, 523 (1934).

$$H = \frac{p^2}{2M} - V \cos^2(k_L z) + Fz$$

[3] BenDahan *et. al.*, Phys. Rev. Lett. **76**, 4508 (1996).

[4] M. G. Raizen, C. Salomon, and Qian Niu, Physics Today **50**(8), 30 (1997).

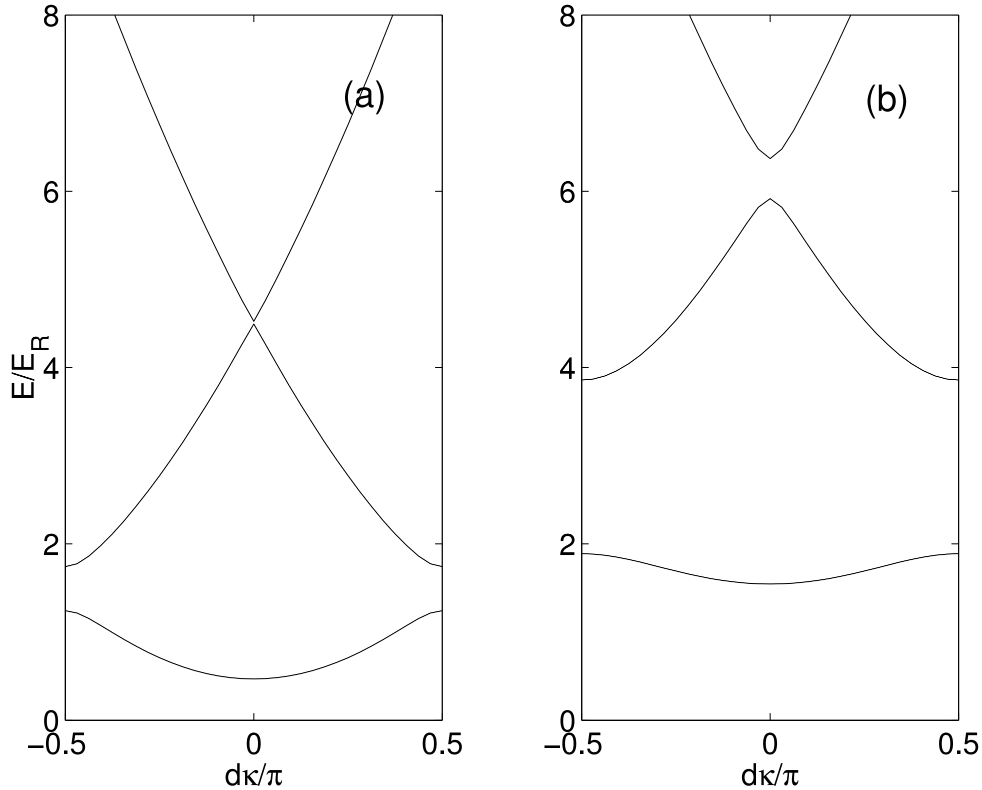
[5] B. P. Anderson and M. A. Kasevich, Science **282**, 1686 (1998).

[6] O. Morsch *et. al.*, Phys. Rev. Lett. **87**, 140402 (2001).

[7] M. Greiner *et. al.*, Nature **415**, 39 (2002).

Bloch oscillations

$$\hat{H}_0 = \frac{\hat{p}^2}{2M} - V_0 \cos^2(k_L x) \ , \quad \hat{H} = \hat{H}_0 + Fx$$



Bloch band spectrum of the atoms in the optical lattice with the depth $V_0 = E_R$ (a) and $V_0 = 4E_R$ (b).

Floquet-Bloch solutions: $\psi_\kappa(x,t) \sim \psi_{\kappa+Ft/\hbar}(x)$.

$T_B = 2\pi\hbar/dF$ – Bloch period

Tight-binding model

$$\psi(x,t) = \sum_l c_l(t)\phi_l(x) \equiv \sum_l c_l|l\rangle \ ,$$

where $\phi_l(x)$ are the Wannier functions

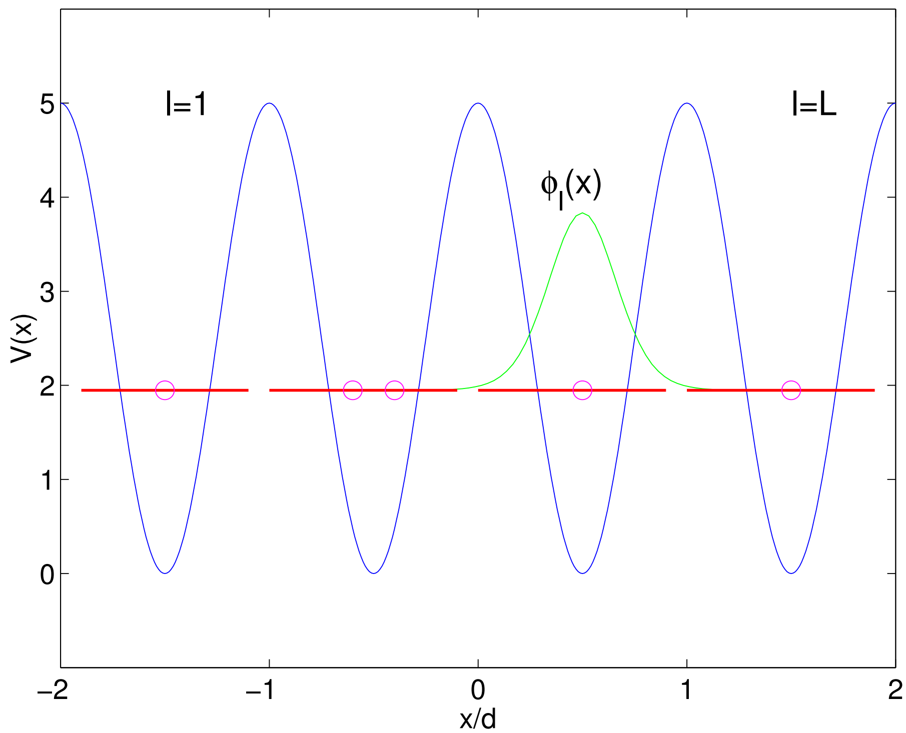
$$\widehat{H}_{tb} = E_0 \sum_l |l\rangle\langle l| - \frac{J}{2}\sum_l (|l+1\rangle\langle l| + h.c.) + dF \sum_l l|l\rangle\langle l| \ .$$

Gauge transformation

$$\widetilde{H}_{tb}(t) = -\frac{J}{2}\sum_l \left(e^{i\omega_B t}|l+1\rangle\langle l| + h.c.\right) \ , \quad \omega_B = \frac{dF}{\hbar}$$

Bose-Hubbard model

$$\widehat{H}(t) = -\frac{J}{2}\left(\sum_l e^{i\omega_B t}\hat{a}^\dagger_{l+1}\hat{a}_l + h.c.\right) + \frac{W}{2}\sum_l \hat{n}_l(\hat{n}_l - 1)$$



Parameters:

ω_B – Bloch frequency;

J – hopping matrix element;

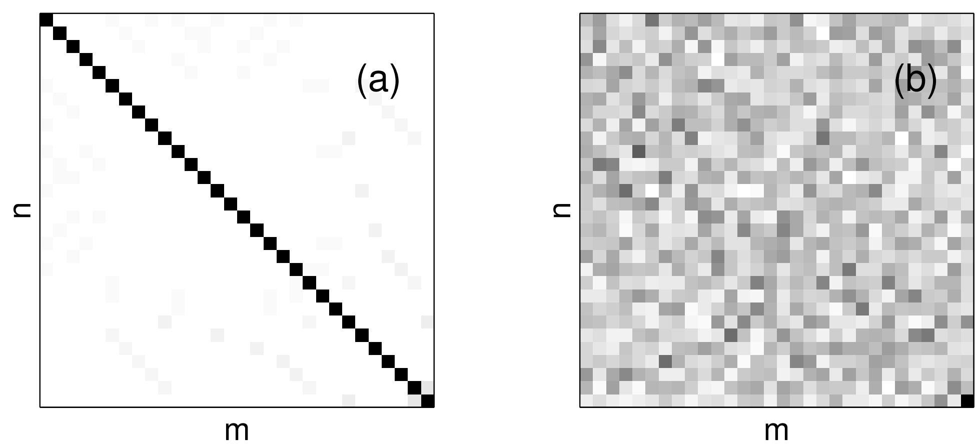
W – interaction constant;

$\bar{n}$ – mean occupation number (filling factor).

Floquet-Bloch operator

$$\widehat{U} = \widehat{\exp}\left[-\frac{i}{\hbar}\int_0^{T_B} \widehat{H}(t)dt\right] .$$

[4] A.R.Kolovsky and A.Buchleitner, Phys. Rev. E **68** (2003), 056213.



Matrix of the Floquet-Bloch operator for strong (a) and weak (b) static field ($W \sim J$, $\bar{n} \sim 1$).

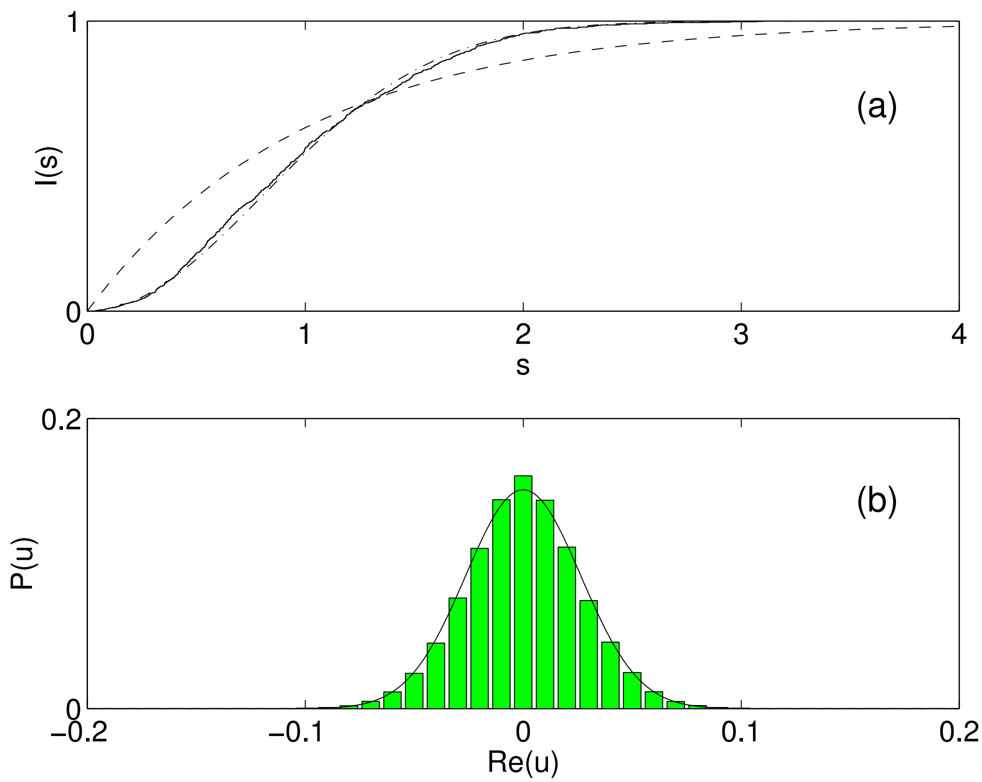
Quantum chaos

Poisson distribution (integrable systems):

$$P(s) = \exp(-s)$$

Wigner-Dyson distribution (chaotic systems):

$$P(s) = (\pi^2/6) s \exp(-\pi s^2/4)$$



Integrated level spacing distribution (upper panel) and distribution of the matrix elements (lower panel) of the Floquet-Bloch operator in the weak field regime.

Mean field approach

$$\hat{a}/\bar{n} \to a(t) \ , \quad \hat{a}^\dagger/\bar{n} \to a^*(t)$$

Discrite nonlinear Schrödinger equation

$$i\hbar\dot{a}_l = -\frac{J}{2}\left(e^{i\omega_B t}a_{l+1} + e^{-i\omega_B t}a_{l-1}\right) + g|a_l|^2 a_l \ , \quad g = \bar{n}W \ .$$

$$\begin{aligned} i\hbar\dot{b}_\kappa &= -J\cos(d\kappa - \omega_B t)b_\kappa \\ &+ \frac{g}{L}\sum_{\kappa_1,\kappa_2,\kappa_3} b_{\kappa_1}b^*_{\kappa_2}b_{\kappa_3}\delta(\kappa - \kappa_1 + \kappa_2 - \kappa_3) \ , \end{aligned}$$

Bloch oscillations

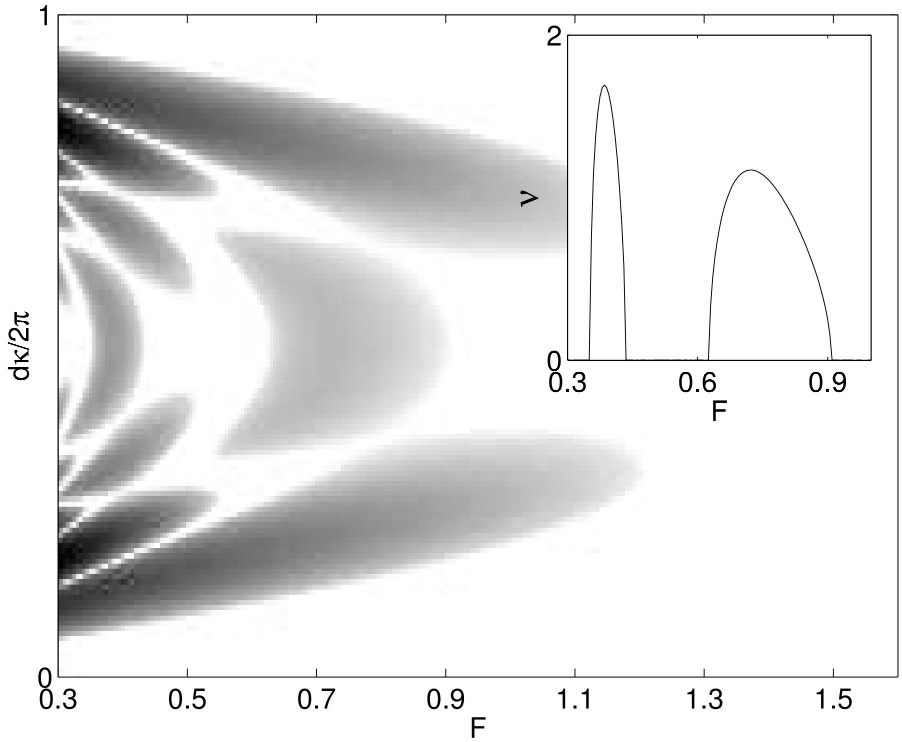
$$b_0(t) = \exp\left(i\frac{J}{dF}\sin(\omega_B t) - i\frac{g}{\hbar}t\right)b_0(0) \ , \quad b_{\kappa\neq 0}(t) = 0$$

Dynamical (modulation) instability

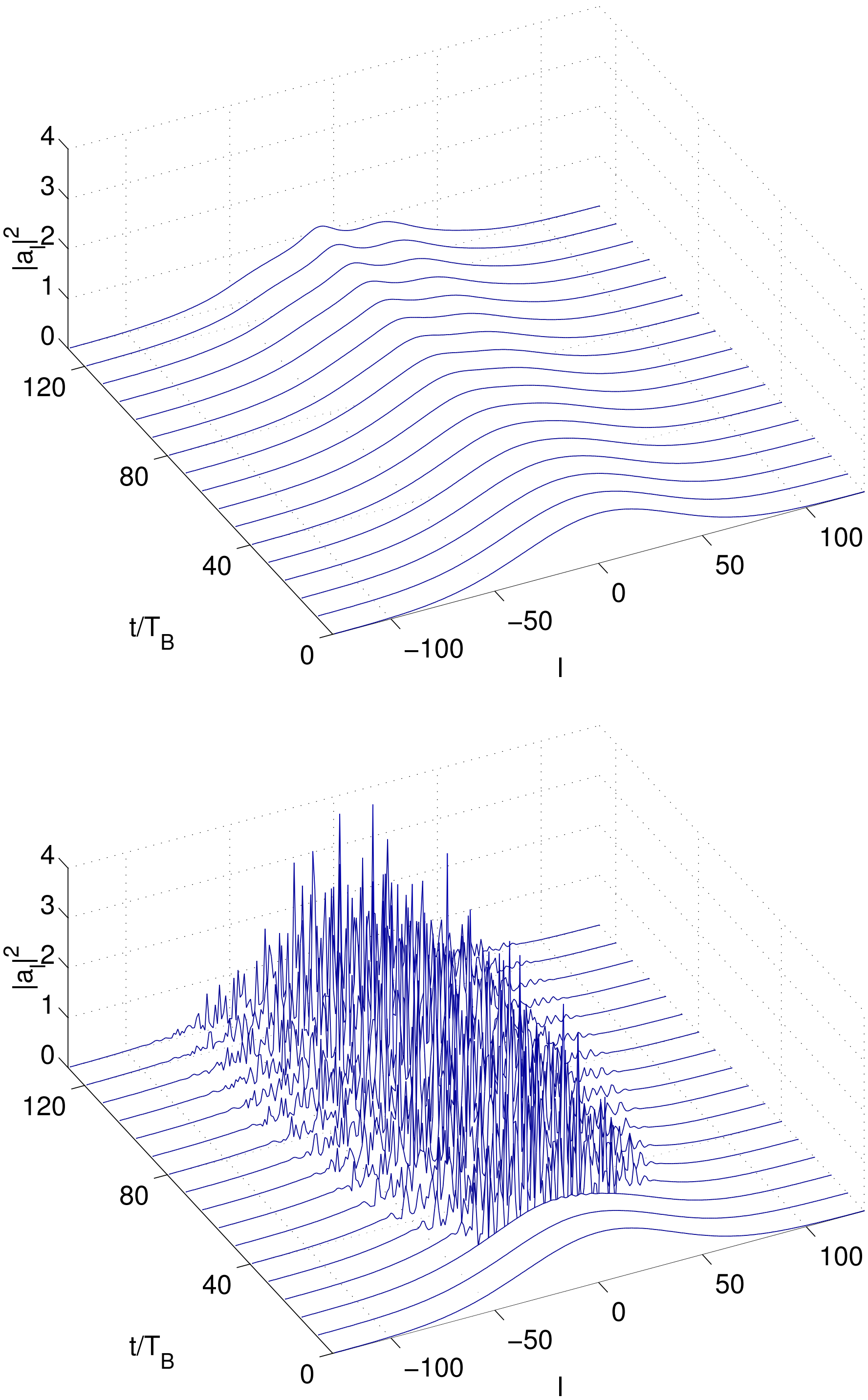
Bloch oscillations

$$b_0(t) = \exp\left(i\frac{J}{dF}\sin(\omega_B t) - i\frac{g}{\hbar}t\right) b_0(0)\ , \qquad b_{\kappa\neq 0}(t) = 0$$

$$b_\kappa(t) \sim \exp(\nu t) b_\kappa(0)$$



Increment of the dynamical (modulation) instability ν as the function of the static force magnitude F and the quasimomentum κ for $g = 0.4$. (The lattice period d and the tunnelling constant J are set to unity). The inset shows $\nu = \nu(F)$ for $\kappa = \pi/d$ (edge of the Brillouin zone).

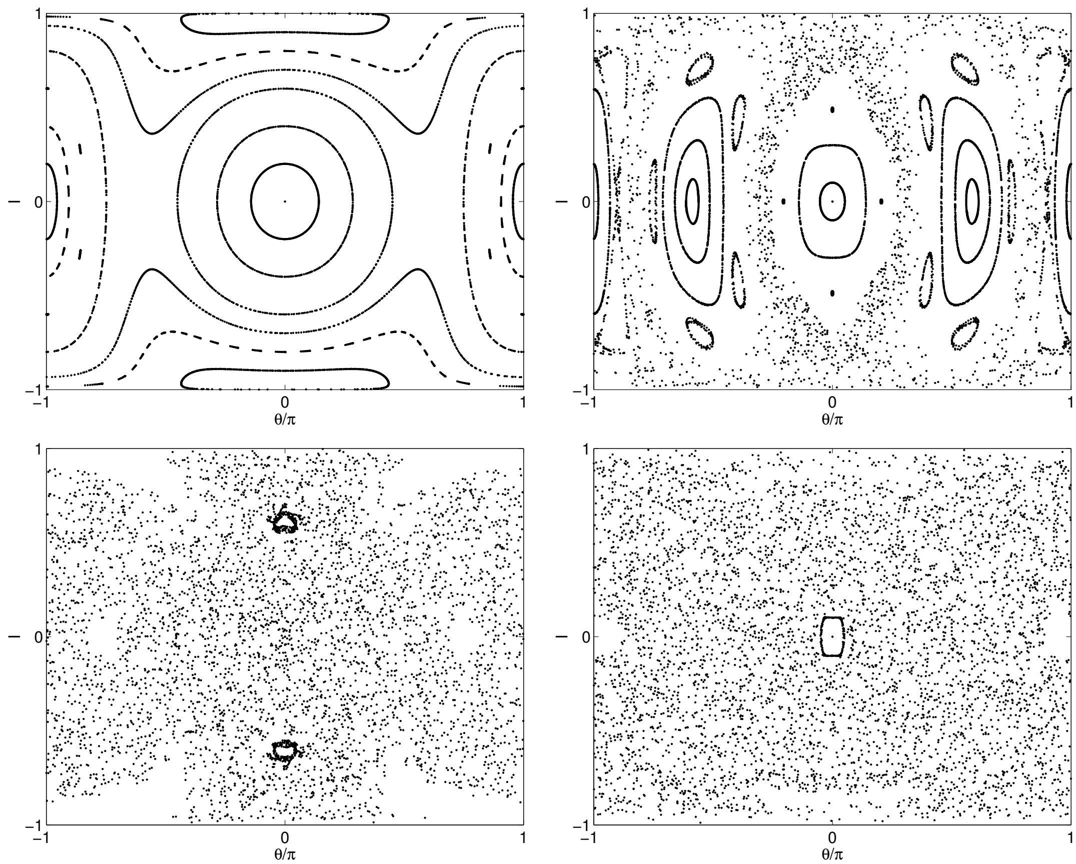


System dynamics (stroboscopic map over Bloch period) for $F < F_{cr}(g)$ (upper panel) and $F > F_{cr}(g)$ (lower panel).

Two-site model

$$H(t) = gI^2 - J\sqrt{1 - I^2}\cos(\omega_B t)\cos\theta \ ,$$

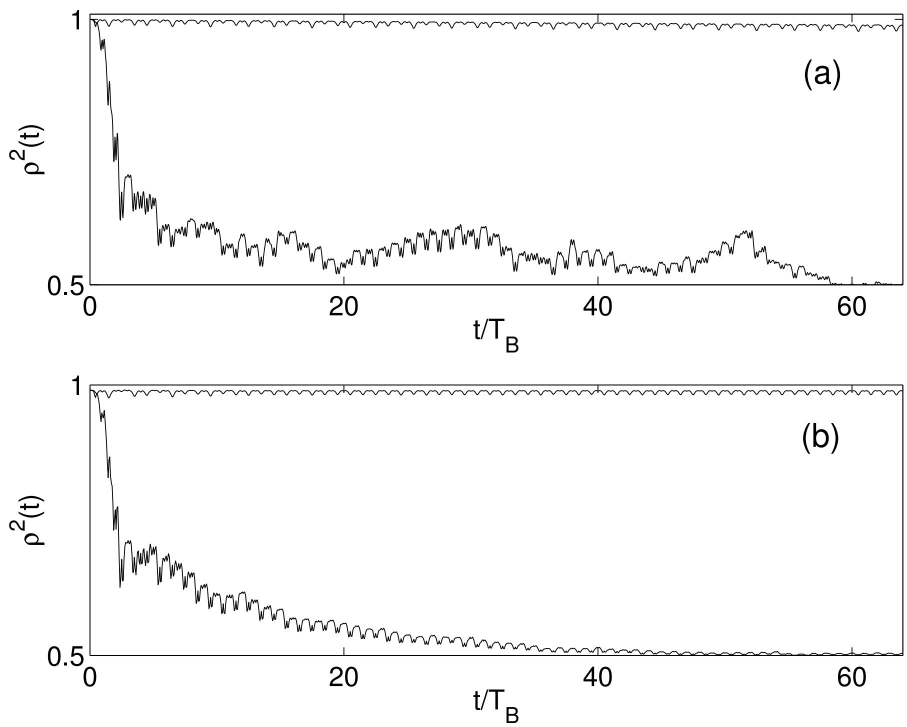
where $a_{1,2} = \sqrt{2I_{1,2}}\exp(i\theta_{1,2})$, $I = I_1 - I_2$, $\theta = \theta_1 - \theta_2$.



Phase portrait (stroboscopic map) of the 'pendulum' for $g = 0.4$ and $F = 2.0$ (upper left panel), $F = 1.3$ (upper right), $F = 0.7$ (lower left), and $F = 0.5$ (lower right panel).

Decoherence due to chaos

$$\begin{aligned}\rho_{l,m}(t) &= N^{-1}\langle\Psi(t)|\hat{a}_l^\dagger\hat{a}_m|\Psi(t)\rangle \\ \rho_{l,m}(t) &= L^{-1}\langle\langle a_l^*(t)a_m(t)\rangle\rangle\end{aligned}$$

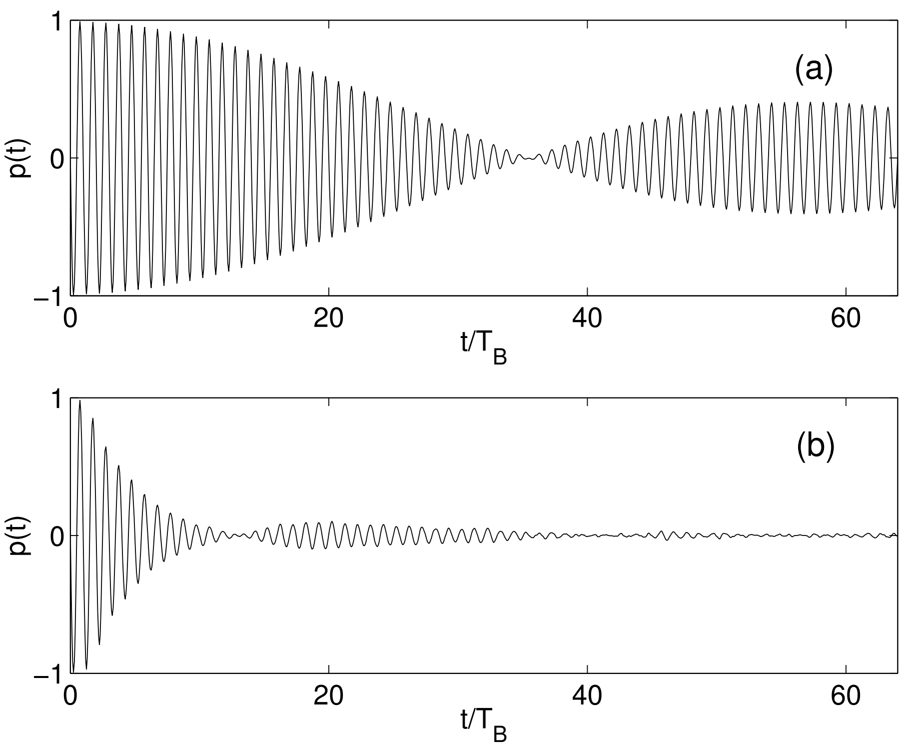


Upper panel: Dynamics of $\rho^2(t) = \mathrm{Tr}[\hat{\rho}^2(t)]$ for the two-site system with $\bar{n} = 50$ and $g = \bar{n}W = 0.4$. Lower panel is the ‘classical’ simulation of the decoherence process.

Bloch oscillations

$$|a_l|^2 = \overline{|a_l|^2} + \xi \left(\frac{\overline{|a_l|^2}}{\bar{n}} \right)^{1/2} \ , \quad \overline{|a_l|^2} = \exp\left(-\frac{l^2}{\sigma^2} \right) \ ,$$

$$\theta_l = \xi \left(4\bar{n}\overline{|a_l|^2} \right)^{-1/2} \ , \quad a_l = |a_l| \exp(i\theta_l)$$



Bloch oscillations of the mean atomic momentum for $F = 1.3 > F_{cr}$ (a) and $F = 0.7 < F_{cr}$ (b).