Atom-Hole BECs in Ultracold Fermi Gases Trappd Within Optical Lattices

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We investigate the Bose-Einstein Condensation (BEC) of particle-hole pairs in a gas of ultracold Fermionic atoms with repulsive interactions and arbitrary polarization, which are trapped within optical lattices. Near a Feshbach resonance, the dynamics of particle-hole pairs can be described by a hard-core Bose-Hubbard model. The insulator - superfluid/BEC and charge-density-wave - spuerfluid/BEC phase transitions can be induced by decreasing and increasing the potential depths, respectively. The parameter and polarization dependence of the critical temperatures for the ordered states is discussed simultaneously.

MOTIVATION

Bose-Einstein Condensation of Particle-Particle Pairs in Fermi Atoms

- Resonance superfluidity
  - BCS Cooper pair formation (M. Holland et al.), resonance pairing in an alkali gas yields a quantum fluid which can undergo a superfluid phase transition.
  - BCS-BEC crossover (A. Griffin et al.), fluctuations in particle-particle channel induces a BCS-BEC crossover.
  - Weakly bound bosonic molecules (L. D. Carr, et al.)
  - Pseudogap and bosonic molecules (J. Kinnunen, et al.)
- Superfluidity in optical lattices
  - $s-$ wave and $d-$ wave superfluidity (W. Hofstetter et al.).
  - quantum critical behavior induced by harmonic potential (M. Rigol et al.)

Bose-Einstein Condensation of Particle-Hole Pairs

- Electron-hole BEC

  In the dilute limit, the electron-hole pairs behave as weakly interacting bosonic particles and are expected to undergo the BEC phase transition.
- Atom-hole BEC

  ? (to be studied)

LATTICE MODEL

- An ensemble of arbitrarily polarized ultracold fermionic atoms occupying two different hyperfine states $|S\rangle$ and $|P\rangle$, which are trapped within the optical lattices, can be described by an asymmetric Fermi-Hubbard Hamiltonian

$$H = -\sum_{\langle i,j\rangle}(t_s f_{si}^{+} f_{sj} + t_p f_{pi}^{+} f_{pj}) + \sum_i(\epsilon_s n_{si} + \epsilon_p n_{pi}) + U\sum_i n_{si} n_{pi}.$$

where

$f_{\sigma i}^{+}(f_{\sigma i})$: fermionic creation (annihilation) operators

$\epsilon_s(\epsilon_p)$: single-atom energy for two occupied states

$t_s(t_p)$: state-dependent hopping

U : on-site interaction

- Filling number

$$n = \sum_i (n_{si} + n_{pi})/N_L$$

The half-filled case ($n = 1$) is considered in the following, i.e., one atom per site. The symbol N_L is the total number of lattice sites.

- Polarization

$$\gamma = \sum_i (n_{si} - n_{pi}) / \sum_i (n_{si} + n_{pi})$$

GROUND STATES FOR 1D LATTICES WITH STATE-INDEPENDENT HOPPING

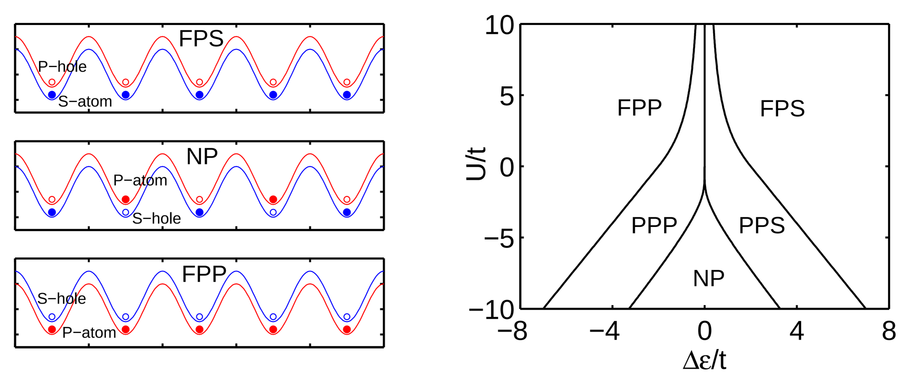
- Polarization regimes
  - Non-polarized states (NP), $\gamma = 0$
  - Partially polarized in state $|S\rangle$ (PPS), $0 < \gamma < 1$
  - Partially polarized in state $|P\rangle$ (PPP), $-1 < \gamma < 0$
  - Fully polarized in state $|S\rangle$ (FPS), $\gamma = 1$
  - Fully polarized in state $|P\rangle$ (FPP), $\gamma = -1$
- Critical values for the energy difference $\Delta\epsilon = \epsilon_p - \epsilon_s$
  - Non-polarized regimes to partially polarized regimes

$$\Delta\epsilon_1^c = \begin{cases} \frac{|U|}{2} - 2t + 4t\int_0^\infty \frac{J_1(w)dw}{w[1+\exp(\frac{|U|w}{2t})]} & \text{for } U < 0, \\ 0 & \text{for } U > 0, \end{cases}$$

  - Partially polarized regimes to fully polarized regimes

$$\Delta\epsilon_2^c = \begin{cases} 2t + |U| & \text{for } U < 0, \\ \sqrt{\frac{U^2}{4} + 4t^2} - \frac{U}{2} & \text{for } U > 0, \end{cases}$$

- Polarization phase diagram



Left: Ultracold fermionic atoms in one-dimensional optical lattices with half-filling and state-independent hopping. The dots and circles denote the atoms and holes (no atoms) respectively.
Right: Polarization regimes of the ground states for the one-dimensional lattices with state-independent hopping.

STRONG REPULSIVE LIMIT

Hard-Core Bose-Hubbard Model for Atom-Hole Pairs

- Strong repulsive limit near a Feshbach resonance

$$0 < t_{s,p} \ll U$$

- Bosonic operators for atom-hole pairs

$$b_j^{+} \Leftrightarrow f_{sj}^{+} f_{pj},\ b_j \Leftrightarrow f_{pj}^{+} f_{sj},$$
$$n_j = b_j^{+} b_j \Leftrightarrow \tfrac{1}{2} + \tfrac{1}{2}(n_{sj} - n_{pj}),$$

The operator $b_j^{+}(b_j)$ creates (annihilates) a pair of S-atom (atom in $|S\rangle$) and P-hole (hole in $|P\rangle$) on cite j.

- Hard-core Bose-Hubbard hamiltonian (up to third order terms of the perturbation parameters)

$$H_B = -\mu\sum_i n_i + J\sum_{\langle i,j\rangle} b_i^{+} b_j + V\sum_{\langle i,j\rangle} n_i n_j.$$

Here,
$t_p = \alpha t_s = \alpha t$,
$J = 4t_p t_s/U = 4\alpha t^2/U$,
$V = 2(t_s^2 + t_p^2)/U = 2(1+\alpha^2)t^2/U$,
$\mu = \Delta\epsilon + ZV/2 = \Delta\epsilon + Z(1+\alpha^2)t^2/U$, and
$Z = 2d$ (for the cubic lattices).

Zero-Temperature Behavior

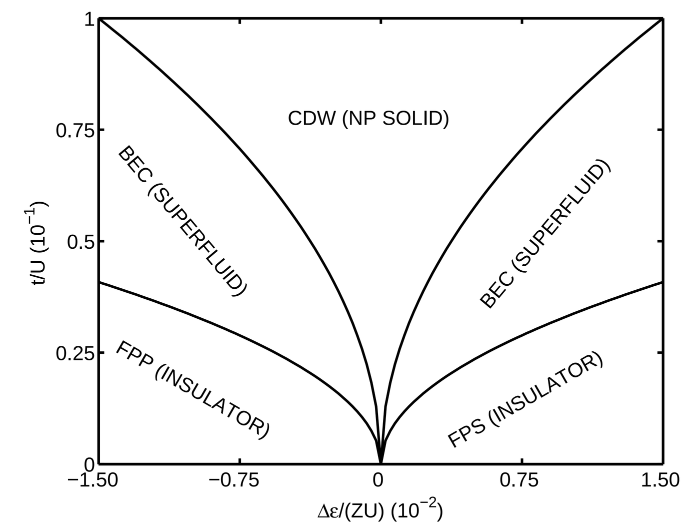
- Ground state phases
  - charge-density-wave (CDW) /solid phase with zero polarization ($\gamma = 0$) corresponds to the half-filled case of the hard-core Bose-Hubbard model ($\langle b^{+}b\rangle = 1/2$)
  - Bose-Einstein condensation (BEC) /superfluid phase with non-zero order parameter $\langle b\rangle$
  - insulator phase with the largest polarization ($|\gamma| = 1$) corresponds to the empty ($\langle b^{+}b\rangle = 0$) or the fully-filled ($\langle b^{+}b\rangle = 1$) case of the hard-core Bose-Hubbard model
- Phase transitions
  - CDW/solid - BEC/supersolid transition occurs at

$$|\Delta\epsilon|/U = (Z/2)(t/U)^2\sqrt{(1-\alpha^2)^2}$$

  - BEC/superfluid - insulator transition occurs at

$$|\Delta\epsilon|/U = Z(t/U)^2(1+\alpha)^2$$

- Phase diagram



Zero-temperature phase diagram of the ground states for the atom-holes in arbitrarily dimensional lattices with $\alpha = 2$.

Finite-Temperature Behavior

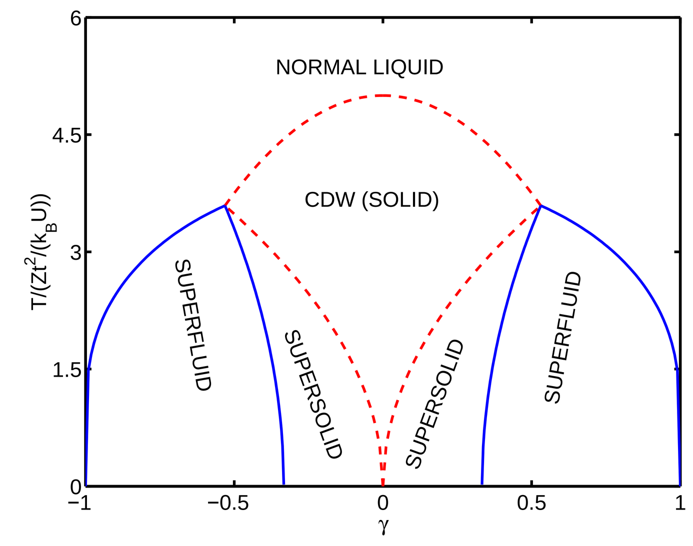
- Critical temperatures
  - critical temperature for CDW states

$$T_{CDW}^{C} = \frac{Z}{k_B}\cdot\frac{(1+\alpha^2)t^2}{U}\cdot(1-\gamma^2)$$

  - critical temperature for superfluid (SF) states

$$T_{SF}^{C} = \frac{Z}{k_B}\cdot\frac{2\alpha t^2}{U}\cdot\frac{\gamma}{arctanh(\gamma)}$$

- Critical polarizations
  - bicritical polarization $\gamma^{BC} = \gamma^{BC}(\alpha) > 0$ corresponds to $T_{SF}^{C} = T_{CDW}^{C}$ and $|\gamma| \neq 1$
  - critical superfluid polarization corresponds to $\gamma^{SFC} = |(1-\alpha)/(1+\alpha)|$ zero-temperature superfluid - supersolid transition
- Phase transition routes
  - CDW - NL (normal liquid) transition for $\gamma = 0$
  - supersolid - CDW - NL transition for $0 < |\gamma| \leq \gamma^{SFC}$
  - superfluid - supersolid - CDW - NL transition for $\gamma^{SFC} < |\gamma| < \gamma^{BC}$
  - superfluid - NL transition for $|\gamma| \geq \gamma^{BC}$
- Phase diagram



Finite-temperature phase diagram for the atom-holes in arbitrarily dimensional lattices with hopping ratio $\alpha = 2$.

CONCLUSIONS AND DISCUSSIONS

- Conclusions
  - hard-core Bose-Hubbard model for atom-hole pairs in the strong repulsive limit
  - demonstration of the existence of atom-hole BEC
  - parameter and polarization dependence of the critical temperatures
- Discussions: experimental possibility
  - ultracold two-component fermionic atoms with fixed polarization
  - optical lattices
  - strong repulsive limit near a magnetic-field induced Feshbach resonance
  - observation (Bragg scattering or stimulated two-photon emission)

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