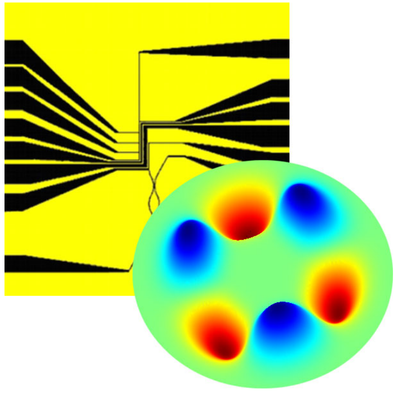


Spectral Properties and Lifetimes of Neutral Spin-1/2-Fermions in a Magnetic Guide

I. Lesanovsky[1] and P. Schmelcher[1,2]

[1]Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, 69120 Heidelberg, Germany

[2]Physikalisch-Chemisches Institut, INF 229, 69120 Heidelberg, Germany

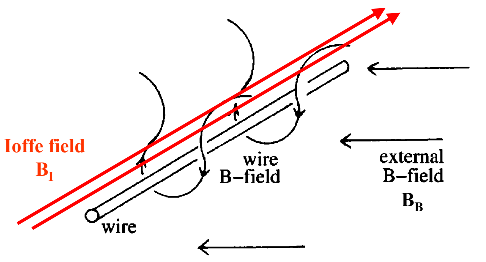
E-mail: igor.lesanovsky@physi.uni-heidelberg.de

The System

- we investigate the resonant motion of neutral spin-1/2-particles in a magnetic guide

Magnetic Guide

- magentic field of a straight current carrying wire superimposed by an external homogeneous bias field
- in order to avoid a line of zero field parallel to the wire a so-called Ioffe field can be applied



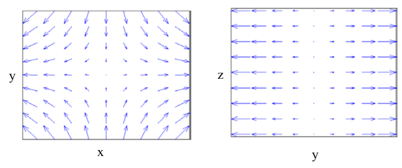
Taylor expansion of the magnetic guide around field minimum

$$\vec{B} \approx \frac{B_B}{\rho_0}\begin{pmatrix} x \\ -y \\ B_I \end{pmatrix} + \frac{B_B}{\sqrt{2}\rho_0^2}\begin{pmatrix} -x^2+2xy+y^2 \\ x^2+2xy-y^2 \\ B_I \end{pmatrix} + \frac{B_B}{\rho_0^3}\begin{pmatrix} y(y^2-3x^2) \\ -x(x^2-3y^2) \\ B_I \end{pmatrix}$$

- ρ_0 ... distance from the wire to the field minimum
- consider only the leading (quadrupole) term

Vectorial Plots of the quadrupole term (B_I=0)

$$\vec{B} = b\begin{pmatrix} x \\ -y \\ 0 \end{pmatrix}$$



- translation invariance along the z-axis

The Hamiltonian

- a neutral particle couples only through its magnetic moment to the magentic field

  → $V_{mag} = -\vec{\mu}\vec{B} = \frac{g}{2}\vec{S}\vec{B}$

- due to its translational invariance along the z-axis a two-dimensional description of the system is sufficient (plain waves in z-direction)

$$H = \frac{1}{2M}\left[p_x^2 + p_y^2 + \frac{bg}{2}\left(x\sigma_x - y\sigma_y\right) + \frac{B_I}{2}\sigma_z\right]$$

Scaling Transformation

$$\bar{x}_i = \left(\frac{bgM}{2}\right)^{-\frac{1}{3}} \quad \bar{p}_i = \left(\frac{bgM}{2}\right)^{\frac{1}{3}} \quad \gamma = B_I\left(\frac{gM}{2b^2}\right)^{\frac{1}{3}}$$ scaled Ioffe field strength

scaled Hamiltonian →

$$\boxed{\bar{H} = \frac{1}{2}\left[\bar{p}_x^2 + \bar{p}_y^2 + \bar{x}\sigma_x - \bar{y}\sigma_y + \gamma\sigma_z\right]}$$

- there are no bound solutions of the Hamiltonian $\bar{H}$
- energies and decay widths of the resonance states are obtained by employing the complex scaling method together with the linear variational principle

Symmetries

- the system exhibits a wealth of symmetries, both unitary and antiunitary
- symmetry properties change depending on whether there is an external Ioffe field applied or not
- Λ_z=L_z-S_z is conserved in both cases: quantum number m
- Σ_z is also conserved in both cases: quantum number κ

P_i	x_i-parity
σ_i	Pauli spin matrices
T	conventional time reversal
I_{xy}	coordinate exchange (x↔y)
S_1	$\sigma_x \to -\sigma_y$ $\sigma_y \to -\sigma_x$ $\sigma_z \to -\sigma_z$
S_2	$\sigma_x \to -\sigma_y$ $\sigma_y \to \sigma_x$ $\sigma_z \to \sigma_z$

Symmetries for γ = 0

$\Sigma_x = \sigma_x P_y$	$\Sigma_y = P_x\sigma_y$	$\Sigma_z = P_xP_y\sigma_z$	
$I_{xy}S_1^*$	$P_yI_{xy}S_2$	$P_xP_yI_{xy}S_1$	$P_xI_{xy}S_2^*$
$T\sigma_x$	$TP_xP_y\sigma_y$	$TP_x\sigma_z$	TP_y
$TP_yI_{xy}S_1^*$	$TI_{xy}S_2$	$TP_xI_{xy}S_1$	$TP_xP_yI_{xy}S_2^*$

→ symmetry structure gives rise to a two-fold degeneracy of any energy level

degenerate pair of states

$$|E,m,\kappa\rangle \quad \text{and} \quad \Sigma_x|E,m,\kappa\rangle = |E,-m,-\kappa\rangle$$

- degenerate states posses opposite κ and m quantum numbers

Symmetries for γ ≠ 0

Σ_z	$P_yI_{xy}S_2$	$P_xI_{xy}S_2^*$	
TP_y	$TP_x\sigma_z$	$TP_xP_yI_{xy}S_2^*$	$TI_{xy}S_2$

- symmetry group simplifies due to the presence of the Ioffe field

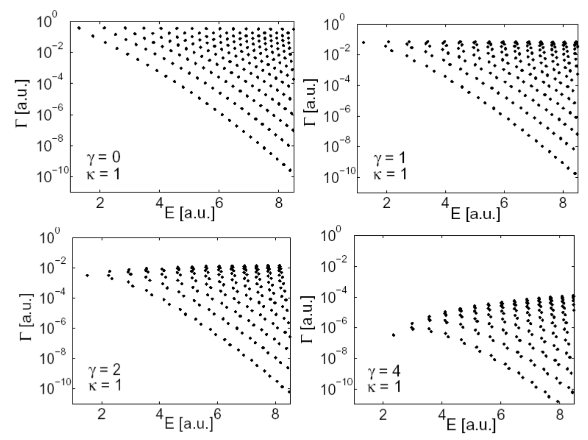
  → no symmetry related degeneracies occur

Resonance Energies and Decay Widths

- without Ioffe field (γ=0) a regular distribution of the resonance positions is observed
- systematic increase of the life times (decrease of the decay widths) with increaing Ioffe field strength

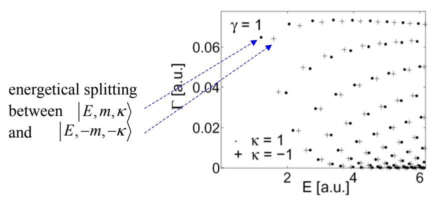
  → lifetime of the energetically lowest resonance decreases exponentially with growing γ

Resonance positions at 4 different Ioffe field strengths γ



Splitting of the degeneracies

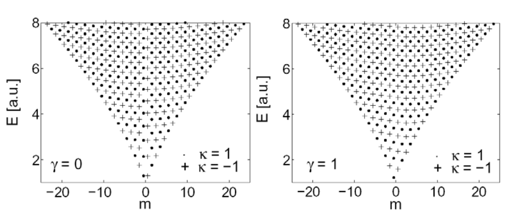
- the presence of a Ioffe field leads to a energetical splitting between formally degenerate pairs of states



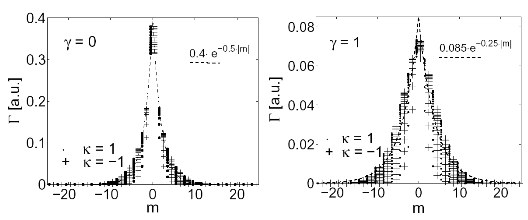
Λ_z-Expectation Value and Resonance Positions

Dependence of the resonance energy on the Λ_z-eigenvalue

- ground state energy in a given m-subspace grows linearly with increasing modulus of m
- pyramid-like distribution
- γ=0:degenerate pairs of states are located opposite to each other
- γ≠0: alternating pattern of states with different κ eigenvalues



Dependence of the decay width on the Λ_z-eigenvalue



- exponentially decreasing decay width (increasing lifetime) with increasing modulus of the Λ_z-eigenvalue
- global decrease of the decay widths if Ioffe field is present

Quasi-Bound States vs. Adiabatic Approximation

Unitary transformation of the Hamiltonian

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{1+\alpha} & \sqrt{1-\alpha} \\ \sqrt{1-\alpha} & -\sqrt{1+\alpha} \end{pmatrix} \qquad \alpha = \frac{\gamma}{\sqrt{\gamma^2+\rho^2}}$$

- diagonalizes the spin-field interaction term
- additional off-diagonal terms occur which involve negative powers of ρ

  → coupling between the bound and unbound solution takes place near the center of the guide

  → for sufficient high angular momenta the wavefunction is localized far away from the center of the guide

  → transitions to unbound states are inhibited

  → states are quasi-bound

Hamiltonian describing quasibound states

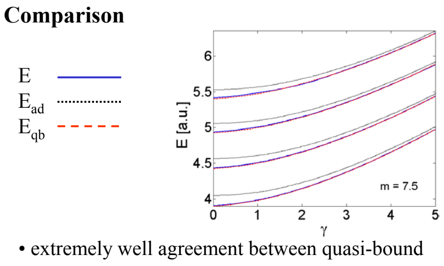
$$H_{qb} = \frac{1}{2}\left[-\frac{\partial^2}{\partial\rho^2} + \frac{m^2}{\rho^2} + \sqrt{\gamma^2+\rho^2} + \frac{m\gamma}{\rho^2\sqrt{\gamma^2+\rho^2}} + \frac{\gamma^2}{4\left(\gamma^2+\rho^2\right)^2}\right]$$

Hamiltonian in the adiabatic approximation

$$H_{ad} = \frac{1}{2}\left[-\frac{\partial^2}{\partial\rho^2} + \frac{l^2-\frac{1}{4}}{\rho^2} + \sqrt{\gamma^2+\rho^2}\right], \; l \in \mathbb{N}$$

- projection of the spin onto the local field direction is assumed to be conserved → scalar potential

Comparison



- extremely well agreement between quasi-bound and exact resonance energies
- with increasing strength of the Ioffe field the adiabatic approximation also begins to work well