Effects of Anisotropy in Control of Ultracold Atom-Atom Collisions by a Light Field

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Problem

controlling atom-atom interaction

at $\boldsymbol{k} = \sqrt{2M\varepsilon} \to 0$

sign of the s-wave scattering length

$$a_0 = -\frac{tan(\delta_0(k))}{k} \quad \textbf{at } k \to 0$$

Feschbach resonances induced by magnetic fields
radio frequency fields
near resonant lasers
static electric fields (10^5 V/cm)

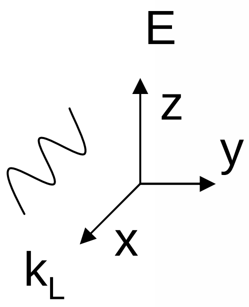
alternative: **nonresonant laser fields**

- strong anisotropy even at $k \to 0$
- nonseparability over scattering angles θ, ϕ

Melezhik & Chi-Yu Hu PRL (2003) 083202

peculiarity 1: $\Rightarrow$ 2D

strong anisotropy even at $k \longrightarrow 0$



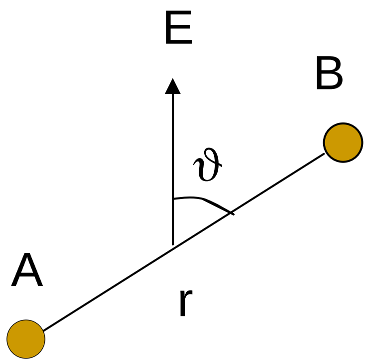
polarized laser

$$\boldsymbol{E}(t) = \boldsymbol{E}\cos(\boldsymbol{k_L r} - \omega_L t)$$

$$\boldsymbol{k}_L = k_L \boldsymbol{n}_X = \frac{\omega_L}{C}\boldsymbol{n}_X = \frac{2\pi}{\lambda_L}\boldsymbol{n}_X$$

quasistatic limit $\Rightarrow$ static electric field (Marinescu & You)

$k_L, \omega_L \longrightarrow 0 : \boldsymbol{E}(t) \Rightarrow E\boldsymbol{n}_Z$
$(\lambda_L \longrightarrow \infty)$



	short range ($\sim \frac{1}{r^6}$) atom-atom		long range electric field
$V(\boldsymbol{r}) =$	$V(\mid r \mid)$	$+$	$\frac{2E^2\alpha_1^A\alpha_1^B}{r^3}P_2(cos\theta)$
$T \sim nK,\ k \sim 10^{-4}$: (Cs-Cs)	$f_{l=0} \longrightarrow const$ $f_{l>0} \longrightarrow k^l \longrightarrow 0$		$f_{l>0} \longrightarrow const$

anisotropic nature $\Rightarrow$ new numerical technique
(storng coupling l, l')

partial-wave analysis: close-coupling Marinescu & You (1998)
multichannel quantum defect Deb & You (2000)

without partial-wave analysis: present work

$$V(\boldsymbol{r}) = V(r) + V_E(r,\hat{r}), \qquad \hat{r} = \{\theta,\phi\}$$

$r \to \infty$:

- $V(r) \to -\frac{C_6}{r^6} + \dots$

peculiarity 2: $\Rightarrow$ 3D

nonseparability over scattering angles θ, ϕ
(finiteness of λ_L, polarization $0 \le \gamma \le 1$)

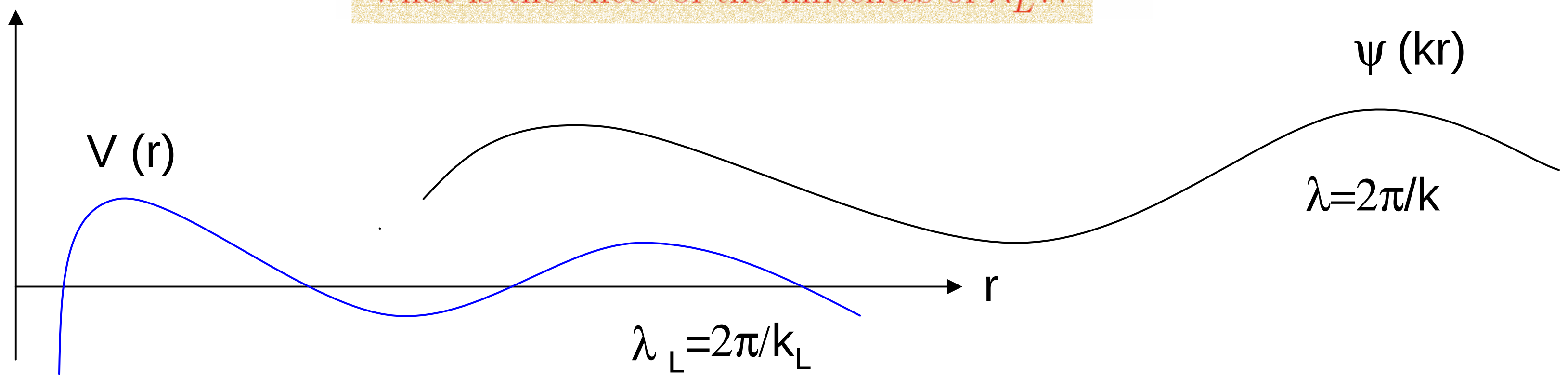
Cs-Cs collisions in ellipticaly polarized laser field

- $V_E(r,\hat{r}) = -\frac{\alpha^2 E^2}{2(1+\gamma^2)r^3}\{\frac{3(z^2+\gamma^2 y^2)}{r^2} - 1 - \gamma^2\}\cos(k_L x)\ \{\cos(k_L r) + k_L r \sin(k_L r)\}$

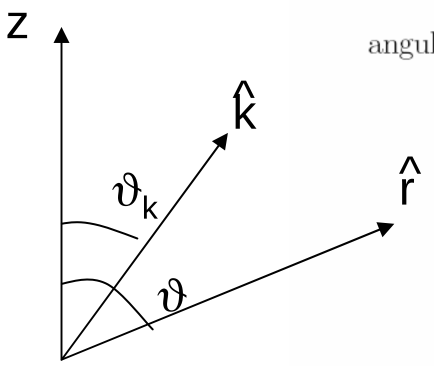
$T \sim nK,\ k \sim 10^{-4}$a.u. : $\Rightarrow \lambda = \frac{2\pi}{k} \sim 5 \times 10^3$nm de Broglie wave-length

optical laser : $\Rightarrow \lambda_L \sim 10^3$nm "modulation" length of Cs-Cs interaction

$\lambda_L \sim \lambda$:
what is the effect of the finiteness of λ_L??



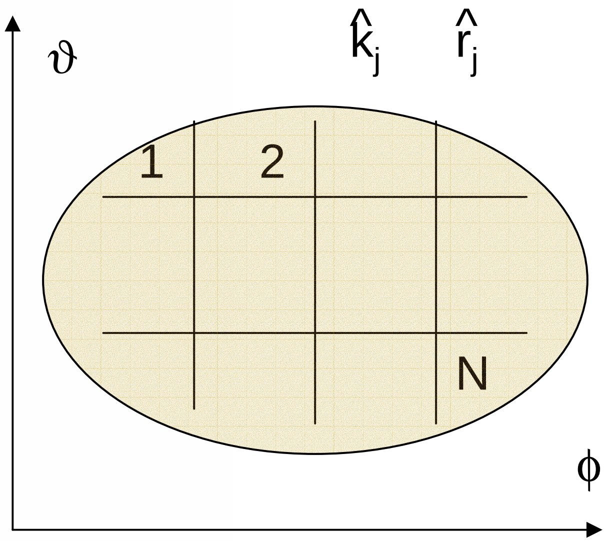
Method



angular space discretization $\Longleftrightarrow$ special 2D basis

$$\boldsymbol{k} = k\hat{\boldsymbol{k}} = \{k, \theta_k, \phi_k\}$$
$$\boldsymbol{r} = r\hat{\boldsymbol{r}} = \{r, \theta, \phi\}$$

$$\hat{\boldsymbol{k}} = \hat{\boldsymbol{k}}_j$$
$$\hat{\boldsymbol{r}} = \hat{\boldsymbol{r}}_j$$



$Y_\nu(\hat{\boldsymbol{r}}) = Y_{lm}(\theta, \phi) \qquad \nu = \{lm\} = 1, 2, ..., N$

$N \times N$ matrix: $\qquad Y_{\nu j} = Y_\nu(\hat{\boldsymbol{r}}_j)$

(1) $\psi(\boldsymbol{r}) = \frac{1}{r} \Sigma_{\nu j}^{N} Y_\nu(\hat{\boldsymbol{r}})(Y_{\nu j})^{-1}\psi_j(r)$ + O(1/N!)

2D interpolation

$\psi(r, \hat{\boldsymbol{r}}_i) = \frac{1}{r} \Sigma_{\nu j}^{N} Y_\nu(\hat{\boldsymbol{r}}_i)(Y_{\nu j})^{-1}\psi_j(r) = \psi_i(r)$

number of grid points N = number of basis functions N

2D Lagrange basis: $f_i(\hat{r}) = \Sigma_\nu Y_\nu(\vartheta,\phi)\,(Y_{\nu i})^{-1} \longrightarrow f_i(\hat{r}_j) = \delta_{ij}$

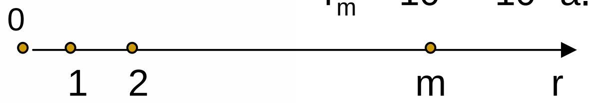
(1D Lagrange basis: Baye, Heenen 1986; 1D DVR: J.C.Light et al ~ 1980)

Algorithm

$\boldsymbol{\psi}(r) = \psi(r, \hat{\boldsymbol{r}}_i) \qquad i = 1, 2, ..., N$

$\hat{H}(r) = H_{ij}(r) = (-\frac{\partial^2}{\partial r^2} + V(r, \hat{\boldsymbol{r}}_i))\delta_{ij} + \frac{1}{r^2}\Sigma_{\nu=lm}^{N} Y_{i\nu} l(l+1)(Y_{\nu j})^{-1}$

$r_m \sim 10^4 – 10^5$ a.u.



boundary-value problem
($N\times$ m linear algebraic equations):

$$\begin{cases} (\hat{H}(r_n) - k^2)\boldsymbol{\psi}(r_n) = 0 & n = 1, 2, ...m-1 \\ \boldsymbol{\psi}(r_m) - \omega(k)\boldsymbol{\psi}(r_{m-1}) = g(k, \hat{\boldsymbol{k}}) & n = m \end{cases}$$

$\omega(k) = \frac{r_{m-1}}{r_m} exp\{ik(r_{m-1} - r_m)\}$

$g(k, \hat{\boldsymbol{k}}) = exp\{i(\boldsymbol{k}\boldsymbol{r}_m - 2kr_m)\}$

$\qquad -\frac{r_{m-1}}{r_m} exp\{i(\boldsymbol{k}\boldsymbol{r}_{m-1} - k(r_{m-1} - r_m)\}$

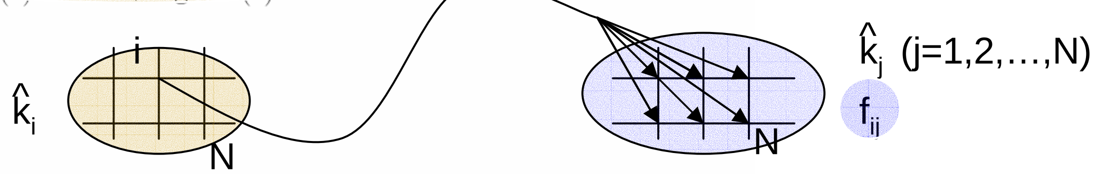
LU

(1) $\hat{k} = \hat{k}_i \qquad i = 1, 2, ...N$

(2) solve ● $\Rightarrow \qquad \boldsymbol{\psi}(r) = \psi(r, \hat{r}_j)$

(3) calculate $f_{ij}(k)$: $\quad f(k, \hat{k}_i, \hat{r}_j) = exp\{-ikr_m\}\psi(r_m, \hat{r}_j)$
$\qquad -r_m exp\{i(\boldsymbol{k}\boldsymbol{r}_m - kr_m)\}$

(4) $i = i + 1$ go to (1)



Results

ultracold Cs-Cs collisions in laser field quasistatic limit — static electric field (Marinescu & You)

$k_L = \frac{\omega_L}{C} = \frac{2\pi}{\lambda_L} \to 0$

$\boldsymbol{E}\cos(\boldsymbol{k_L r} - \omega_L t) \longrightarrow E\boldsymbol{n}_Z$

$$\sigma(k) = \frac{1}{4\pi}\Sigma_{i,j}^{N} \mid f(k, \hat{\mathbf{k}}_i, \hat{\mathbf{k}}_j) \mid^2 w_i w_j$$

$E = 0 \quad \Rightarrow \quad$ s-wave scattering length

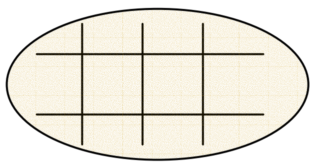
a_0=-2121 resonant

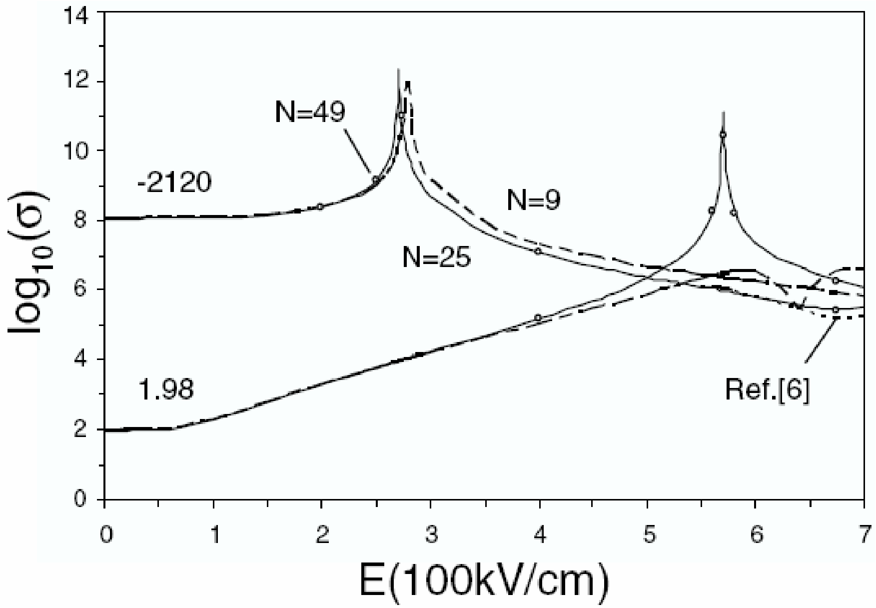
1.98 nonresonant

convergence $\Rightarrow \quad N \to \infty$

$$\psi(r, \hat{\boldsymbol{r}}_i) = \frac{1}{r}\Sigma_{\nu j}^{N} Y_\nu(\hat{\boldsymbol{r}}_i)(Y_{\nu j})^{-1}\psi_j(r)$$

$\hat{\boldsymbol{r}}_j$

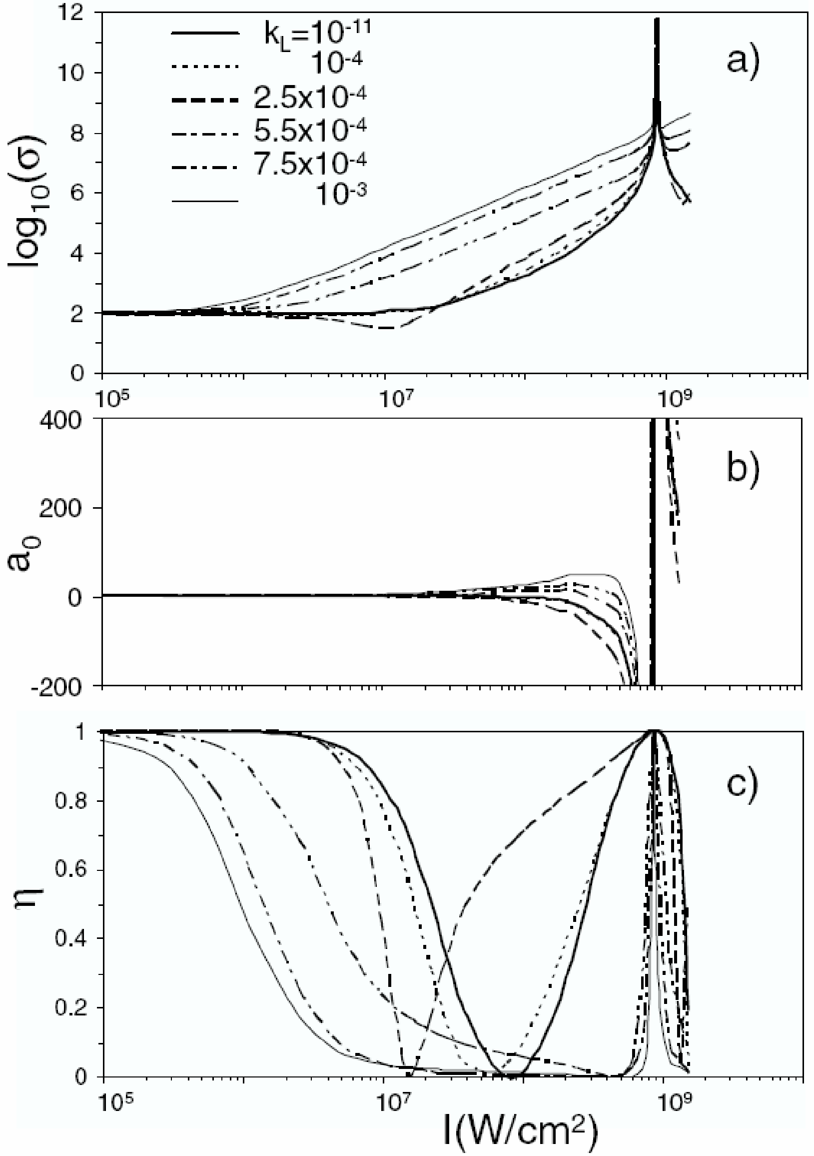




Ref.[6]: Marinescu & You PRL 1998

$k = 5.5 \times 10^{-6}$ (37 pK)

dependence on $I = \frac{cE^2}{8\pi}$ and k_L



$\gamma = 0 \quad k = 5.5 \times 10^{-6}$ (37 pK)

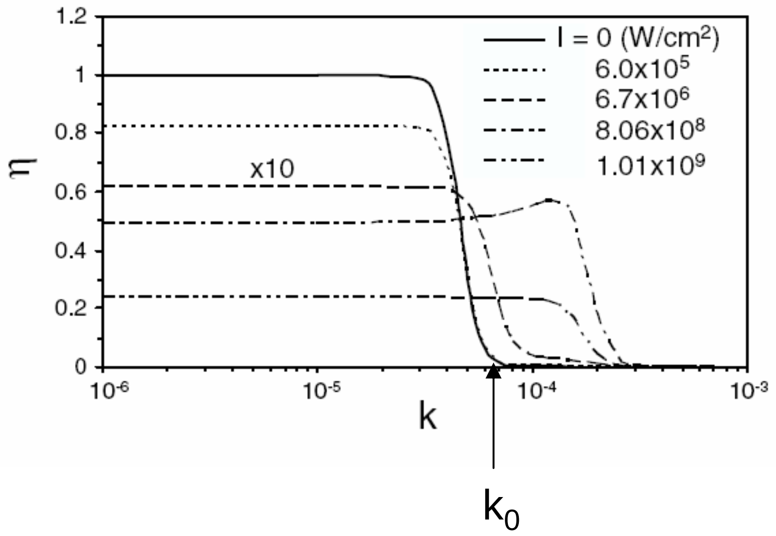
$(k_L = 5.5 \times 10^{-4}$ ⟷ 604 nm)

$$a_0(k) = \Sigma_{i,j}^{N} f(k, \hat{\mathbf{k}}_i, \hat{\mathbf{k}}_j) Y_{00}(\hat{\mathbf{r}}_i) Y_{00}^*(\hat{\mathbf{r}}_j) w_i w_j \xrightarrow[k \to 0]{} a_0$$

$\eta = \frac{8\pi a_0^2(k_L I)}{\sigma(k_L I)}$ anisotropy parameter

anisotropy is essential at:

$I \geq 10^5 \frac{W}{cm^2} \quad k_L \sim 10^{-3}$



violation of the scattering length approach

$$\mathbf{f}(k, \hat{k_i}, \hat{k_j}) \neq -a_0 \quad \text{at } \boldsymbol{k} \to 0$$

$(k_L = 5.5 \times 10^{-4} \longleftrightarrow 604 \text{ nm}, \quad \gamma = 0)$

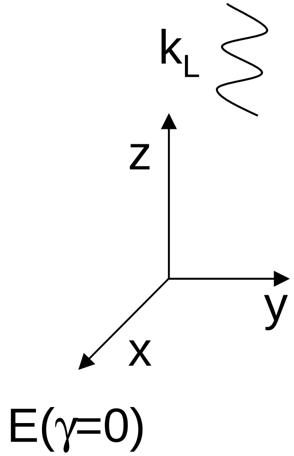
$k < k_0 \sim 10^{-4} \sim 10nK$ the region of s-wave domination as $I = 0$

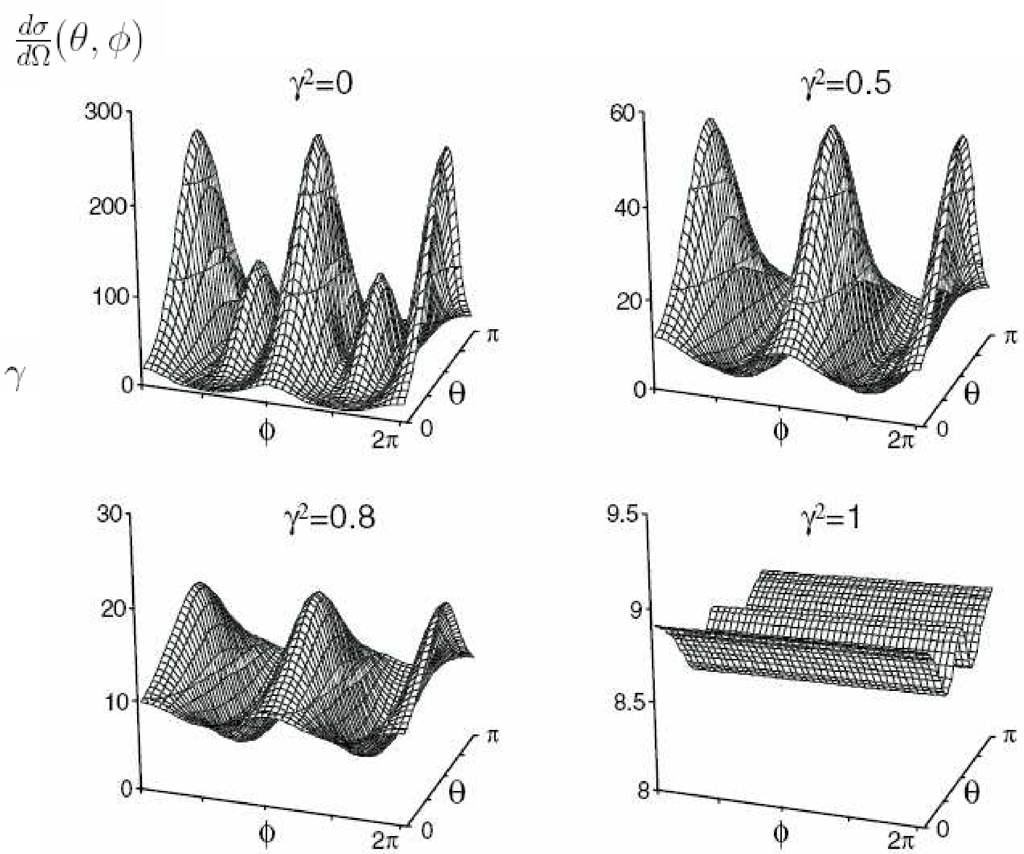
$$I \geq 10^5 \frac{W}{cm^2} \Longrightarrow \eta = \frac{8\pi a_0^2(k_L I)}{\sigma(k_L I)} < 1 \text{ even for } k < k_0 \text{ !}$$

$\eta = \frac{8\pi a_0^2(k_L I)}{\sigma(k_L I)}$ is k-independent value as $k < k_0$

$$\frac{d\sigma}{d\Omega}(\theta_j, \phi_j) = \frac{1}{4\pi}\Sigma_i^N \mid f(\hat{\mathbf{k}}_i, \hat{\mathbf{k}}_j) \mid^2 w_i$$

dependence on θ and ϕ at $k \rightarrow 0$!!





dependence on laser polarization γ

$\mathrm{f}(k, \hat{k_i}, \hat{k_j}) \neq -a_0$

$$I = 1.07 \times 10^6 \frac{W}{cm^2}(28\frac{kV}{cm})$$

$$k_L = 5.5 \times 10^{-4}(604nm)$$

$$k = 10^{-6}$$

Conclusion

- **dependence of Cs-Cs ultracold collisions on I and λ_L**

  at $I \geq 10^5 \frac{W}{cm^2}$ and $\lambda_L \leq 3000$ nm :

  scattering length approach does not work

  $\mathbf{f}(k, \hat{k}_i, \hat{k}_j) \neq -a_0$

- **other possible applications:**

  atom-atom in crossed $\boldsymbol{E}$ and $\boldsymbol{B}$

  metastable alkaline atoms (quadrupoles)

  polar molecules (dipolar interaction)

  three-body problem without partial-wave analysis

  ...