Deformed Fermi surfaces in ultracold Fermi gases

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The superfluid transition in an ultracold two-component atomic Fermi gas is analyzed in the case where the two components have different densities. We describe a superfluid state which spontaneously breaks the rotational-symmetry by deforming the Fermi surfaces of both species into ellipsoidal form. At relatively large hyperfine-spin asymmetries (e.g., around 20% for $k_F|a| = 1$), this deformation is shown to help the appearance of pairing, which in the rotationally-symmetric (BCS) case would be forbidden by Pauli blocking. The prospects for experimental detection of such a deformed Fermi surface (DFS) phase are discussed.

Introduction

The present capabilities of cooling of fermionic atomic ensembles to temperatures that are a fraction $\sim 0.1-0.3$ of their Fermi temperature [1] allow for reasonable **expectations to observe a superfluid transition in ultracold fermionic systems**, in direct analogy to BCS superconductivity [2], as the trapped atoms are in the quantum-degenerate regime where an attractive interaction can drive the Cooper instability. Moreover, the strength of the two-body interactions can be tuned using a Feshbach resonance by varying an external magnetic field [3]; thus the entire range from weak to strong couplings can be probed.

At the very low temperatures and densities reached in the experiments, only s-wave collisions (characterized by the scattering length a), are relevant for the description of these systems. Since Pauli's principle forbids s-wave interactions between indistinguishable fermions, the pairing should appear between atoms in different hyperfine states, as in the experiments realized with ^{6}Li and ^{40}K [1,4,5].

The **BCS theory predicts a suppression of the pairing correlations** when the Fermi energies or, equivalently, the densities of the two hyperfine states are different. In the low density limit $k_F|a| \ll 1$ the dependence of the pairing gap Δ at the Fermi surface (FS) on the total density ρ and density asymmetry $\alpha = (\rho_1 - \rho_2)/\rho$ is given by [6]

$$\frac{\Delta_{FS}(\alpha)}{\Delta_0} = \sqrt{1 - \frac{4\mu}{3\Delta_0}\alpha}, \qquad (1)$$

where $\Delta_0 \simeq 8e^{-2}\mu \exp\left[-\pi/(2k_F|a|)\right] \ll 1$ is the gap in symmetric matter and μ is the chemical potential. Therefore, **the (BCS) gap disappears for asymmetries $\alpha > \alpha_{\rm max} = 3\Delta_0/(4\mu)$, which in this limit is a very small number**. For example, for the pairing of ^{6}Li atoms in the hyperfine states $|1\rangle = |F = 3/2, m_F = 3/2\rangle$ and $|2\rangle = |3/2, 1/2\rangle$, for which the triplet scattering length is $a = -2160a_B$ (a_B = Bohr radius), at a density $\rho = 3.8 \times 10^{12}$ cm^{-3} ($\Rightarrow k_F|a| = 0.55$), the maximum asymmetry at which BCS pairing is possible is only $\alpha_{\rm max} = 0.07 \equiv 7\%$.

In this work we show that the superfluid state in ultracold atomic gases can persist for density asymmetries $\alpha > \alpha_{\rm max}$ due to a spontaneous deformation of the FS's of the two-hyperfine states in momentum space, thus breaking the global rotational symmetry of the space from O(3) down to O(2) [7].

The model

Consider a uniform gas of Fermi atoms with two hyperfine states, which we assign labels 1 and 2. The model Hamiltonian that describes our system is

$$\hat H = \sum_{\boldsymbol p,\sigma} \epsilon_{\boldsymbol p} \hat a^\dagger_{\boldsymbol p\sigma}\hat a_{\boldsymbol p\sigma} - g\sum_{\boldsymbol p\boldsymbol p'} \hat a^\dagger_{\boldsymbol p',1}\hat a^\dagger_{-\boldsymbol p',2}\hat a_{-\boldsymbol p,2}\hat a_{\boldsymbol p,1}, \qquad (2)$$

where $\hat a^\dagger_{\boldsymbol p\sigma}$ and $\hat a_{\boldsymbol p\sigma}$ are the creation and annihilation operators of a state with momentum $\boldsymbol p$, pseudospin $\sigma(=1,2)$ and energy $\epsilon_{\boldsymbol p} = p^2/2m$, where m is the atom bare mass. The coupling constant is determined by the s-wave scattering length $a < 0$ as $g = 4\pi\hbar^2|a|/m$.

The mean-field solutions for the Hamiltonian (2) can be obtained by diagonalizing it with the familiar Bogolyubov transformations: $\hat b_{\boldsymbol p,1} = u_p \hat a_{\boldsymbol p,1} + v_p \hat a^\dagger_{-\boldsymbol p,2}$ and $\hat b_{\boldsymbol p,2} = u_p \hat a_{\boldsymbol p,2} - v_p \hat a^\dagger_{-\boldsymbol p,1}$, with $u^2_{\boldsymbol p} + v^2_{\boldsymbol p} = 1$. A variational minimization of the energy with respect to the parameter $u_{\boldsymbol p}$ (or $v_{\boldsymbol p}$) leads to the gap equation

$$\Delta = g\int \frac{d\boldsymbol p}{(2\pi)^3}\, u_{\boldsymbol p}\, v_{\boldsymbol p}\left[1 - f(E_1) - f(E_2)\right], \qquad (3)$$

where $f(E) = [1 + \exp(E/T)]^{-1}$, T is the temperature. The quasiparticle spectra are defined as

$$E_{1/2} = \sqrt{\xi_S^2 + \Delta^2} \pm \xi_A, \qquad (4)$$

where the symmetrized $\xi_S = \frac{1}{2}(\varepsilon_1 + \varepsilon_2)$ and anti-symmetrized $\xi_A = \frac{1}{2}(\varepsilon_1 - \varepsilon_2)$ spectra are written in terms of the normal state spectra $\varepsilon_\sigma = \epsilon_{\boldsymbol p} - \mu_\sigma$, and $u^2_{\boldsymbol p} = [1 + 2\xi_S/(E_1 + E_2)]$. The occupations of the states in the superfluid phase, $n_{\boldsymbol p,\sigma} = u^2_{\boldsymbol p} f(E_\sigma) + v^2_{\boldsymbol p}[1 - f(E_{-\sigma})]$, obey the normalization conditions $\rho_\sigma = \sum_{\boldsymbol p} n_{\boldsymbol p,\sigma}$, which are ensured in the calculations by adjusting the chemical potentials μ_σ of the hyperfine states.

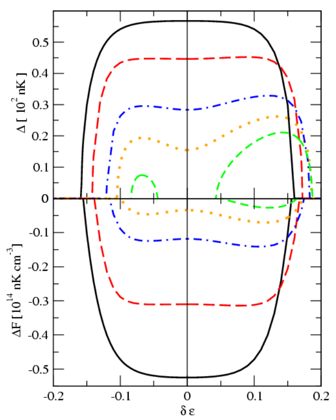

***FIG. 1** Dependence of the gap (upper panel) and the free-energy difference (lower panel) on $\delta\epsilon$ for $k_Fa = 0.55$, at $T = 10$ nK, $\rho = 3.8 \times 10^{12}$ cm^{-3}, and $\alpha = 0.0$ (black line), $\alpha = 0.02$ (red), $\alpha = 0.04$ (blue), $\alpha = 0.05$ (yellow) and $\alpha = 0.057$ (green).*

Deforming the Fermi surfaces

Fermi surfaces (FS's) are defined by $\varepsilon_\sigma = \epsilon_{\boldsymbol p} - \mu_\sigma = 0$. When the chemical potentials are isotropic in $\boldsymbol k$-space, the FS's are spherical. Relaxing this assumption, we expand the quasiparticle spectra in spherical harmonics $\varepsilon_\sigma = \sum_l \varepsilon_{l\sigma} P_l(x)$, where x is the cosine of the angle formed by the particle momentum and a randomly chosen symmetry breaking axis; $P_l(x)$ are the Legendre polynomials. The $l = 1$ terms break the translational symmetry by shifting the FS's without deforming them; they are ignored below. Truncating the expansion at $l = 2$, we rewrite the spectra as [7]

$$\varepsilon_\sigma = \epsilon_{\boldsymbol p} - \mu_\sigma\left(1 + \eta_\sigma x^2\right), \qquad (5)$$

where the parameters η_σ describe the quadrupolar deformation of the FS's. It is convenient to work with $\delta\epsilon = (\eta_1 - \eta_2)/2$ and $\Xi = (\eta_1 + \eta_2)/2$.

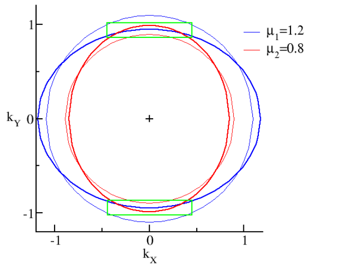

***FIG. 2** FS's for an asymmetric system in the undeformed (thin) and deformed (thick) cases. The green boxes mark the regions where pairing occurs.*

We study the energy of the superfluid state at finite deformations to see whether they lower the energy of the system and lead to a new stable ground state. We work at $\Xi = 0$ and fixed T, and look into the free energies of the deformed superfluid state and the undeformed normal one, as a function of the single parameter $\delta\epsilon$. For the superfluid phase we have $F_S = E_{\rm kin} + E_{\rm pot} - TS_S$, with the entropy S_S defined by the well-known combinatorial expression, $E_{\rm kin} = \sum_{\boldsymbol p,\sigma}\epsilon_{\boldsymbol p} n_{\boldsymbol p,\sigma}$ and, for a contact interaction, $E_{\rm pot} = -\Delta^2/g$. For the undeformed normal state, set $\Delta = 0 = \delta\epsilon$. As the interaction is of the contact form, the gap equation and the superfluid kinetic energy need a regularization. The regularized gap equation is

$$\frac{2}{g} = \int_0^\Lambda \frac{dpp^2}{(2\pi)^2}\int_{-1}^1 dx\left(\frac{1}{\sqrt{\xi_S^2 + \Delta^2}} - \frac{\gamma}{\epsilon_{\boldsymbol p}}\right)\times \left[1 - f(E_1) - f(E_2)\right]; \qquad (6)$$

$\gamma = 1$ and $\Lambda \to \infty$ corresponds the common practice [2] while $\gamma = 0$ and finite Λ corresponds to the cut-off regularization of the gap equation, with which $E_{\rm kin}$ is evaluated. Λ is fixed by requiring both regularization schemes to give the same gap.

Results and detection

Eq. (6) was solved numerically for given ρ, α for a coupling $k_Fa = 0.55$ at density $\rho = 3.8 \times 10^{12}$ cm^{-3} ($T_F = 942$ nK) and $T = 10$ nK. For $\delta\epsilon = 0$, ξ_A reduces in Eq. (3) the phase space coherence between the quasiparticles that pair. This blocking is responsible for the reduction of the gap with increasing asymmetry and its disappearance above $\alpha \simeq 7\%$. **Allowing for deformations introduces a modulation of ξ_A with the cosine x of the polar angle restoring the phase space coherence for some values of x (and lowering it for the remainder values). The result (Fig. 1) is an *increase* of the gap for finite deformations.** At extreme large asymmetries the gap exists only for the deformed state, with lower and upper critical deformations marking the pairing regions. [$\delta\epsilon > 0$ corresponds to a cigar-like deformation of the majority and pancake-like deformation of the minority population's FS's (Fig. 2).]

The same calculations were carried out **for larger couplings $k_Fa = 1$ and $k_Fa = 2$, obtaining larger gaps and allowed density asymmetries: the gaps found in the symmetric case are 193 and 375 nK**, respectively, and the reentrance effect is observed in each case for **asymmetries around 18% and 30%**, which are rather large values that should be possible to observe experimentally by the anisotropy of the momenta distributions of both species in typical time-of-flight measurements (Fig. 3). The pairing disappears above asymmetries 22% and 43%, respectively.

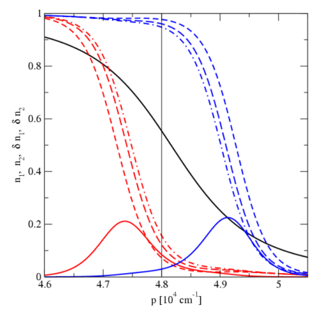

***FIG. 3** Momentum occupations for $k_Fa = 0.55$ for $\alpha = 0 = \delta\epsilon$ (solid); $\alpha = 0.05$ and $\delta\epsilon = 0$ (dashed), $\delta\epsilon = 0.1$, $x = 0$ (dashed-dotted) and $x = 1$ (short-dashed). The lower curves show the difference between the $x = 1$ and $x = 0$ occupations for $\alpha = 0.05$, $\delta\epsilon = 0.1$.*

[1] B. DeMarco and D. S. Jin, Science (1999); [2] H. T. C. Stoof *et al.*, PRL (1996). [3] W. C. Stwalley, PRL (1976); E. Tiesinga *et al.*, PRA (1993); Ph. Courteille *et al.*, PRL (1998). [4] C. A. Regal *et al.*, PRL (2004). [5] M. W. Zwierlein *et al.*, PRL (2004). J. Kinast *et al.*, PRL (2004). [6] JMP *et al.*, PLA (2001); U. Lombardo *et al.*, PRC (2001). [7] H. Müther and A. Sedrakian, PRL (2002); PRC (2003).