The superfluidity phenomenon:
Is everything understood ???

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Poll on the superfluidity phenomenon

Do you think that the absence of damping of a relative motion between the normal fluid and superfluid is fully understood ?

	Yes	◯
Answer:	No	◯
	I don't know	◯

Abstract:

We present the microscopic kinetic theory of a homogeneous dilute Bose condensed gas in the generalized random phase approximation (GRPA), which satisfies the following requirements: 1) the mass, momentum and energy conservation laws; 2) the H-theorem; 3) the superfluidity property and 4) the recovery of the Bogoliubov theory at zero temperature. Contrary to previous approaches, these requirements impose a totally different understanding on the superfluidity phenomenon. Indeed, as long as the Bose gas is stable, no binary collision happens between condensed and normal atoms due to the ability of the condensate wavefunction to attenuate totally the interatomic forces. As a consequence, no relaxation of any initial relative velocity between the normal and superfluid occurs and the superfluid moves without any friction. Furthermore, the condensate influences the binary collisional process between the two normal atoms, in the sense that their interaction force results from the mediation of a Bogoliubov collective excitation traveling throughout the condensate. In this paper, we discuss about a 'time of flight' experiment which could allow to validate the GRPA approach.

cond-mat/0406033, cond-mat/0309319

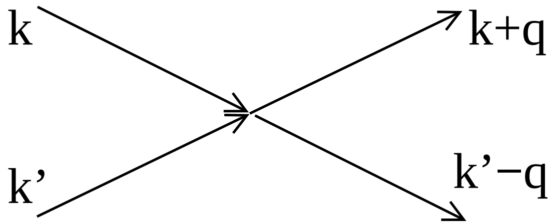
" Do not forget Boltzmann when describing a BEC "

$$S = k \ln W$$

$$\frac{dH(t)}{dt} \leq 0$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v_1}.\nabla_{\mathbf{r}} + \frac{\mathbf{F}}{m}.\nabla_{\mathbf{v_1}}\right) f_1 = \int d\Omega \int d^3\mathbf{v_2}\sigma(\Omega)|\mathbf{v_1} - \mathbf{v_2}|(f_2' f_1' - f_2 f_1)$$

Previous works: 1) Boltzmann-Nordheim QKE



Bose Enhancement → Stimulated scattering

$$\frac{\partial}{\partial t} n_{\mathbf{k}} = \sum_{\mathbf{q},\mathbf{k}'} \left(\frac{8\pi a}{mV}\right)^2 \pi\delta(\epsilon_{\mathbf{k}+\mathbf{q}}+\epsilon_{\mathbf{k}'-\mathbf{q}}-\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}'})[n_{\mathbf{k}+\mathbf{q}}n_{\mathbf{k}'-\mathbf{q}}(n_{\mathbf{k}}+1)(n_{\mathbf{k}'}+1)-n_{\mathbf{k}}n_{\mathbf{k}'}(n_{\mathbf{k}+\mathbf{q}}+1)(n_{\mathbf{k}'-\mathbf{q}}+1)]$$

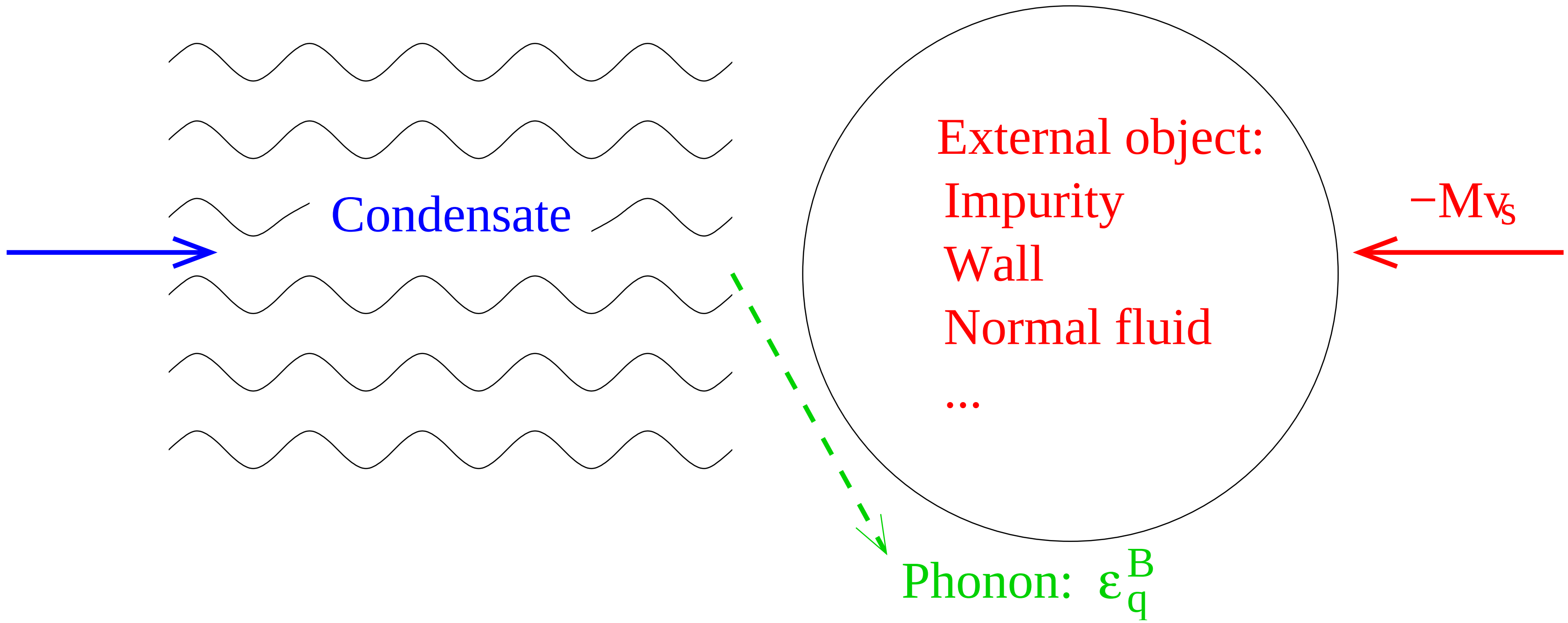
where $\epsilon_{\mathbf{k}} = \mathbf{k}^2/(2m)$

- Superfluidity NOT explained !!! always damping of relative velocity

- H-theorem $\Rightarrow S_0 \sim \ln n_0 \Rightarrow$ Huge fluctuations $\delta n_0/n_0 \sim 1$
  $\longrightarrow$ GC ensemble pathology

$\Rightarrow$ Need higher order theory !!!

Landau criterion $\longrightarrow$ necessary condition



Moving ext. obj.: $-\mathbf{v}_s \longrightarrow -\mathbf{v}'_s$

Energy-momentum conservation: $M{\mathbf{v}_s}^2/2 - M{\mathbf{v}'_s}^2/2 = \epsilon^B_{\mathbf{q}} \quad -M\mathbf{v}_s = -M\mathbf{v}'_s + \mathbf{q}$

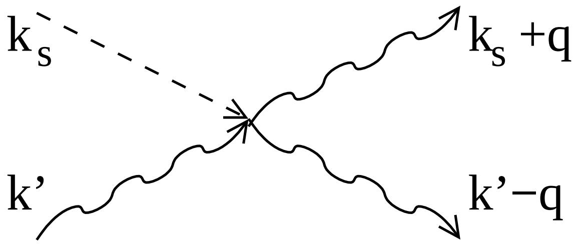
$\Rightarrow$ Moving condensate frame: $\omega_{\mathbf{q}} = \epsilon^B_{\mathbf{q}} + \mathbf{v}_s.\mathbf{q} \geq 0$

$\longrightarrow$ normal fluid considered as a WHOLE

2) Bogoliubov-like kinetic theory

Spontaneous symmetry breaking $U(1)$: $\epsilon^B_{\mathbf{q}} = \sqrt{c^2\mathbf{q}^2 + (\frac{\mathbf{q}^2}{2m})^2} \Rightarrow \omega_{\mathbf{q}} = \epsilon^B_{\mathbf{q}} + \mathbf{k}_s.\mathbf{q}$

Beliaev damping: $\omega_{\mathbf{k}'} = \omega_{\mathbf{k}_s+\mathbf{q}} + \omega_{\mathbf{k}'-\mathbf{q}}$ $\rightarrow$ balanced process



$\Rightarrow$ persistent relative velocity if $\omega_{\mathbf{q}} \geq 0$
i.e. $|\mathbf{v}_s| = |\mathbf{k}_s|/m \leq c = \sqrt{4\pi a N/m^2 V}$ (sound velocity)
otherwise instability

Kinetic theory involving ONLY Goldstone bosons
Imamovic-Tomasovic & Griffin, J. Low Temp. Phys. **122**, 617 (2001)

Do not forget Lavoisier (1743-1794)
when describing a BEC

"Matter is neither created or destroyed"

Problem with conservation law for particle number

If $f_{\mathbf{k}}$ Bogoliubov excitations distribution
$u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are the parameters of the Bogoliubov transformation

Beliaev process conserves

$$n_{\mathbf{k}_s} + \sum_{\mathbf{k}}{}' f_{\mathbf{k}}$$

incompatible with the conservation of the total particle number

$$N = \sum_{\mathbf{k}} n_{\mathbf{k}} = n_{\mathbf{k}_s} + \sum_{\mathbf{k}}{}' (u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2) f_{\mathbf{k}} + v_{\mathbf{k}}^2$$

Kinetic theory of a Bose condensed gas (BEC)

"The situation is not entirely clear" A. Leggett, Rev.Mod.Phys. **73**, 307(2001)
(Nobel Prize 2003)

THE CHALLENGE $\rightarrow$ Derivation in a systematic way of kinetic equations (QKE)
for quasi-particle distribution function valid for a dilute Bose gas and verifying:

1. Conservation laws: mass, momentum and energy density
   $\Rightarrow$ Hydrodynamic equations

2. Second principle of thermodynamics (H-theorem)

3. Superfluidity

4. Bogoliubov theory at zero temperature

Do not forget Hohenberg and Martin (Ann. Phys. **34**, 291 (1965)) when describing a BEC at finite temperature

"Gapless vs. Conserving"

1. Girardeau & Arnowitt (HFB), not gapless
2. Bogoliubov, not conserving
3. Beliaev, not conserving
4. Popov approximation, not conserving
5. Fliesser, Reidl, Szépfalusy, Graham (RPA), OK ! ! !

   Phys. Rev. A **64** 013609 (2001)

RPA $\rightarrow$ far field limit: $na^3/V \ll 1$

Excitation operator: $\rho_{\mathbf{q}} = \sum_{\mathbf{k}} \rho_{\mathbf{k},\mathbf{q}} = \sum_{\mathbf{k}} c^{\dagger}_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}} \rightarrow \rho_{\mathbf{k},\mathbf{q}\neq 0} \ll \rho_{\mathbf{k},0} = \hat{n}_{\mathbf{k}}$

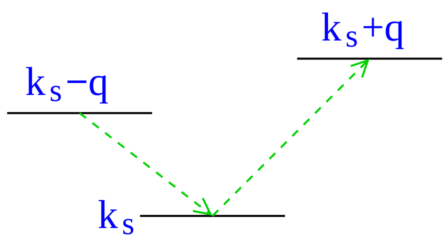
At equilibrium: $\langle \rho_{\mathbf{q}} \rangle^{eq} = \delta_{\mathbf{q},\mathbf{0}} n_{\mathbf{k_s}}$

Response to an external perturbation: $\phi_{ext}(\mathbf{q},\omega)$

$$\delta n(\mathbf{q},\omega) = \langle \delta\rho_{\mathbf{q}} \rangle = \langle \rho_{\mathbf{q}} \rangle - \langle \rho_{\mathbf{q}} \rangle^{eq} = \chi_0(\mathbf{q},\omega)\phi_{tot}(\mathbf{q},\omega) = \frac{\chi_0(\mathbf{q},\omega)}{1-\chi_0(\mathbf{q},\omega)U_{\mathbf{q}}/V}\phi_{ext}(\mathbf{q},\omega)$$

Susceptibility:

$$\chi_0(\mathbf{q},\omega) = \frac{n_{\mathbf{k_s}}}{\omega - (\epsilon_{\mathbf{k_s}+\mathbf{q}} - \epsilon_{\mathbf{k_s}}) + i0_+} - \frac{n_{\mathbf{k_s}}}{\omega + (\epsilon_{\mathbf{k_s}} - \epsilon_{\mathbf{k_s}-\mathbf{q}}) - i0_+}$$



Induced potential: $\phi_{tot}(\mathbf{q},\omega) = \phi_{ext}(\mathbf{q},\omega) + (U_{\mathbf{q}}/V)\langle \delta\rho_{\mathbf{q}} \rangle$

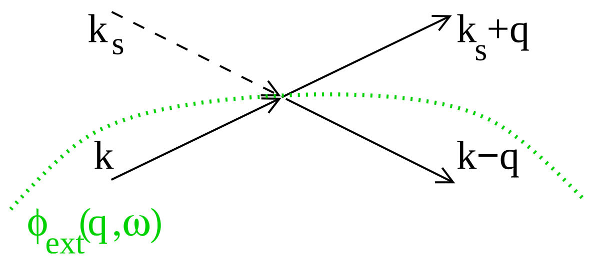
Dynamic dielectric function:

$$\tilde{\mathcal{K}}(\mathbf{q},\omega) = \frac{\phi_{ext}(\mathbf{q},\omega)}{\phi_{tot}(\mathbf{q},\omega)} = 1 - \frac{U_{\mathbf{q}}}{V}\chi_0(\mathbf{q},\omega) \quad \rightarrow \begin{cases} 0 & \text{resonance at Bog. frequency} \\ \infty & \text{no response} \end{cases}$$

Assume

- $\phi_{ext}(\mathbf{q},\omega)$ originates from
  an excited particle transition $\mathbf{k} \rightarrow \mathbf{k} - \mathbf{q}$

- Golden rule $\epsilon_{\mathbf{k_s}} + \epsilon_{\mathbf{k}} = \epsilon_{\mathbf{k_s}+\mathbf{q}} + \epsilon_{\mathbf{k}-\mathbf{q}}$



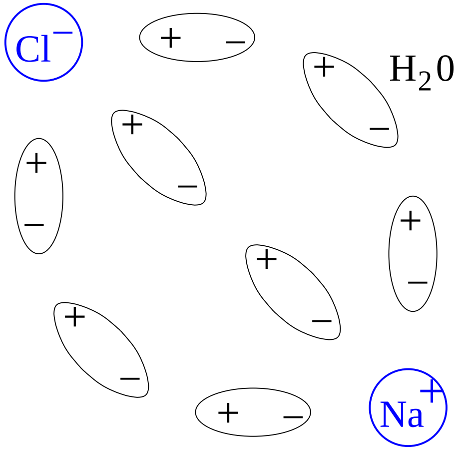
$$\Rightarrow \tilde{\mathcal{K}}(\mathbf{q},\omega) \rightarrow \infty \Rightarrow \phi_{tot}(\mathbf{q},\omega) \rightarrow 0$$

No interaction potential anymore !!! ⇒ NO COLLISION

Remarks:

a) Same results for condensed outgoing particle
b) Two conditions for this attenuation:
  -Reservoir of potential energy of the condensate
  -Macroscopic and coherent condensate

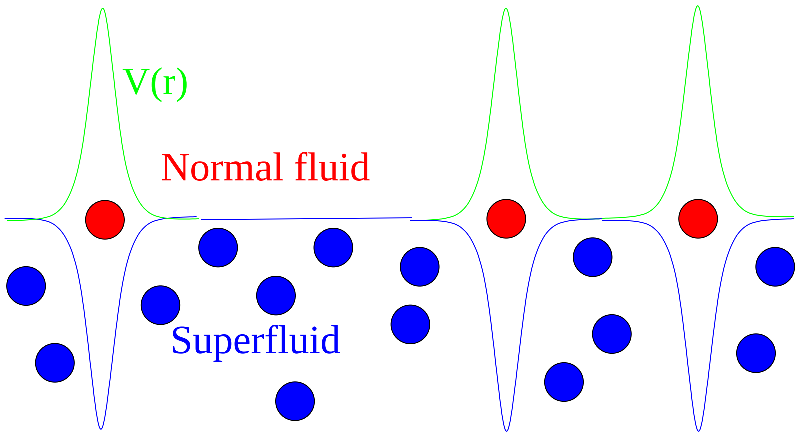
RPA −> Dielectric => Attenuation of interaction forces



$V_{eff}(r)=V_{Coul}(r)/\ \kappa$

Water: $\kappa = 80$

=> Dissociation of NaCl



$\kappa \to \infty$

$V_{eff}(r)=V(r)/\ \kappa \to 0$

No Collisions !!!

Generalized RPA (see Nozières and Pines)

$i\frac{\partial}{\partial t}\rho_{\mathbf{k},\mathbf{q}} = [\rho_{\mathbf{k},\mathbf{q}}, H] \Rightarrow$ Two equations of motion for operators:

1) Quasi-particle number equation

$$i\frac{\partial}{\partial t}\widehat{n}_{\mathbf{k}} = \sum_{\mathbf{q}'} \frac{U_{\mathbf{q}'}}{2V}[\rho_{\mathbf{k},\mathbf{q}'} - \rho_{\mathbf{k}-\mathbf{q}',\mathbf{q}'}, \rho^{\dagger}_{\mathbf{q}'}]_{+}$$

2) Excitation equation in GRPA: $\rho_{\mathbf{k},\mathbf{q}} \ll n_{\mathbf{k}}$ (→ linearized Q Vlassov Eq.)

$$\left[i\frac{\partial}{\partial t} - (\epsilon^{HFA}_{\mathbf{k}+\mathbf{q}} - \epsilon^{HFA}_{\mathbf{k}})\right]\rho_{\mathbf{k},\mathbf{q}} = (n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{q}})\sum_{\mathbf{k}'\neq\mathbf{k}} \frac{U_{\mathbf{q}}}{V}\rho_{\mathbf{k}',\mathbf{q}}$$

$$+ n_{\mathbf{k}} \sum_{\mathbf{k}'\neq\mathbf{k}-\mathbf{q}} \frac{U_{\mathbf{k}-\mathbf{k}'}}{V}\rho_{\mathbf{k}',\mathbf{q}} - n_{\mathbf{k}+\mathbf{q}} \sum_{\mathbf{k}'\neq\mathbf{k}+\mathbf{q}} \frac{U_{\mathbf{k}-\mathbf{k}'}}{V}\rho_{\mathbf{k}',\mathbf{q}}$$

where $\langle\widehat{n}_{\mathbf{k}}\rangle = n_{\mathbf{k}}$ and

$$\epsilon^{HFA}_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m} + \sum_{\mathbf{k}'} \frac{U_0}{V} n_{\mathbf{k}'} + \sum_{\mathbf{k}'} \frac{U_{\mathbf{k}-\mathbf{k}'}}{V} n_{\mathbf{k}'}$$

$\longrightarrow$ 1) scattering solution $\omega_{\mathbf{k},\mathbf{q}} = \epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}}$

2) scattering solution $\omega_{\mathbf{k_s},\mathbf{q}}$ and $\omega_{\mathbf{k_s}-\mathbf{q},\mathbf{q}}$ $\longrightarrow$ collective solution $\Delta(\mathbf{q},\omega) = 0$

For a contact potential:

$$\Delta(\mathbf{q},\omega) = \mathcal{K}_n(\mathbf{q},\omega)[(\omega + i0_+ - \frac{\mathbf{k_s}.\mathbf{q}}{m})^2 - {\epsilon_{\mathbf{q}}^B}^2] + (\mathcal{K}_n(\mathbf{q},\omega) - 1)\frac{8\pi a n_{\mathbf{k_s}}}{mV}\frac{\mathbf{q}^2}{m}$$

where

$$\epsilon_{\mathbf{q}}^B = \sqrt{\frac{4\pi a n_{\mathbf{k_s}}}{mV}\frac{\mathbf{q}^2}{m} + (\frac{\mathbf{q}^2}{2m})^2} \quad \longrightarrow \text{Bogoliubov excitation energy}$$

$$\mathcal{K}_n(\mathbf{q},\omega) = 1 - \frac{8\pi a}{mV}\sum_{\mathbf{k}} \frac{n'_{\mathbf{k}} - n'_{\mathbf{k}+\mathbf{q}}}{\omega + i0_+ - \frac{\mathbf{k}.\mathbf{q}}{m} - \frac{\mathbf{q}^2}{2m}} \quad \longrightarrow \text{Dielectric function of normal fluid}$$

Collective solution $\omega = \omega_{\mathbf{q}} + i\gamma_{\mathbf{q}}$ for weak depletion $\Rightarrow \gamma_{\mathbf{q}} \ll \epsilon_{\mathbf{q}}^B$:

$$\omega_{\mathbf{q}}^{\pm} \simeq \frac{\mathbf{k_s}.\mathbf{q}}{m} \pm \epsilon_{\mathbf{q}}^B \qquad \gamma_{\mathbf{q}}^{\pm} \simeq \pm \mathrm{Im}\mathcal{K}_n(\mathbf{q},\omega_{\mathbf{q}}^{\pm})\frac{\frac{4\pi a n_{\mathbf{k_s}}}{mV}\frac{\mathbf{q}^2}{2m}}{\epsilon_{\mathbf{q}}^B} \longrightarrow \text{Landau damping}$$

Kinetic Equation for a stable gas ($\gamma_{\mathbf{q}} \geq 0$)

Approximations:

1) Homogeneous Bose gas
2) Thermodynamic limit
3) Generalized RPA
4) Instantaneous collisions (Markovian QKE)
5) No fragmentation of the condensate
   (one macroscopic mode)

Correlation function: $\langle \rho_{\mathbf{k}',-\mathbf{q}} \rho_{\mathbf{k},\mathbf{q}} \rangle = (n_{\mathbf{k}}+1) n_{\mathbf{k}'} \delta_{\mathbf{k}'-\mathbf{k},\mathbf{q}} + n_{\mathbf{k}} n_{\mathbf{k}'} \delta_{\mathbf{q},0} (1-\delta_{\mathbf{k},\mathbf{k}'}) - \delta_{\mathbf{q},0} \delta_{\mathbf{k},\mathbf{k}'} n_{\mathbf{k}} + g_{\mathbf{q}}(\mathbf{k},\mathbf{k}')$

Average on operator equations of motion $\Rightarrow$:

$$i\frac{\partial}{\partial t} n_{\mathbf{k}} = \sum_{\mathbf{q},\mathbf{k}'} \frac{2\pi a}{mV} [g_{\mathbf{q}}(\mathbf{k},\mathbf{k}') - g_{\mathbf{q}}(\mathbf{k}-\mathbf{q},\mathbf{k}') + g_{\mathbf{q}}(\mathbf{k}',\mathbf{k}) - g_{\mathbf{q}}(\mathbf{k}',\mathbf{k}+\mathbf{q})]$$

$$[i\frac{\partial}{\partial t} + \frac{(\mathbf{k}'-\mathbf{k}).\mathbf{q}}{m} - \frac{\mathbf{q}^2}{m}] g_{\mathbf{q}}(\mathbf{k},\mathbf{k}') = Q_{\mathbf{q}}(\mathbf{k},\mathbf{k}') + \frac{4\pi a}{mV} \sum_{\mathbf{k}''} [n_{\mathbf{k}}(2 - \delta_{\mathbf{k},\mathbf{k}''} - \delta_{\mathbf{k}-\mathbf{q},\mathbf{k}''})$$

$$-n_{\mathbf{k}+\mathbf{q}}(2 - \delta_{\mathbf{k},\mathbf{k}''} - \delta_{\mathbf{k}+\mathbf{q},\mathbf{k}''})] g_{\mathbf{q}}(\mathbf{k}'',\mathbf{k}') + [n_{\mathbf{k}'}(2 - \delta_{\mathbf{k}',\mathbf{k}''} - \delta_{\mathbf{k}'+\mathbf{q},\mathbf{k}''}) - n_{\mathbf{k}'-\mathbf{q}}(2 - \delta_{\mathbf{k}',\mathbf{k}''} - \delta_{\mathbf{k}'-\mathbf{q},\mathbf{k}''})] g_{\mathbf{q}}(\mathbf{k},\mathbf{k}'')$$

where

$$Q_{\mathbf{q}}(\mathbf{k},\mathbf{k}') = \frac{8\pi a}{mV} [(n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{q}})(n_{\mathbf{k}'-\mathbf{q}} + 1) n_{\mathbf{k}'} + (n_{\mathbf{k}'} - n_{\mathbf{k}'-\mathbf{q}})(n_{\mathbf{k}} + 1) n_{\mathbf{k}+\mathbf{q}}]$$

$$-\frac{4\pi a n_{\mathbf{k}_s}^2}{mV} (\delta_{\mathbf{k},\mathbf{k}_s} \delta_{\mathbf{k}',\mathbf{k}_s} - \delta_{\mathbf{k},\mathbf{k}_s-\mathbf{q}} \delta_{\mathbf{k}',\mathbf{k}_s+\mathbf{q}}) \left[(1 + n_{\mathbf{k}_s}) n_{\mathbf{k}_s+\mathbf{q}} + (1 + n_{\mathbf{k}_s-\mathbf{q}}) n_{\mathbf{k}_s}\right]$$

We obtain

$$\frac{\partial}{\partial t}n_{\mathbf{k}} = \mathcal{C}^T_{\mathbf{k}}[n_{\mathbf{k}'};\mathbf{k_s}] = \mathcal{C}_{\mathbf{k}}[n_{\mathbf{k}'};\mathbf{k_s}] + \tilde{\mathcal{C}}_{\mathbf{k}}[n_{\mathbf{k}'};\mathbf{k_s}]$$

where

$$\mathcal{C}_{\mathbf{k}}[n_{\mathbf{k}'};\mathbf{k_s}] = \sum_{\mathbf{q},\mathbf{k}'} \left| \frac{\frac{8\pi a}{mV}}{\mathcal{K}(\mathbf{q},\epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}})} \right|^2 (1 - \delta_{\mathbf{k},\mathbf{k_s}} - \delta_{\mathbf{k}+\mathbf{q},\mathbf{k_s}} - \delta_{\mathbf{k}',\mathbf{k_s}} - \delta_{\mathbf{k}'-\mathbf{q},\mathbf{k_s}})\pi\delta(\epsilon_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{k}'-\mathbf{q}} - \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'})$$
$$[n_{\mathbf{k}+\mathbf{q}} n_{\mathbf{k}'-\mathbf{q}}(n_{\mathbf{k}}+1)(n_{\mathbf{k}'}+1) - n_{\mathbf{k}} n_{\mathbf{k}'}(n_{\mathbf{k}+\mathbf{q}}+1)(n_{\mathbf{k}'-\mathbf{q}}+1)]$$

$$\tilde{\mathcal{C}}_{\mathbf{k}}[n_{\mathbf{k}'};\mathbf{k_s}] = \sum_{\mathbf{q},\mathbf{k}'} \frac{\left(\frac{8\pi a}{mV}\right)^2}{\left|\mathcal{K}^*(\mathbf{q},\epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}})\tilde{\mathcal{K}}(\mathbf{q},\epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}})\right|} (\delta_{\mathbf{k},\mathbf{k_s}} + \delta_{\mathbf{k}+\mathbf{q},\mathbf{k_s}} + \delta_{\mathbf{k}',\mathbf{k_s}} + \delta_{\mathbf{k}'-\mathbf{q},\mathbf{k_s}})$$
$$\pi\delta(\epsilon_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{k}'-\mathbf{q}} - \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'})[n_{\mathbf{k}+\mathbf{q}} n_{\mathbf{k}'-\mathbf{q}}(n_{\mathbf{k}}+1)(n_{\mathbf{k}'}+1) - n_{\mathbf{k}} n_{\mathbf{k}'}(n_{\mathbf{k}+\mathbf{q}}+1)(n_{\mathbf{k}'-\mathbf{q}}+1)]$$

Effective potential controlled by dielectric functions:

$$\mathcal{K}(\mathbf{q},\omega) = \frac{\Delta(\mathbf{q},\omega)}{(\omega + i0_+ - \frac{\mathbf{k_s}.\mathbf{q}}{m})^2 - (\frac{\mathbf{q}^2}{2m})^2 + \frac{4\pi a n_{\mathbf{k_s}}}{mV}\frac{\mathbf{q}^2}{m}}$$

$$\tilde{\mathcal{K}}(\mathbf{q},\omega) = \frac{\Delta(\mathbf{q},\omega)}{(\omega + i0_+ - \frac{\mathbf{k_s}.\mathbf{q}}{m})^2 - (\frac{\mathbf{q}^2}{2m})^2} \overset{\omega=\omega_{\mathbf{k_s},\mathbf{q}},-\omega_{\mathbf{k_s}-\mathbf{q},\mathbf{q}}}{\longrightarrow} \infty$$

Bose condensed gas $\Rightarrow$ Dielectric with huge power to annihilate the interaction potential for $n_{\mathbf{k}_s}$ macroscopic

$\Rightarrow$ COLLISION BLOCKADE !!!

$\Rightarrow$ $n_{\mathbf{k}_s}$ constant of motion whatever $\mathbf{k}_s$ provided stable gas

1) QKE conserves particle number, momentum and kinetic energy and verifies the H-theorem $\Rightarrow$ Zero entropy of the condensate

2) Opposite case $n_{\mathbf{k}_s}/V \to 0 \Rightarrow \mathcal{K}(\mathbf{q},\omega) = \tilde{\mathcal{K}}(\mathbf{q},\omega) = \mathcal{K}_n(\mathbf{q},\omega)$

3) Equilibrium solution:

$$n'_{\mathbf{k}} = n'^{eq}_{\mathbf{k}} = \frac{1}{\exp\left[\beta(\epsilon_{\mathbf{k}} - \mathbf{k}.\mathbf{v_n} - \mu)\right] - 1}$$

$\Rightarrow$ superfluidity possible $\mathbf{v_n} \neq \mathbf{k}_s/m$!!!

Landau criterion

$$\text{Stable condensate} \Longleftrightarrow \gamma_{\mathbf{q}} \geq 0$$

For weak depletion and $n'^{eq}_{\mathbf{k}} = \dfrac{1}{\exp(\beta\epsilon_{\mathbf{k}}) - 1}$

$\Rightarrow \epsilon^B_{\mathbf{q}} - \frac{\mathbf{k}_s.\mathbf{q}}{m} \geq 0 \Rightarrow |\mathbf{k}_s|/m \leq c$

Otherwise, the condensate is unstable $\Rightarrow$ NOT superfluid !!!

$\rightarrow$ QKE is different and allows particle exchange

Weakly inhomogeneous kinetic equations: $l_{hyd} \gg V/(N_e\sigma)$

$$\Psi(\mathbf{r},t) = \sqrt{n_{\mathbf{k}_s}(\mathbf{r},t)}e^{i\theta(\mathbf{r},t)} \quad \text{with} \quad \mathbf{k}_s(\mathbf{r},t) = \nabla_{\mathbf{r}}\theta(\mathbf{r},t)$$

$$\begin{aligned}
i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) &= \left[-\frac{\nabla_{\mathbf{r}}^2}{2m} + V_c(\mathbf{r},t)\right]\Psi(\mathbf{r},t) \\
\frac{\partial}{\partial t}n'_{\mathbf{k}}(\mathbf{r},t) &= \left[-\frac{\mathbf{k}}{m}.\nabla_{\mathbf{r}} + \nabla_{\mathbf{r}}V_e(\mathbf{r},t).\nabla_{\mathbf{k}}\right] n'_{\mathbf{k}}(\mathbf{r},t) + \mathcal{C}_{\mathbf{k}}[n_{\mathbf{k}'}(\mathbf{r},t);\mathbf{k}_s(\mathbf{r},t)]
\end{aligned}$$

where

$$V_c(\mathbf{r},t) = V_{ext}(\mathbf{r}) + \frac{4\pi a}{mV}\left[|\Psi(\mathbf{r},t)|^2 + 2\sum_{\mathbf{k}} n'_{\mathbf{k}}(\mathbf{r},t)\right]$$

$$V_e(\mathbf{r},t) = V_{ext}(\mathbf{r}) + \frac{4\pi a}{mV}\left[2|\Psi(\mathbf{r},t)|^2 + 2\sum_{\mathbf{k}} n'_{\mathbf{k}}(\mathbf{r},t)\right]$$

Hartree-Fock energy $\Rightarrow$ Forbidden gap (in principle compatible with Hugenholtz-Pines theorem)

Superfluid universe at finite temperature for stable gas

Stationary solution: $\Psi(\mathbf{r},t) = e^{-i\mu_c t}\Psi_c(\mathbf{r})$

$$\left[-\frac{\nabla_{\mathbf{r}}^2}{2m} + V_{ext}(\mathbf{r}) + \frac{4\pi a}{mV}(|\Psi_c(\mathbf{r})|^2 + 2N_e^{eq}(\mathbf{r}))\right]\Psi_c(\mathbf{r}) = \mu_c \Psi_c(\mathbf{r})$$

$$N_e^{eq}(\mathbf{r}) = \sum_{\mathbf{k}} n_{\mathbf{k}}^{\prime eq}(\mathbf{r}) = V\left(\frac{m}{2\pi\beta}\right)^{3/2} g_{3/2}\left(e^{\beta[\mu_e - V_{ext}(\mathbf{r}) + \frac{8\pi a}{mV}(|\Psi_c(\mathbf{r})|^2 + N_e^{eq}(\mathbf{r}))]}\right)$$

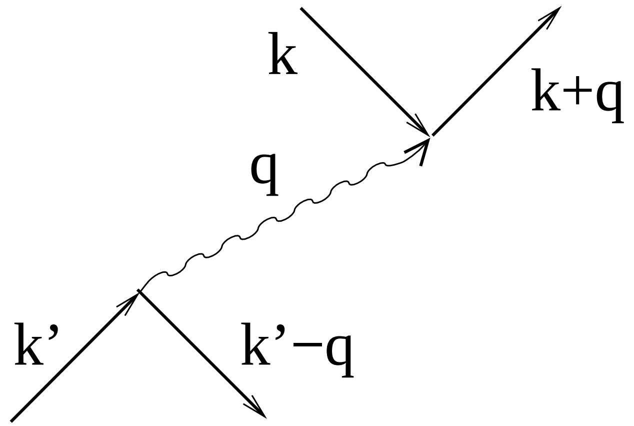
- describe any non dissipative structures like vortices
- $\mu_c = \mu_e$ not required !!! >< ensemble statistical physics

e.g.: For a non zero relative velocity: $\mathbf{v}_{\mathrm{s}} = \mathbf{k}_{\mathrm{s}}/m \Rightarrow \mu_c - \mu_e = m\mathbf{v}_{\mathrm{s}}{}^2/2$

Analogy with plasmon theory: Wyld-Pines interpretation

Plasma collective excitation → plasmon

Condensate collective excitation → . . . why not a “ condenson ”



Feymann diagram of the interaction of two quasi-particles mediated by the collective excitation

$$\frac{\partial}{\partial t} n'_{\mathbf{k}} = \frac{8\pi a}{mV} \sum_{\mathbf{q}} \frac{8\pi^2 a n_0 \mathbf{q}^2}{m^2 V \omega_{\mathbf{q}}} [\delta(\omega_{\mathbf{q}} + \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k+q}}) \left((n'_{\mathbf{k}} + 1) n'_{\mathbf{k+q}} (f_{\mathbf{q}} + 1) - n'_{\mathbf{k}} (n'_{\mathbf{k+q}} + 1) f_{\mathbf{q}}\right)$$

$$+\delta(\omega_{\mathbf{q}} - \epsilon_{\mathbf{k}} + \epsilon_{\mathbf{k-q}}) \left((n'_{\mathbf{k}} + 1) n'_{\mathbf{k-q}} f_{\mathbf{q}} - n'_{\mathbf{k}} (n'_{\mathbf{k-q}} + 1)(f_{\mathbf{q}} + 1)\right)]$$

$$\frac{\partial}{\partial t} f_{\mathbf{q}} = -2\gamma_{\mathbf{q}} f_{\mathbf{q}} + \frac{8\pi a}{mV} \frac{8\pi^2 a n_0 \mathbf{q}^2}{m^2 V \omega_{\mathbf{q}}} \sum_{\mathbf{k}} \delta(\omega_{\mathbf{q}} - \epsilon_{\mathbf{k}} + \epsilon_{\mathbf{k-q}}) n'_{\mathbf{k}} (n'_{\mathbf{k-q}} + 1)$$

For a weakly depleted gas, the condenson has the Bogoliubov energy spectrum $\omega_{\mathbf{q}} = \epsilon^B_{\mathbf{q}}$ and a decay rate $2\gamma_{\mathbf{q}}$

$$n'_{\mathbf{k}} = n^{eq}_{\mathbf{k}} = \frac{1}{\exp\left[\beta(\epsilon_{\mathbf{k}} - \mu)\right] - 1} \quad \Rightarrow \quad \text{quasi} - \text{particle}$$

$$f^{eq}_{\mathbf{q}} = \frac{1}{\exp\left(\beta\omega_{\mathbf{q}}\right) - 1} \quad \Rightarrow \quad \text{condenson} = \text{boson}$$

Does Bogoliubov excitation still correspond to the quasi-particle ???

Adiabatic approximation $\rightarrow$ eliminate $f_{\mathbf{q}}$ and recover the QKE for $n'_{\mathbf{k}}$ only

valid if $\tau_{col}^{-1} \sim \gamma_{\mathbf{q}} \gg \tau_{rel}^{-1} \sim N_e \sigma / [V(\beta m)^{1/2}]$

Summary

-GRPA $\rightarrow$ Different theory to be compared with others:

1. collision blockade
2. three fluids model: condensate, mediating condenson, quasi-particle
3. allows to recover Bogoliubov theory at $T = 0$, dynamic structure factor $S(\mathbf{q}, \omega)$, collective excitations
4. fulfill conservation laws
5. H-theorem allowing to distinguish between superfluid and dissipative behaviors
6. $\neq$ ensemble theory in equilibrium
7. number-conserving $\neq$ spontaneous $U(1)$ breaking

Conditions for superfluidity:

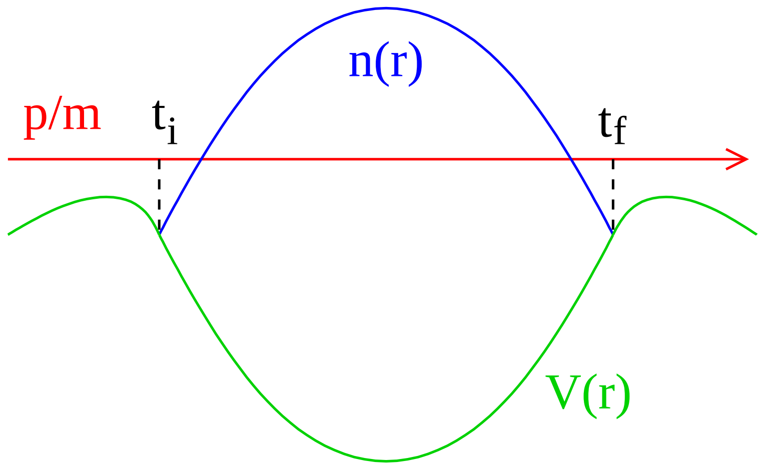
1. Collision blockade $\Rightarrow$ Reservoir of potential energy, Macroscopic coherent condensate

2. Stable gas $\longrightarrow$ Landau criterion

Normal fluid can be considered as a whole

Are there only the Bogoliubov collective excitations?

Assume an atom with low momentum p traveling through the condensate at $T = 0$ given μ, $V(r) = m\omega^2 r^2/2$, $c(r) = \sqrt{4\pi a n(r)}/m$



What is the time of flight?

$$\Delta t = t_f - t_i = \int_{r_i}^{r_f} \frac{dr}{v(r)} \quad \rightarrow$$

1) Standard theory $\rightarrow v(r) = \dfrac{d\epsilon^B_{\mathbf{p}}}{dp}(r) \sim c(r)$

2) GRPA theory $\rightarrow v(r) = p/m$

Defining $s = p^2/(4m\mu)$, we get

$\Delta t_{GRPA} - \Delta t_S = F(s)/\omega \sim 10ms$