EPR CORRELATIONS VIA BEC DISSOCIATION

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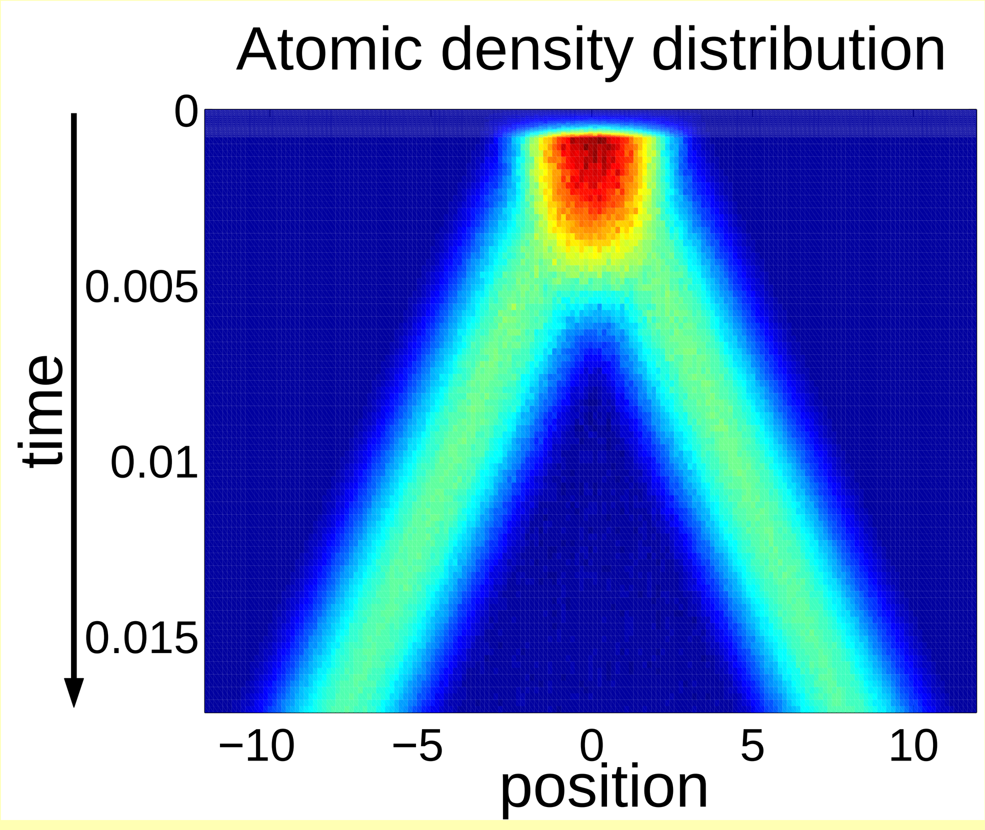
Introduction

- **Continuous variable demonstration of quantum mechanical paradoxes with massive particles**.
- **Will correlations be the same for massive atoms and massless photons?**
- **We use field quadratures: directly analogous to position and momentum of original paradox**.
- **We extend quantum optical experiments into the atom-molecular domain: will interactions and increased noise destroy correlations?**.

BEC Dissociation

- Molecular BEC via Feshbach resonance or photoassociation of atomic BEC.
- Dissociation (with appropriate detunings) gives correlated atomic outputs
- Previously shown to be strongly correlated in intensities [K. V. Kheruntsyan and P. D. Drummond, PRA **66**, 031602 (2002)]



Quantum Hamiltonian

$$\hat{H} = \hat{H}_{kin} + \hbar \int dz \left\{ V_2(z)\hat{\Psi}_2^\dagger \hat{\Psi}_2 + \frac{1}{2}\sum_{i \geq j} U_{ij} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i - i\frac{\chi(t)}{2}\left[\hat{\Psi}_2^\dagger \hat{\Psi}_1^2 - \hat{\Psi}_1^{\dagger 2}\hat{\Psi}_2\right]\right\}.$$

- $\hat{H}_{kin}$ is kinetic energy
- $\hat{\Psi}_{1,2}(z,t)$ – field operators for bosonic atoms (1) and bosonic dimers (2)
- $V_2(z)$ is harmonic trap for molecules
- U_{ij} – s-wave scatterings
- χ – atom-molecule coupling ($2A \rightleftharpoons M$)
- Atoms not trapped
- Similarities with well-known optical parametric amplifier Hamiltonian
- **DIFFERENCES ARE IMPORTANT**

EPR with Quadratures

- Two quadrature correlated (entangled) outputs with different momenta
- We can choose noncommuting properties to measure
- Field quadratures:

$$\hat{X}_q = \hat{a}_q + \hat{a}_q^\dagger,$$
$$\hat{Y}_q = -i\left(\hat{a}_q - \hat{a}_q^\dagger\right).$$

- $\left[\hat{a}_q, \hat{a}_{q'}^\dagger\right] = \delta(q-q') \longrightarrow V(\hat{X})V(\hat{Y}) \geq 1.$

$$V^{inf}(\hat{X}_{\pm q}) = V(\hat{X}_{\pm q}) - \frac{\left[V(\hat{X}_{+q}, \hat{X}_{-q})\right]^2}{V(\hat{X}_{\mp q})}.$$

- EPR signature: apparent violation of Heisenberg uncertainty principle

$$V^{inf}(\hat{X}_{\pm q})V^{inf}(\hat{Y}_{\pm q}) < 1$$

[M. D. Reid, PRA **40**, 913 (1989)]

Quantum Optics Approach

- Undepleted molecular field in $k = 0$ mode
- Only two atomic momentum modes populated, at $\pm q_0$
- **EXACT** analytical solutions possible, but miss some physics

$$V^{inf}(\hat{X}_{+q_0})V^{inf}(\hat{Y}_{+q_0}) = 1/\cosh^2(2g\tau) \leq 1.$$

- **A TRAPPED CONDENSATE IS NOT A SINGLE MODE**
- Analysis is simple, but misleading

One Dimensional Analysis

- Positive-P representation equations

$$\frac{\partial \psi_1}{\partial \tau} = i\frac{\partial^2 \psi_1}{\partial \xi^2} - (\gamma_1 + i\delta)\psi_1 + \kappa\psi_2\psi_1^+ + \sqrt{\kappa\psi_2}\,\eta_1,$$
$$\frac{\partial \psi_2}{\partial \tau} = \frac{i}{2}\frac{\partial^2 \psi_2}{\partial \xi^2} - i\upsilon\psi_2 - \frac{\kappa}{2}\psi_1^2 + \sqrt{-i\upsilon\psi_2}\,\eta_2.$$

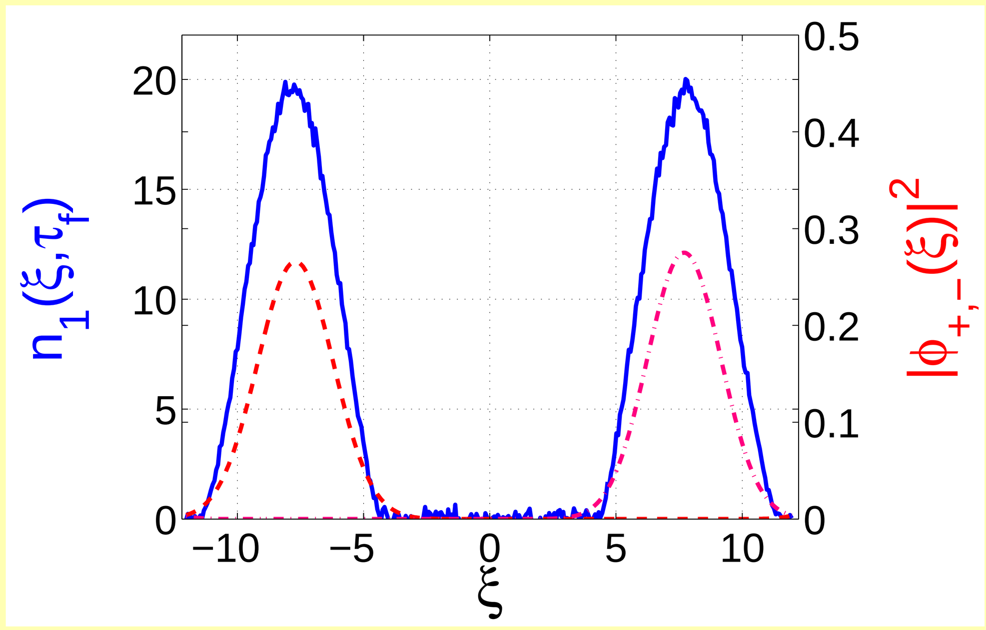
- Plus equations for ψ_1^+, ψ_2^+
- Four independent variables: can model QM with c-number fields
- Use stochastic integration; take trajectory averages
- Stochastic averages give normally-ordered operator moments

Mode matching

- Naive calculation as in single-mode case shows no EPR correlations
- Different momentum modes are mixed
- Solution: mode-matched local oscillators
- Four mode-matched quadrature operators:

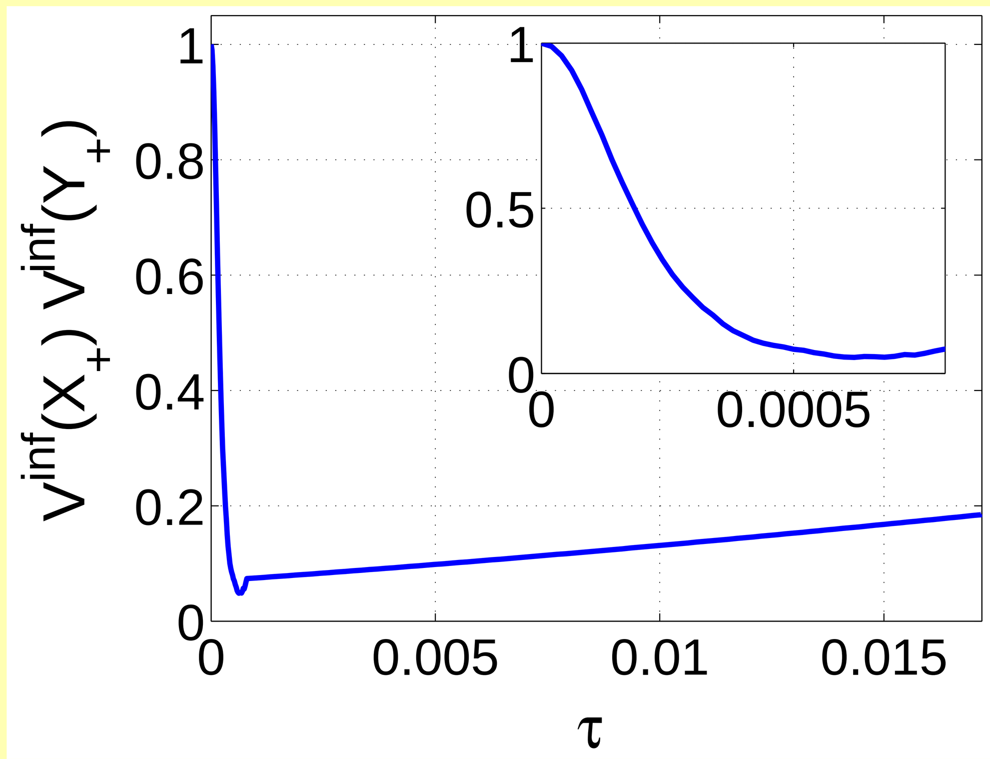
$$\hat{X}_\pm(\tau) = \int d\xi \left[\phi_\pm^*(\xi)\hat{\Psi}_1(\xi,\tau) + h.c\right],$$
$$\hat{Y}_\pm(\tau) = -i\int d\xi \left[\phi_\pm^*(\xi)\hat{\Psi}_1(\xi,\tau) - h.c\right].$$

- $\phi_\pm(\xi) = |\phi_\pm(\xi)|\exp(\mp i q_0 \xi)$ are Gaussian local oscillator fields



- $n_1(\xi,\tau_f) = \overline{\Psi_1^+(\xi,\tau_f)\Psi_1(\xi,\tau_f)}$

RESULTS



- Inset: dissociation coupling on
- Clear demonstration of EPR paradox

Summary

- Homodyne measurements of BEC could open new lines of investigation
- Macroscopic test of quantum mechanics with massive particles
- Important to use realistic condensates

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