Superfluid Fermi gases in optical lattices

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[1] Collisional (non superfluid) regime: role of umklapp processes
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[2] Superfluid regime: sound propagation and collective modes
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MODEL

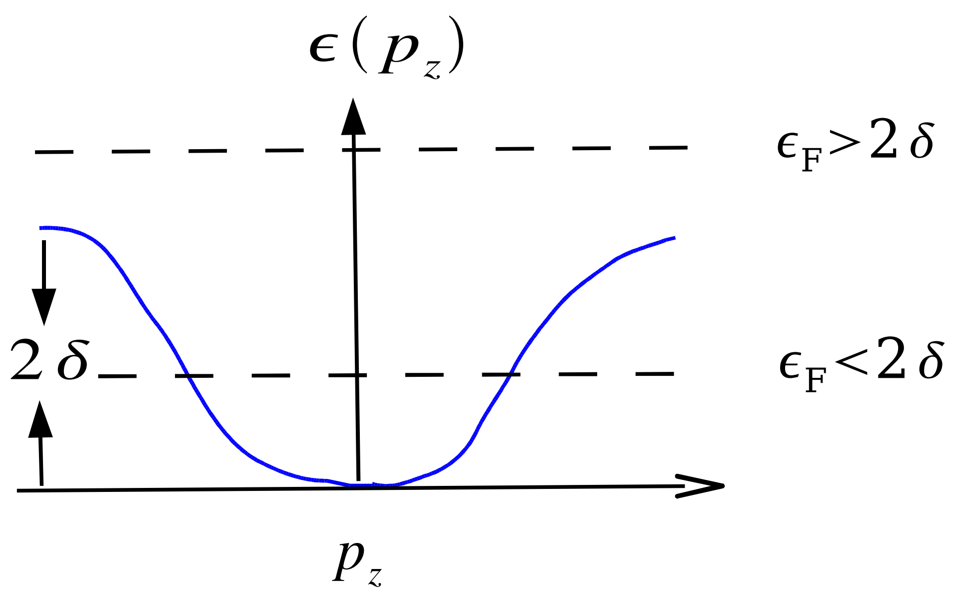
1D optical lattice $V_{opt}(z) = s\,E_R \sin^2(q_B z)$

E_R recoil energy $q_B = \frac{\pi}{d}$ Bragg momentum

Motion is free in transverse directions: $\epsilon(\boldsymbol{p}) = \frac{(p_x^2 + p_y^2)}{2m} + \epsilon(p_z)$

lowest Bloch band

(neglect higher bands)



$\delta =$ tunneling term between consecutive wells

Collisional (non superfluid) regime

s-wave scattering $U(\boldsymbol{r}) = g\,\delta(\boldsymbol{r})$ $g = \dfrac{4\pi\hbar^2 a}{m}$

<u>Normal</u> collisions

$$p_{1z} + p_{2z} - p_{3z} - p_{4z} = 0$$

$P = p_{1z} + p_{2z}$ is conserved

<u>Umklapp</u> collisions

$$p_{1z} + p_{2z} - p_{3z} - p_{4z} = \pm 2\hbar q_B$$

$P = p_{1z} + p_{2z}$ is **not** conserved

↙

In presence of harmonic trapping,
the center of mass oscillation is **damped**

Relaxation time τ_{uk}

In tight-binding limit
(s >> 1)

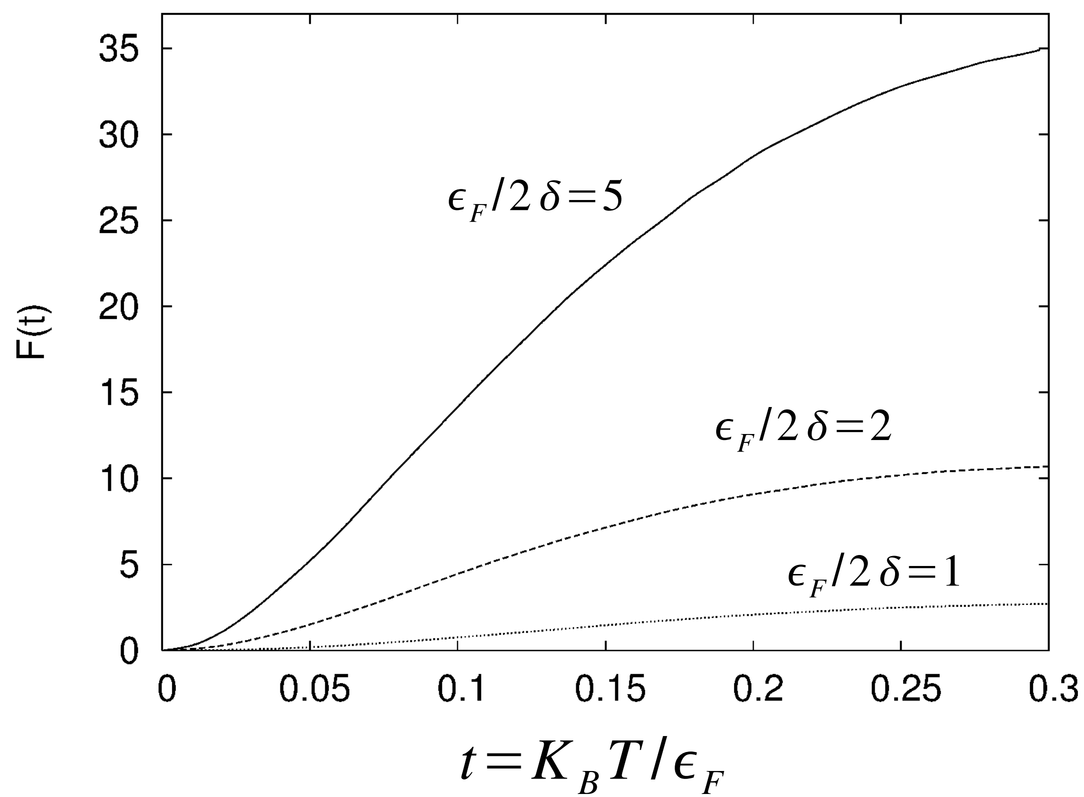
$$\frac{1}{\tau_{uk}} = \alpha^2 \frac{a^2}{d^2} \frac{\delta}{\hbar} F\left(\frac{K_B T}{\epsilon_F}, \frac{\epsilon_F}{2\delta}\right)$$

$$\alpha \approx \sqrt{\pi/2}\, s^{1/4}$$

For typical parameters

$a \ll d$ unless very close to a Feshbach resonance

$\delta \gg \hbar \omega_z$ shallow trap



The oscillation is <u>overdamped</u> if

$$\omega_z \tau_{uk} \ll 1$$

This condition can be satisfied at $K_B T \approx 0.05\epsilon_F$ for $\epsilon_F > 2\delta$ **without** violating the diluteness assumption.

In the superfluid phase, however, the system exhibits persistent Josephson-like oscillations [see below] .

Therefore the study of the center of mass oscillation of a trapped Fermi gas in a 1D optical lattice can be used as a tool to detect superfluidity.

Superfluid phase

Hydrodynamic theory at zero temperature

$$\boldsymbol{v} = \frac{\hbar}{2m} \nabla \phi$$ superfluid velocity

$$\frac{\partial n}{\partial t} + \partial_x (n v_x) + \partial_y (n v_y) + \partial_z \left(\frac{m}{m_s} n v_z\right) = 0$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{m} \nabla \left[\mu(n) + V_{ho} + \frac{m}{2} v_x^2 + \frac{m}{2} v_y^2 + \frac{\partial}{\partial n}\left(\frac{m}{m_s} n\right) \frac{m}{2} v_z^2 \right] = 0$$

Chemical potential

Trapping potential

New effective mass $(m_s > m)$

inertia of the gas increased by the optical lattice

Bogolubov-Anderson modes

$$V_{ho}=0$$

HD equations support phonons $\omega = c\,q$

$$c_z=\sqrt{\frac{n}{m_s}\frac{\partial\mu}{\partial n}} \qquad c_x, c_y=\sqrt{\frac{n}{m}\frac{\partial\mu}{\partial n}}$$

Effective mass and compressibility derived from BCS theory

$$\frac{1}{m_s}=\frac{\int\frac{\partial^2\epsilon}{\partial k_z^2}n_0(\boldsymbol{k})\,d\boldsymbol{k}}{\int n_0(\boldsymbol{k})\,d\boldsymbol{k}}$$

Quasi-momentum distribution

$$n_0(\boldsymbol{k})=2\,\Theta(\epsilon_F-\epsilon(\boldsymbol{k}))$$

$$\left(\frac{\partial\mu}{\partial n}\right)^{-1}$$ = density of states at Fermi level

Exact results in tight-binding limit

$$\epsilon(p_z)=\delta(1-\cos(p_z d/\hbar))$$

For $\epsilon_F \ll 2\delta$

$$m_s=\frac{\hbar^2}{\delta d^2}$$ (independent of density)

$$c_z^2=\frac{2\epsilon_F}{3m_s}\propto n^{2/3}$$

For $\epsilon_F > 2\delta$ situation is *reversed*:

$$m_s=\frac{2\pi\hbar^4 n}{m\delta d^2}\propto n$$

$$c_z^2=\frac{\delta^2 d^2}{2\hbar^2}$$ (independent of density)

Center of mass oscillation frequency ω_{CM}

$$V_{ho} = \frac{m}{2}\omega^2(x^2+y^2) + \frac{m}{2}\omega_z^2 z^2$$

$$\omega_{CM} = \omega_z \sqrt{\frac{m}{m_{CM}}}$$

$$\frac{1}{m_{CM}} = \frac{\delta d^2}{\hbar^2} f\left(\frac{\epsilon_F}{2\delta}\right)$$