Mott-insulator phase of coupled 1D atomic gases in a 2D optical lattice

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Abstract

We consider the physics of coupled 1D bosonic gases in a 2D optical lattice in the Mott-insulator phase at zero temperature. We discuss the effects of the 1D strongly-interacting regime on the momentum distribution of the gas. We show that the momentum distribution in the lattice plane becomes very sensitive to the on-site interaction regime. In particular, we find that the disappearance of interference fringes in time of flight experiments is not a direct signature of the MI phase, but a clear consequence of the strong correlations along the sites. This effect can be tested in current on-going experiments.

Model

The action includes an on site dynamics and a coupling between different sites

$$S = \sum_j S_j - t\sum_{<ij>} \int_0^{\hbar\beta} d\tau \int_0^L dx \ \left(\bar\psi_i \psi_j + \bar\psi_j \psi_i\right)$$

the on site action reads

$$S_j = \int_0^{\hbar\beta} d\tau \int_0^L dx \ \bar\psi_j \left(\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \mu\right)\psi_j + \frac{g}{2}\bar\psi_j\bar\psi_j\psi_j\psi_j$$

where ψ and $\bar\psi$ are bosonic fields, t is Josephson coupling, m the mass and g is coupling constant

Results

In absence of tunneling the sites are not coupled, the Green function reads

$$G_{ij}(x_1-x_2, \tau_1-\tau_2) = \langle\psi_i(x_1,\tau_1)\bar\psi_j(x_2,\tau_2)\rangle = \delta_{ij} G_0(x_1-x_2, \tau_1-\tau_2).$$

where the on-site Green function is calculated using Luttinger liquid theory

$$G_0(k,\omega) = \frac{1}{nv_S}\left(\frac{N}{2\pi}\right)^{2-2\Delta} I\left(\frac{kL}{2\pi}, \frac{\omega L}{2\pi v_S}\right)$$

where $\Delta = 1/4K$, K is the Luttinger parameter (Tonks $K = 1$, mean-field $K \to \infty$) and

$$I(p,\Omega) = \frac{4\pi}{p!}\frac{\Gamma(\Delta+p)}{\Gamma(\Delta)}\Re\left[\frac{{}_3F_2\left(\Delta,\Delta+p,\frac{\Delta+p-i\Omega}{2};1+p,1+\frac{\Delta+p-i\Omega}{2};1\right)}{\Delta+p-i\Omega}\right]$$

If we consider the coupling between sites the Green function reads

$$G(\vec q, k, \omega) = \frac{G_0(k,\omega)}{1-T(\vec q)G_0(k,\omega)}$$

From the previous expression it is possible to calculate the critical tunneling of the Mott-insulator to superfluid transition

$$\frac{t_C}{\mu} = \frac{nv_S\hbar}{4\mu}\left(\frac{N}{2\pi}\right)^{2\Delta-2}\frac{1}{I(0,0)}$$

Since, in a good approximation, only for $k = 0$ the Green function is affected by the tunneling we write

$$G(\vec q, k, \omega) \simeq G_0(k,\omega) + \frac{T(\vec q)G_0^2(0,\omega)}{1-T(\vec q)G_0(0,\omega)}\delta_{k,0}$$

and the momentum distribution in the plane of the lattice reads

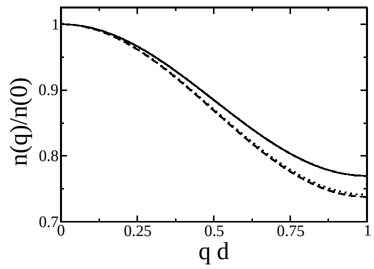
$$\frac{N(\vec q)}{N} = 1 + \frac{t}{t_C}\frac{f(\Delta)}{N^{2\Delta}}\left(\sum_{j=x,y}\cos(q_j d)\right)$$

where

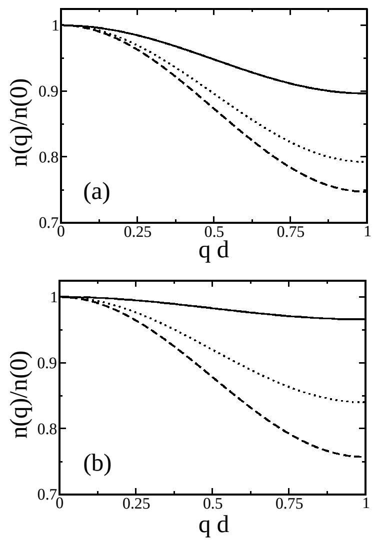
$$f(\Delta) = \frac{(2\pi)^{2\Delta-2}}{2I(0,0)}\int d\Omega I(0,\Omega)^2$$

assumes values of the order of one. Two effects are responsable of the flatness of the momenum distribution: (i) the ratio t/t_C (ii) the factor $N^{-2\Delta}$ that is related to the regime of interaction through the parameters Δ and the number of particles per site N.

Figures



Transversal momentum distribution for $k = 0$ for $t/t_c = 0.3$ for $K = 1$ (solid), $K = 4$ (dotted), and $K = 25$ (dashed). In the figure we have chosen $q_x = q_y = q$



Transversal momentum distribution for $t/t_c = 0.3$, for $K = 1$ (solid), $K = 4$ (dotted), and $K = 25$ (dashed), for $N = 50$ (a) and $N = 500$ (b)

Conclusions

- As pointed out from [1] one can have a non-flat momentum distribution inside the insulating phase.
- The flatness of the momentum distribution is related to the strongly interacting regime.

References

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[2] A. M. Tsvelik, *"Quantum field theory in condensed matter physics"* Cambridge University Press, 1995