Ultracold atoms in one-dimensional optical lattices approaching the Tonks-Girardeau regime

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Abstract

Recent experiments on ultracold atomic alkali gases in an one-dimensional optical lattice have demonstrated the transition from a gas of soft-core bosons to a Tonks-Girardeau gas[3] in the hard-core limit, where one-dimensional bosons behave just like fermions[4]. We have studied the underlying many-body physics through numerical simulations which accommodate both the soft-core and hard-core limits in one single framework[1]. We find that the Tonks-Girardeau gas is reached only at the strongest optical lattice potentials. Results for slightly higher densities, where the gas develops a Mott-like phase[2] already at weaker optical lattice potentials, show that these Mott-like short range correlations do not enhance the convergence to the hard-core limit.

References


Homogeneous system

We calculated the energy density for a homogeneous system (ε=0) consisting of 128 sites at half filling for various values of U/J. For comparison, we also calculated the energy with the Bogoliubov approach valid at low values of U/J and for a system of interacting fermions, which is valid at high values of U/J.

Density

For the homogeneous system, the system develops a gapped Mott phase at integer filling. A trapped system on the contrary, can contain a Mott region[2], i.e. a couple of sites in the center of the trap with integer density where the local compressibility tends to vanish. Can this property be exploited in order to reach the Tonks-Girardeau regime at lower values of the optical potential strength? The answer is no: although the particle densities in coordinate space for soft-core (red curves) and hard-core bosons (green curves) coincide, their momentum profiles do not. \[ V_0/E_R = 7.0 \] and \[ \beta/J = 1 \].

Trapped system

For the quadratically trapped system (ε= -\(\omega_p^2 / \omega_z^2\)), we calculated the local densities in coordinate space and the experimentally measurable momentum profiles for soft-core (red) and hard-core (green) bosons at various optical potential strengths, (a) \( V_0/E_R = 1.0, \) \( U/J = 1.75 \), (b) \( V_0/E_R = 5.0, U/J = 7.8 \), (c) \( V_0/E_R = 9.5, U/J = 28.6 \), (d) \( V_0/E_R = 12.0, U/J = 52.2 \); and (e) \( V_0/E_R = 20.0, U/J = 288 \). There are \( <N> = 1.5 \) particles and the inverse temperature is estimated to be \( \beta/J = 1 \).

Method

The physics of the one-dimensional atoms is accurately described by the Bose-Hubbard Hamiltonian with nearest-neighbor hopping only,

\[ H = -\sum_{i=1}^{U} \sum_{\sigma} (\epsilon_i - \mu) n_{i\sigma} + J \sum_{i=1}^{U} \sum_{\sigma} (n_{i\sigma} - 1)(n_{i+1\sigma} - 1) \]

The tunneling amplitude \( J \), the magnetic trapping \( \epsilon_i \), and the strength of the on-site repulsion \( U \) follow numerically from a lowest band calculation using Wannier orbitals. The Bose-Hubbard Hamiltonian is then simulated by the stochastic series expansion Monte Carlo method (SSE) using directed loops and our locally optimal updating scheme. We can impose a particle cut-off of \( n = 1 \) or \( n = 2,3,... \) per site, corresponding to hard-core and soft-core bosons, respectively. The calculation is based on one single framework for the entire range of optical potential strengths: all input parameters are known or experimentally measurable (no fitting).

Temperature

We also examined how momentum profiles change when lowering the temperature. The relevant momentum scales in a trapped system are different from the momentum scales in the homogeneous case: the harmonic trap sets a lower momentum scale \( p_T = (m \omega_T)^{1/2} \) below which the momentum distribution is flattened due to the suppression of long-range correlations.


For the homogeneous system, the system develops a gapped Mott phase at integer filling. A trapped system on the contrary, can contain a Mott region[2], i.e. a couple of sites in the center of the trap with integer density where the local compressibility tends to vanish. Can this property be exploited in order to reach the Tonks-Girardeau regime at lower values of the optical potential strength? The answer is no: although the particle densities in coordinate space for soft-core (red curves) and hard-core bosons (green curves) coincide, their momentum profiles do not. \[ V_0/E_R = 7.0 \] and \[ \beta/J = 1 \].