Ultracold atoms in one-dimensional optical lattices approaching the Tonks-Girardeau regime

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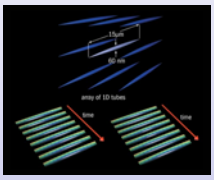
Abstract

Recent experiments on ultracold atomic alkali gases in an one-dimensional optical lattice have demonstrated the transition from a gas of soft-core bosons to a Tonks-Girardeau gas[3] in the hard-core limit, where one-dimensional bosons behave just like fermions[4]. We have studied the underlying many-body physics through numerical simulations which accommodate both the soft-core and hard-core limits in one single framework[1]. We find that the Tonks-Girardeau gas is reached only at the strongest optical lattice potentials. Results for slightly higher densities, where the gas develops a Mott-like phase[2] already at weaker optical lattice potentials, show that these Mott-like short range correlations do not enhance the convergence to the hard-core limit.



In the experiment[4], ^{87}Rb atoms are cooled and effectively confined to one dimension, resulting in a two-dimensional optical lattice of one-dimensional tubes. The axial optical potential strength is then increased in order to increase the repulsion between the atoms.

Method

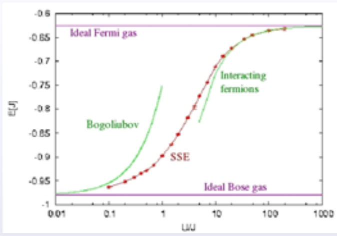
The physics of the one-dimensional atoms is accurately described by the Bose-Hubbard Hamiltonian with nearest-neighbor hopping only,

$$H = -t\sum_{\langle i,j\rangle} b_i^\dagger b_j + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \left(\varepsilon_i - \mu\right) n_i$$

The tunneling amplitude J, the magnetic trapping ε_i , and the strength of the on-site repulsion U follow numerically from a lowest band calculation using Wannier orbitals. The Bose-Hubbard Hamiltonian is then simulated by the stochastic series expansion Monte Carlo method (SSE) using directed loops and our locally optimal updating scheme. We can impose a particle cut-off of $n=1$ or $n=2,3,\ldots$per site, corresponding to hard-core and soft-core bosons, respectively. The calculation is based on one single framework for the entire range of optical potential strengths: all input parameters are known or experimentally measurable (no fitting).

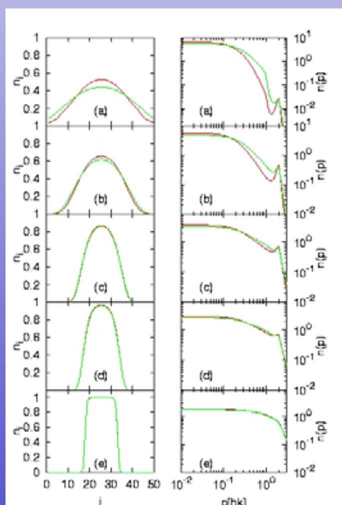
Homogeneous system

We calculated the energy density for a homogeneous system ($\varepsilon_i=0$) consisting of *128* sites at half filling for various values of U/J. For comparison, we also calculated the energy with the Bogoliubov approach valid at low values of U/J and for a system of interacting fermions, which is valid at high values of U/J.



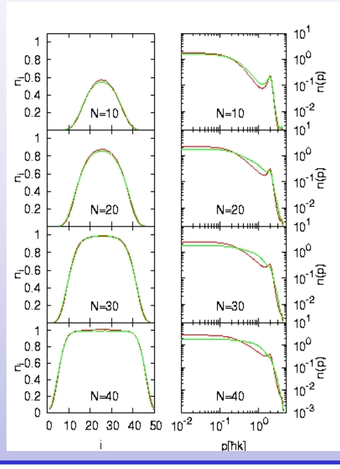
Trapped system

For the quadratically trapped system ($\varepsilon_i \sim i^2$), we calculated the local densities in coordinate space and the experimentally measurable momentum profiles for soft-core (red) and hard-core (green) bosons at various optical potential strengths, (a) $V_0/E_R = 1.0$, $U/J = 1.75$, (b) $V_0/E_R = 5.0$, $U/J = 7.8$, (c) $V_0/E_R = 9.5$, $U/J = 28.6$, (d) $V_0/E_R = 12.0$, $U/J = 52.2$, and (e) $V_0/E_R = 20.0$, $U/J = 258$. There are $\langle N\rangle=15$ particles and the inverse temperature is estimated to be $\beta J=1$.



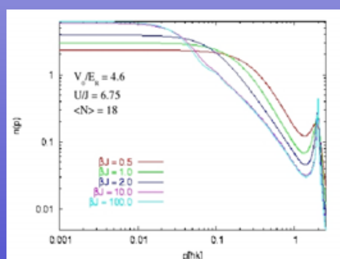
Density

For the homogeneous system, the system develops a gapped Mott phase at integer filling. A trapped system on the contrary, can contain a Mott region[2], i.e. a couple of sites in the center of the trap with integer density where the local compressibility tends to vanish. Can this property be exploited in order to reach the Tonks-Girardeau regime at lower values of the optical potential strength? The answer is no: although the particle densities in coordinate space for soft-core (red curves) and hard-core bosons (green curves) coincide, their momentum profiles do not. [$V_0/E_R = 7.0$ and $\beta J=1$].



Temperature

We also examined how momentum profiles change when lowering the temperature. The relevant momentum scales in a trapped system are different from the momentum scales in the homogeneous case: the harmonic trap sets a lower momentum scale $p_T = (m\hbar\omega)^{1/2}$, below which the momentum distribution is flattened due to the suppression of long-range correlations.



References

1. L. Pollet, S. M. A. Rombouts and P. J. H. Denteneer, cond-mat/0408596 (2004) *and references therein*.

2. M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, and I. Bloch, Nature **415**, 39 (2002).

3. M. Girardeau, J. Math. Phys. **1**, 516 (1960).

4. B. Paredes, A. Widera, V. Murg, O. Mandel, S. Fölling, I. Cirac, G. V. Shlyapnikov, T. W. Hänsch, and I. Bloch, Nature **429**, 277 (2004).