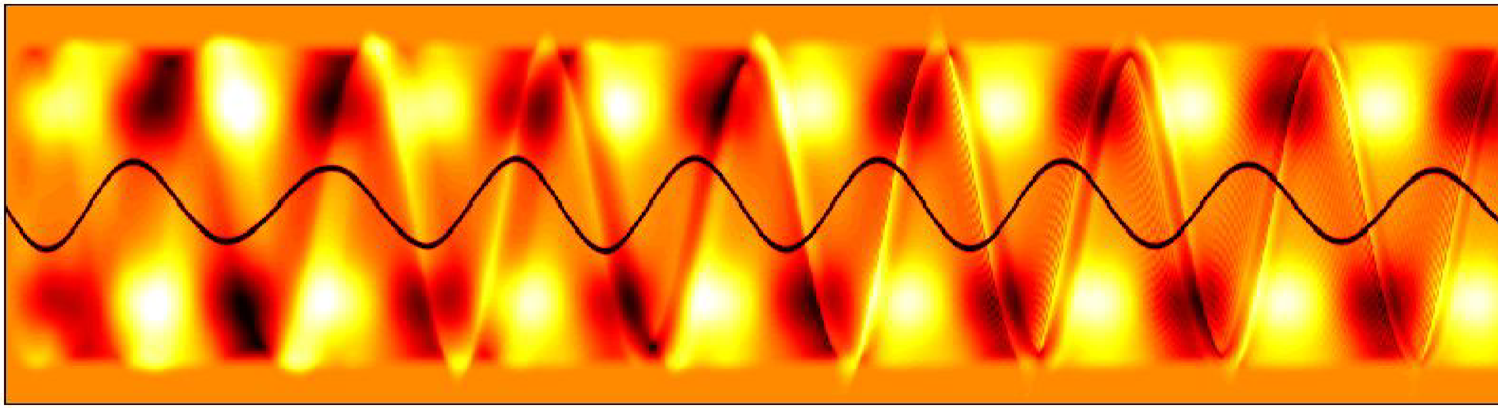
Long-lived Dark Solitons

In Atomic Bose-Einstein Condensates



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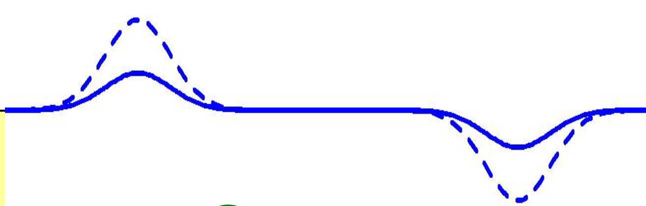
http://massey.dur.ac.uk



Current Collaboration:

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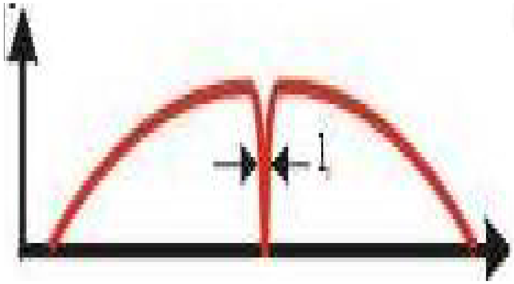
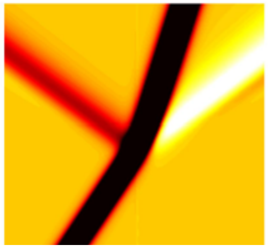
Joachim Brand (Dresden)

Panos Kevrekides (Amherst)

OUTLINE



1. Intro to Dark Solitons & Decay Mechanisms

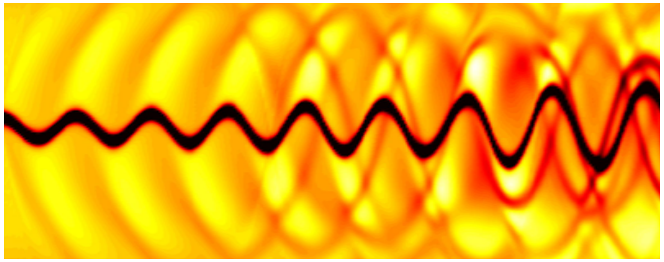


2. Quasi-1D Systems ($T \ll T_C$):
Soliton Decays via Sound Emission
Due to LONGITUDINAL Confinement

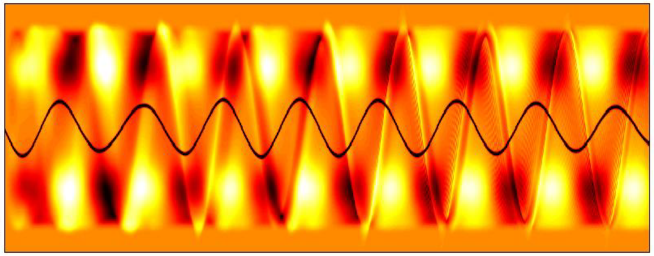
KEY POINT: Soliton Dynamics Are Sensitive to Soliton – Sound Interactions

3. How to Control Soliton Dynamics ?

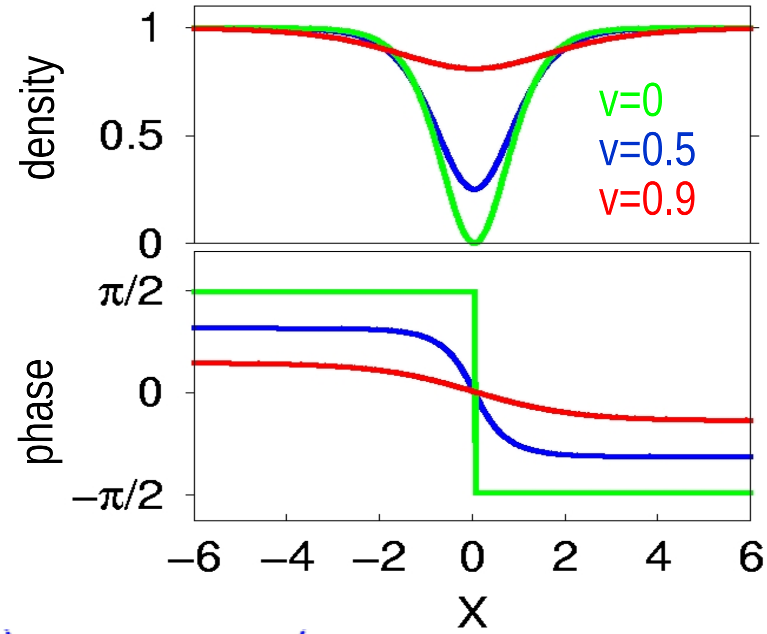
Manipulate Strength of Soliton-Sound Interactions



By Parametric Driving → 'Long-lived Solitons'



DARK SOLITONS IN ATOMIC BEC



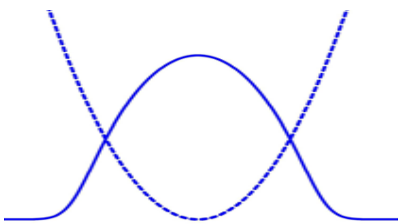
Density Dips Propagating without Dispersion

SOLUTIONS OF HOMOGENEOUS 1D NONLINEAR SCHROEDINGER EQUATION

$$i\frac{\partial\Phi}{\partial t} = -\frac{1}{2}\frac{\partial^2\Phi}{\partial z^2} + |\Phi|^2\,\Phi$$

T → 0 ATOMIC BEC

Gross-Pitaevskii Equation



HARMONIC CONFINEMENT :

$$i\hbar\frac{\partial\Phi}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m\omega_\perp{}^2\left(x^2+y^2\right) + \frac{1}{2}m\omega_z{}^2 z^2 - \mu + g|\Phi|^2\right]\Phi$$

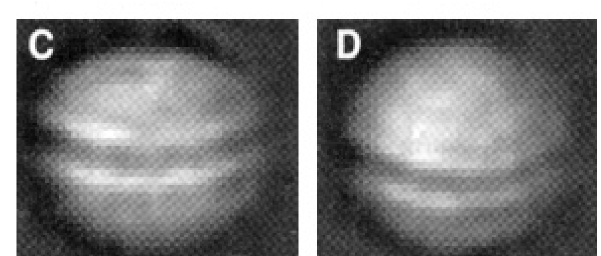
Transverse Longitudinal **Repulsive** Interactions

BEC Dark Soliton Experiments:

NIST: Science 287, 97 (1999)

Hannover: PRL, 83, 5198 (1999)

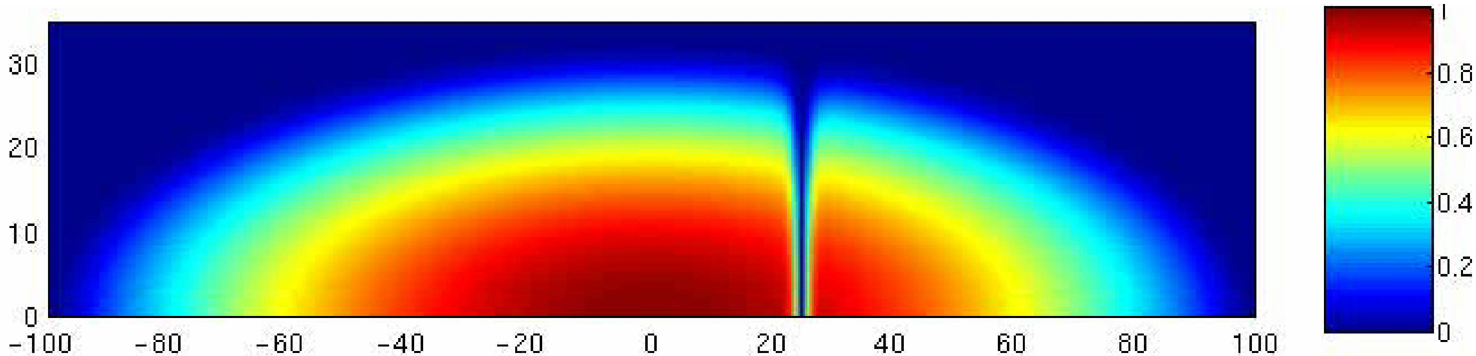
Rowlands Institute, JILA



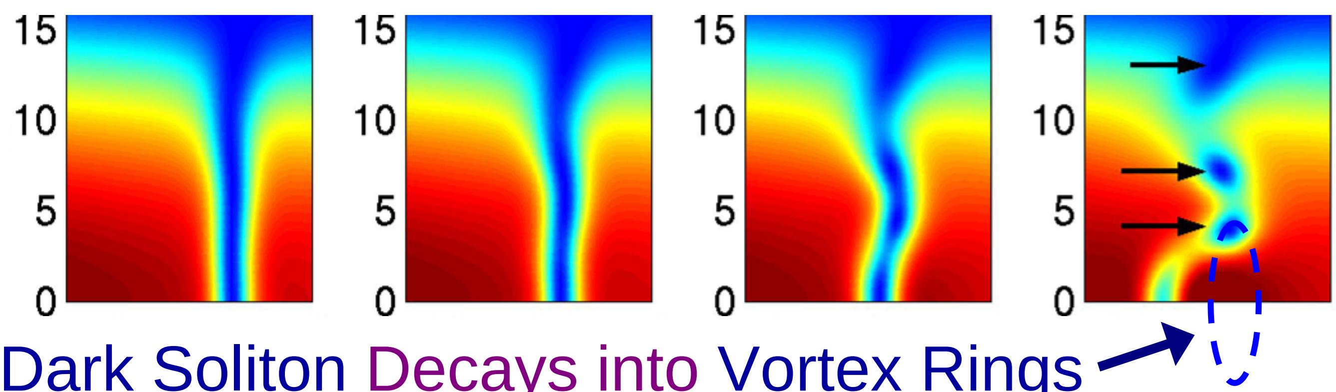
DARK SOLITON DECAY MECHANISMS

1. Thermal Fluctuations — Muryshev et al, PRL 89, 110401 (2002)
2. Quantum Fluctuations — Dziarmaga et al., J. Phys. B 36, 1217 (2003)
3. Transverse Excitations (3D Systems) — Feder et al. , PRA 62, 053606 (2000); Brand & Reinhardt , PRA 65, 043612 (2002); Komineas & Papanikolaou, PRA 68, 043617 (2003); Proukakis et al., J. Opt. B 37, S175 (2004)

2D DENSITY CROSS SECTION



SOLITON CLOSE-UP



→ Dark Soliton Decays into Vortex Rings

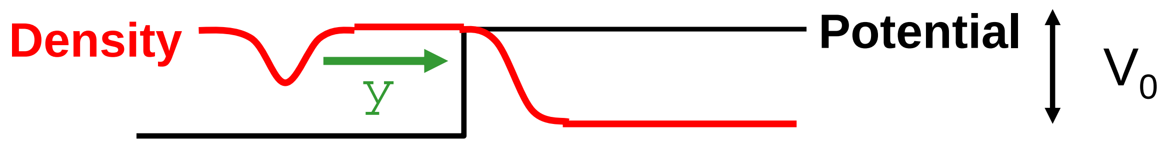
'SNAKE INSTABILITY' OBSERVED EXPERIMENTALLY: **Anderson et al., PRL 86, 2926 (2001)**

→ Suppressed in Highly-Elongated 'Quasi-1D' Systems

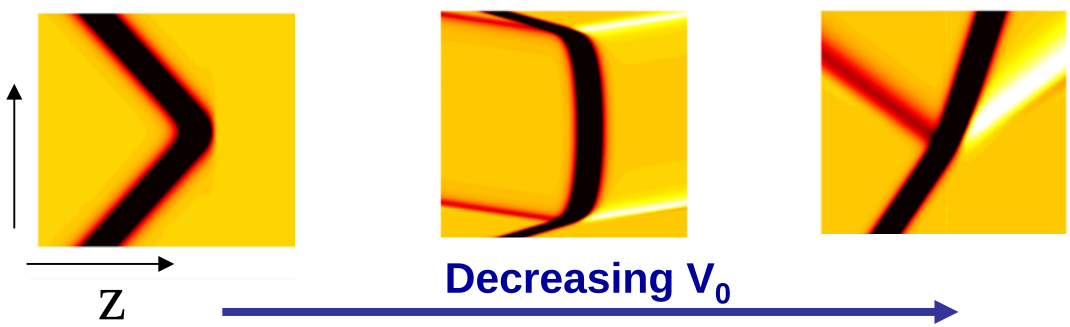
DARK SOLITON DECAY MECHANISMS

4. Longitudinal Background Density Gradient (1D Limit, $T \ll T_c$)

Simplest Case: HOMOGENEOUS 1D SYSTEM



RENORMALIZED DENSITY PLOTS



Sound Emission

Decreasing V_0

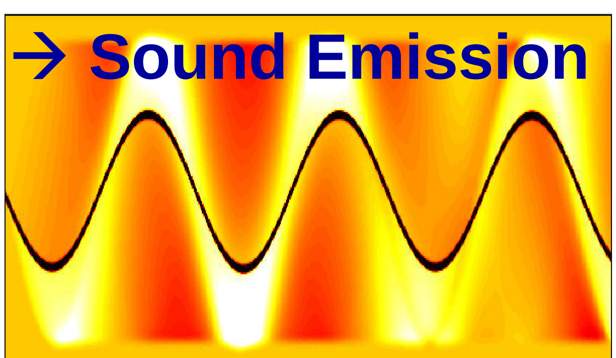
Emitted Sound Energy: Up to 50% of Initial Soliton Energy

N.G. Parker, N.P. Proukakis, M. Leadbeater, and C.S. Adams, J. Phys. B 36, 2891 (2003)

Experiments: QUASI-1D TRAPPED SYSTEM

Soliton Oscillates In Trap

$$\omega_{SOL} \approx \frac{\omega_{TRAP}}{\sqrt{2}}$$



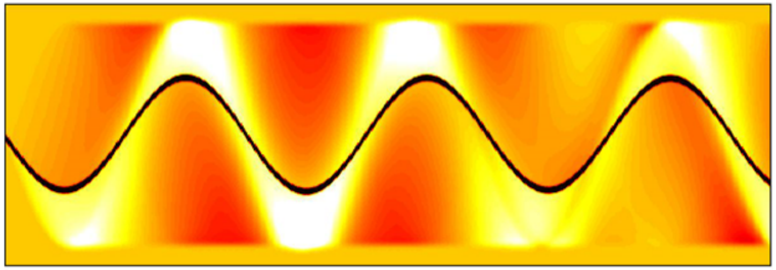
THIS TALK:

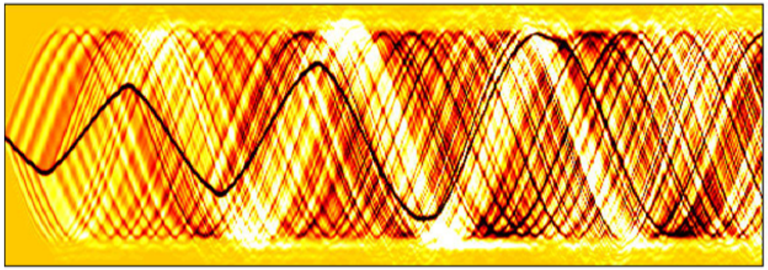
Quasi-1D Systems

$T \ll T_c$

CONTROLLING DARK SOLITON DYNAMICS

(A)
Manipulate Interactions Between Soliton & Background Sound Field





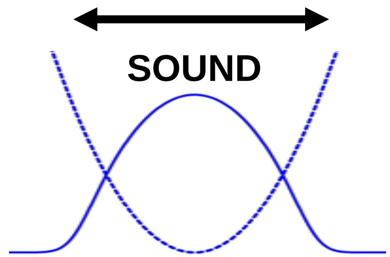
Soliton Dynamics Controlled By:

(Longitudinal) Confining Geometry

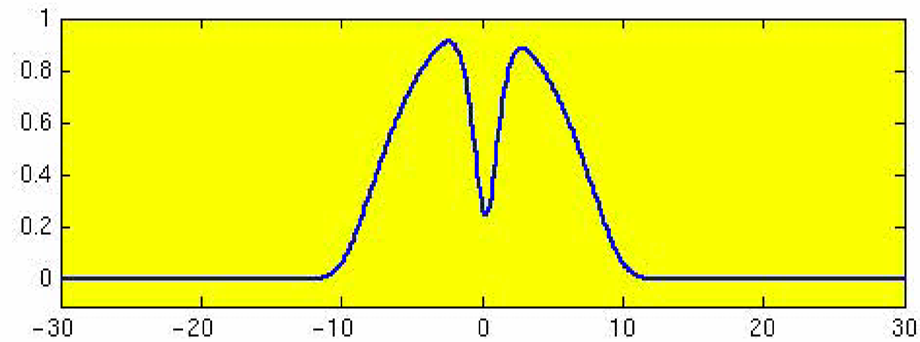
CONTROLLING SOLITON – SOUND INTERACTIONS

Parker, Proukakis, Leadbeater & Adams, PRL 90, 220401 (2003)

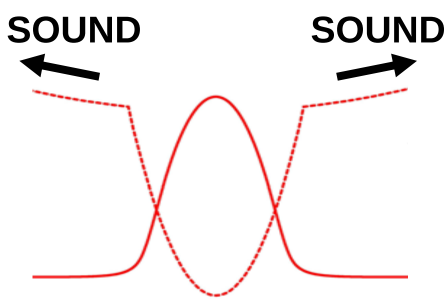
'INFINITE' TRAP



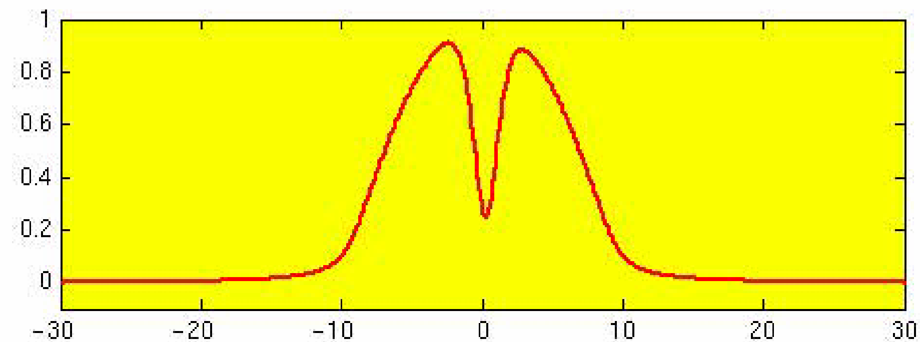
-Emitted Sound Fully Confined
-Continuous Soliton-Sound Interactions



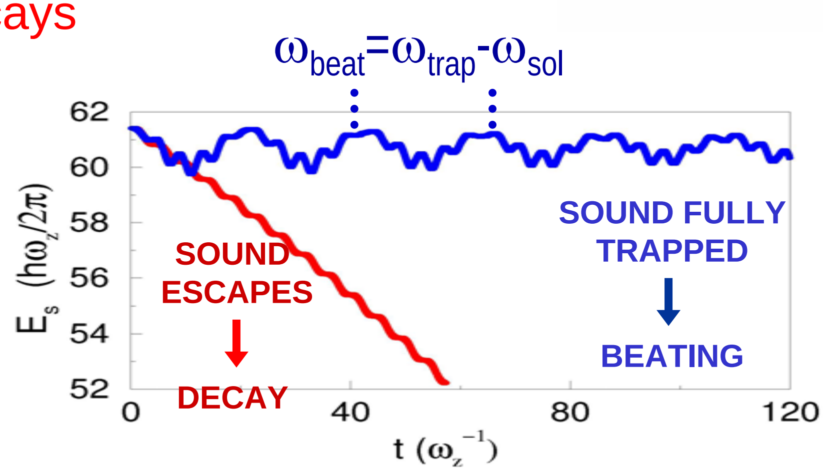
DOUBLE TRAP WITH LOW CUT-OFF



- Emitted Sound Escapes Inner Trap
- Soliton Decays



'ON-AXIS' SOLITON ENERGY



RATE OF SOUND EMISSION

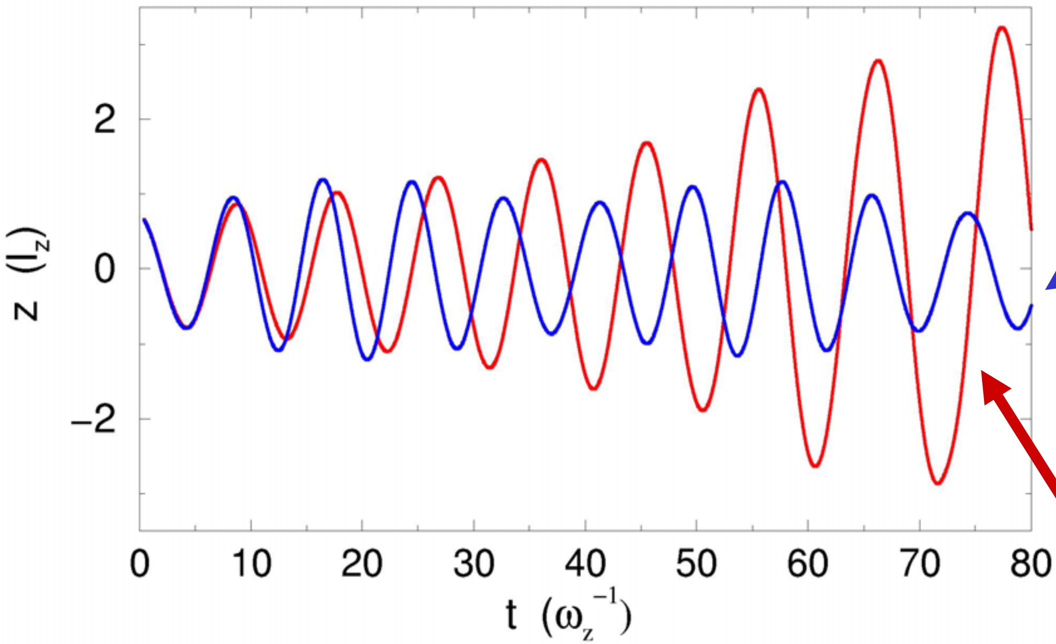
$\propto$

(Acceleration)^2 (Deformation)^2

EXPERIMENTAL SIGNATURE OF SOUND EMISSION

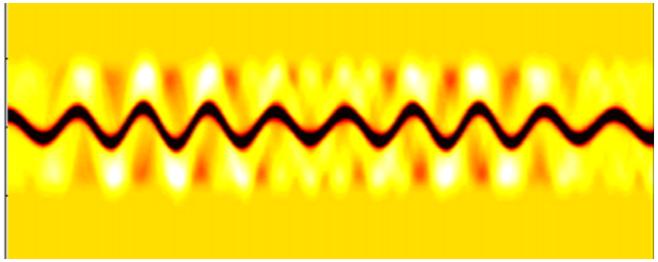
Observe Change in Soliton Oscillations for Different Cut-offs

SOUND DAMPED OUT



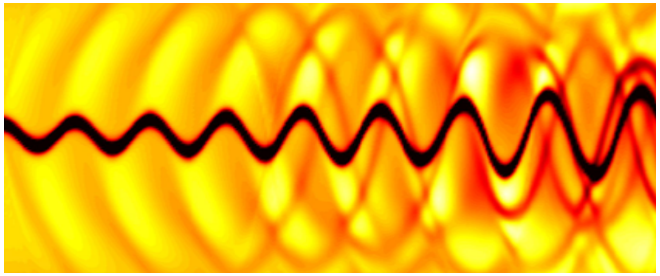
HIGH CUT-OFF ($V_0 = 5\mu$)

Soliton Stability Enhanced
(Reabsorption of Sound)



LOW CUT-OFF ($V_0 = \mu$)

Soliton Decays Rapidly
(Emitted Sound Escapes)



Parker , Proukakis, Leadbeater, Adams PRL 90, 220401 (2003)

$\tau = 200ms$

TYPICAL NUMBERS:

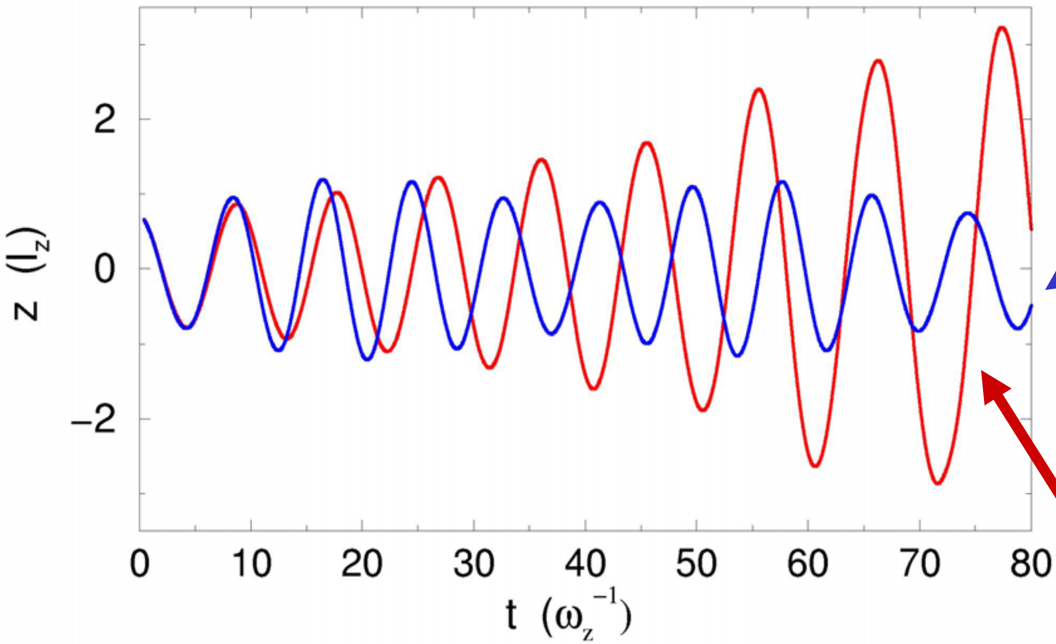
$$\omega_{OUTER} = 2\pi \times 5Hz$$
$$\omega_{DIMPLE} = 2\pi \times 50Hz$$
$$\omega_{\perp} = 2\pi \times 1250Hz$$

(^{23}Na)
$$N = 18,000$$
$$n_{1D} = 5 \times 10^{7} m^{-1}$$

EXPERIMENTAL SIGNATURE OF SOUND EMISSION

Observe Change in Soliton Oscillations for Different Cut-offs

SOUND DAMPED OUT



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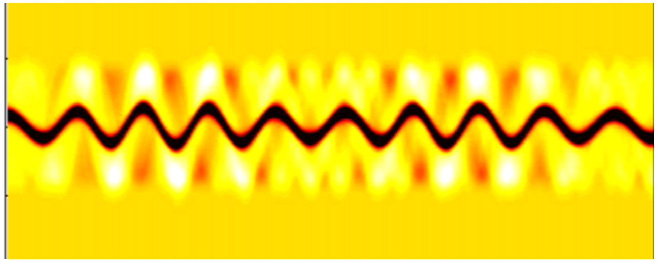
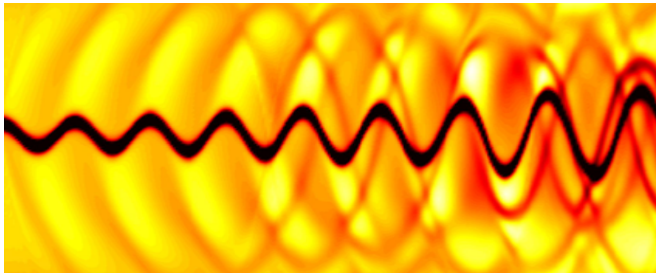


LOW CUT-OFF ($V_0 = \mu$)

Soliton Decays Rapidly
(Emitted Sound Escapes)

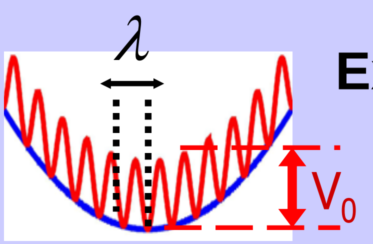


Parker , Proukakis, Leadbeater, Adams PRL 90, 220401 (2003)

Alternatively:

SOUND 'DEPHASED'

Parker, Proukakis, Barenghi, Adams
J. Phys. B 37, S175 (2004)

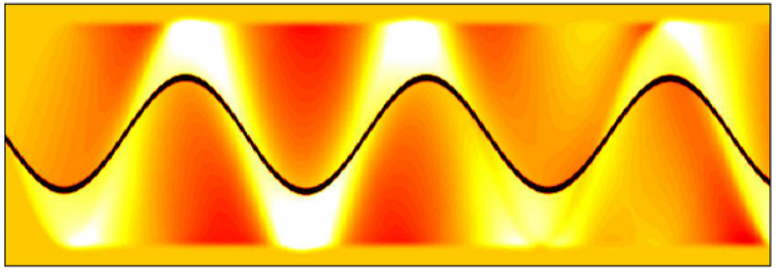


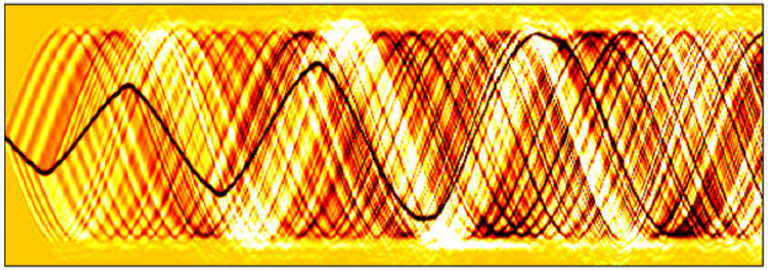
Experimental Handle:
Lattice Height
& Period

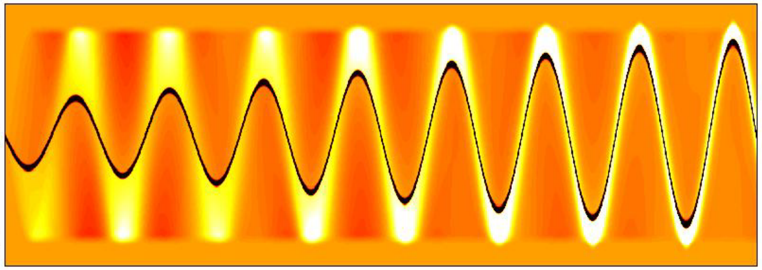
CONTROLLING DARK SOLITON DYNAMICS

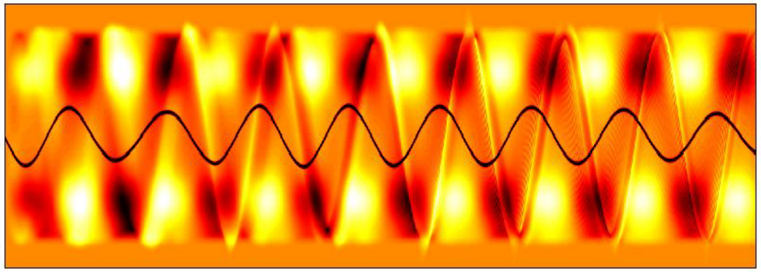
SO FAR:
Manipulate Interactions Between Soliton & Background Sound Field

ALTERNATIVE:
Engineer / Enhance Background Sound Field (Parametric Driving)









Soliton Dynamics Controlled By:

(Longitudinal) Confining Geometry

External Drive

PARAMETRICALLY DRIVEN DARK SOLITON

Dark Solitons → Generally Prone to Dissipative Losses

COMPENSATE AGAINST LOSSES BY
Periodically Pumping Energy Into System (Parametric Driving)

e.g. NONLINEAR OPTICS, SHALLOW LIQUIDS, FERROMAGNETS etc.
[e.g. Barashenkov et al., PRL 90, 054103 (2003)]

ATOMIC CONDENSATES ?

→ Pump Energy Into 'Soliton' By Manipulating Sound Excitations
Artificially Create Sound Field which Enhances Soliton Energy

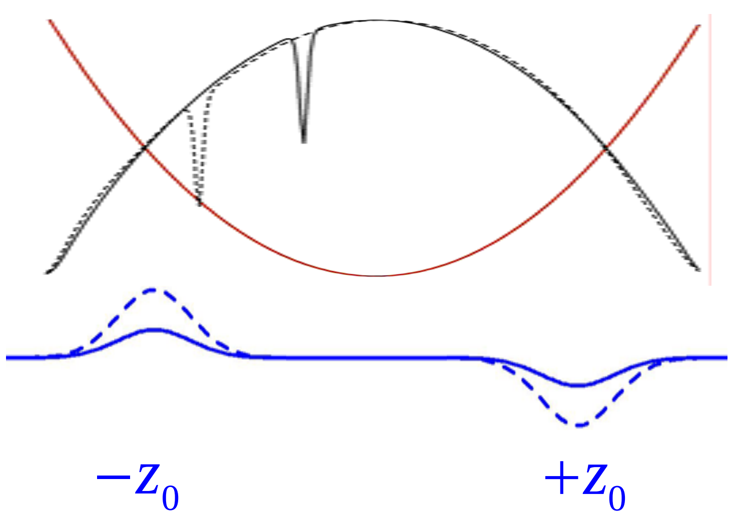
CREATE DENSITY PERTURBATION AT SOLITON FREQUENCY

Proukakis et al.
PRL 93, 130408 (2004)

HARMONIC TRAP

DENSITY

DRIVE POTENTIAL



$$V(z) = \frac{1}{2}\omega_z^{\,2} z^2 + \alpha \sin(\omega_D t)\left[e^{-(z+z_0)^2/w^2} - e^{-(z-z_0)^2/w^2} \right]$$

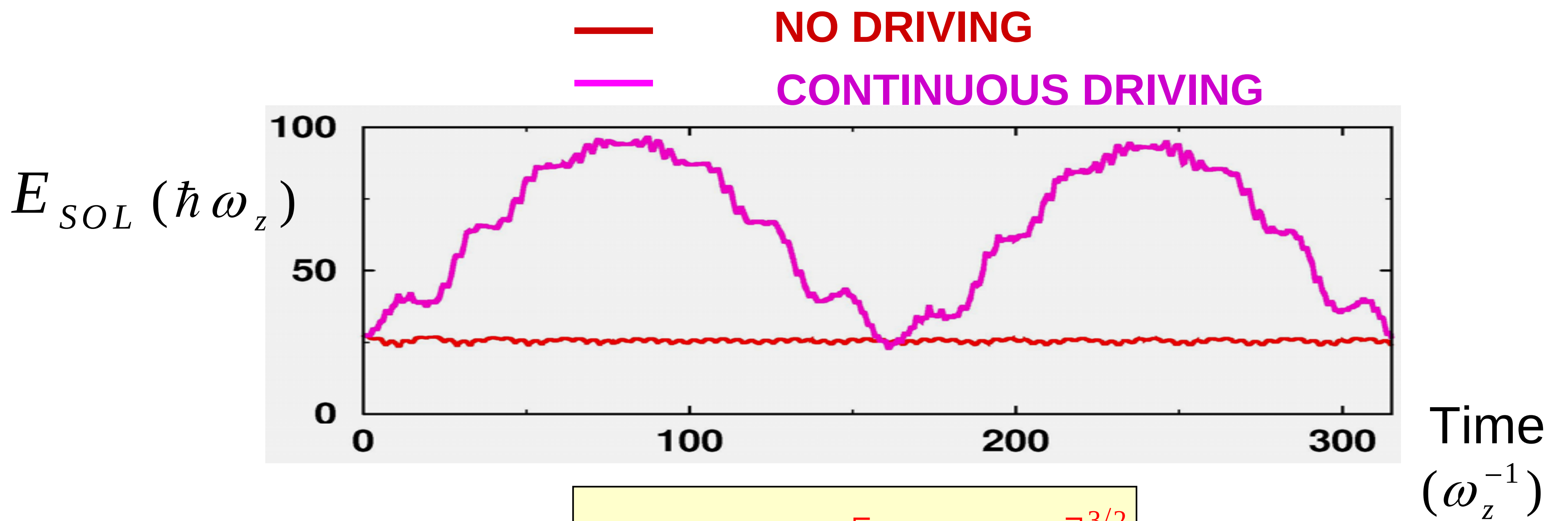
CHOOSE $\omega_D = \dfrac{\omega_z}{\sqrt{2}}$

(I)

EFFECT OF DRIVING:

NO DISSIPATION

DISSIPATIONLESS REGIME (GPE):



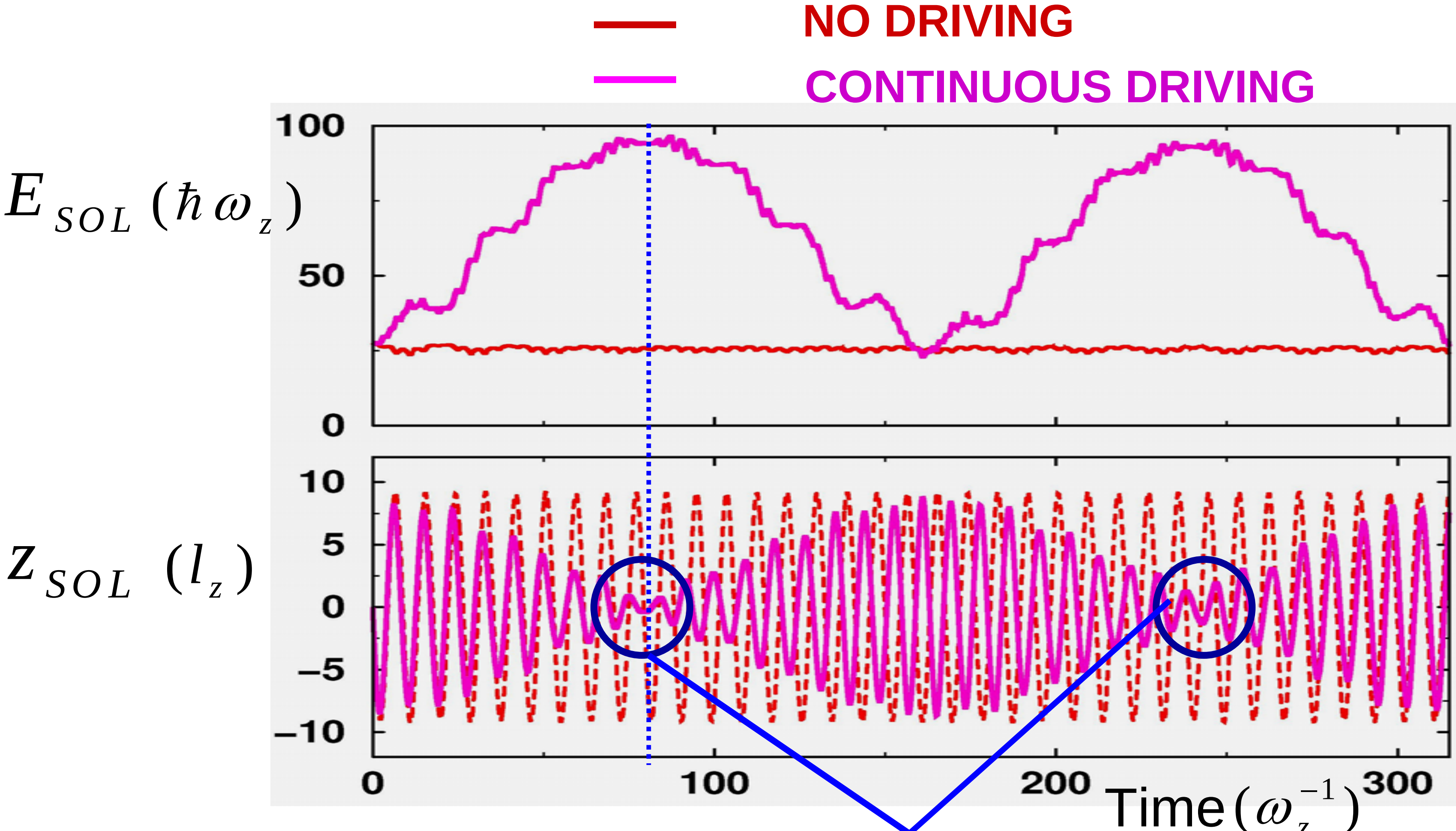
SOLITON ENERGY (Homogeneous NLSE)

$$E_{SOL} = \frac{4}{3}\hbar n c\left[1-\left(\frac{v_0}{c}\right)^2\right]^{3/2}$$

$(c \propto \sqrt{n})$

High Energy Soliton = Slow Soliton Speed at Trap Centre
(i.e. Soliton with Reduced Oscillation Amplitude)

DISSIPATIONLESS REGIME (GPE):



→ Parametric Driving Periodically Slows Down & Localizes Dark Soliton

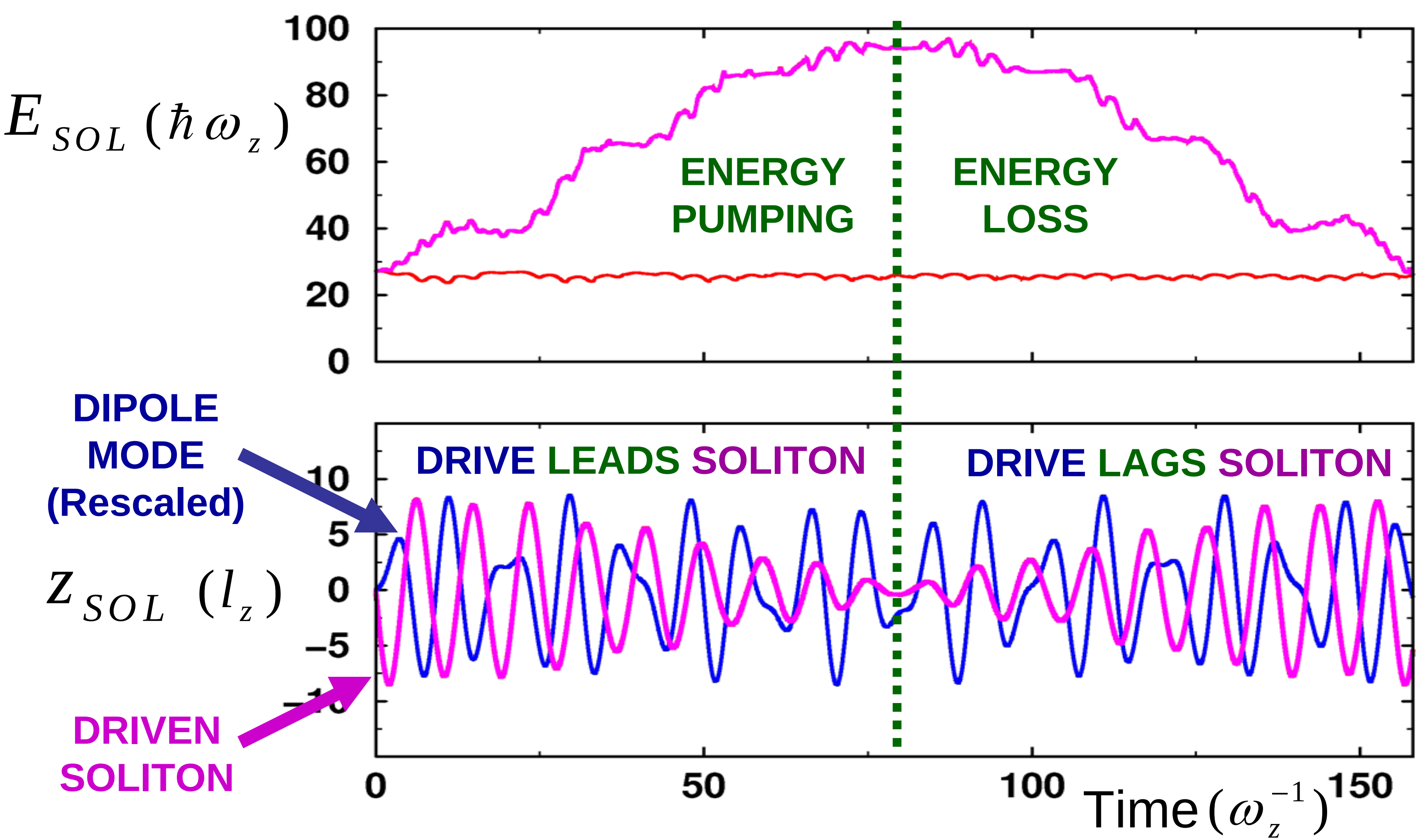
Visualization: Periodic Cycling Between 2 Soliton States of Different Energy

cf. Cycling of Driven Condensate Between 'No Vortex' & 'Single Vortex' Configurations

Caradoc-Davies, Ballagh & Burnett
PRL 83, 895 (1999)

DISSIPATIONLESS REGIME (GPE):

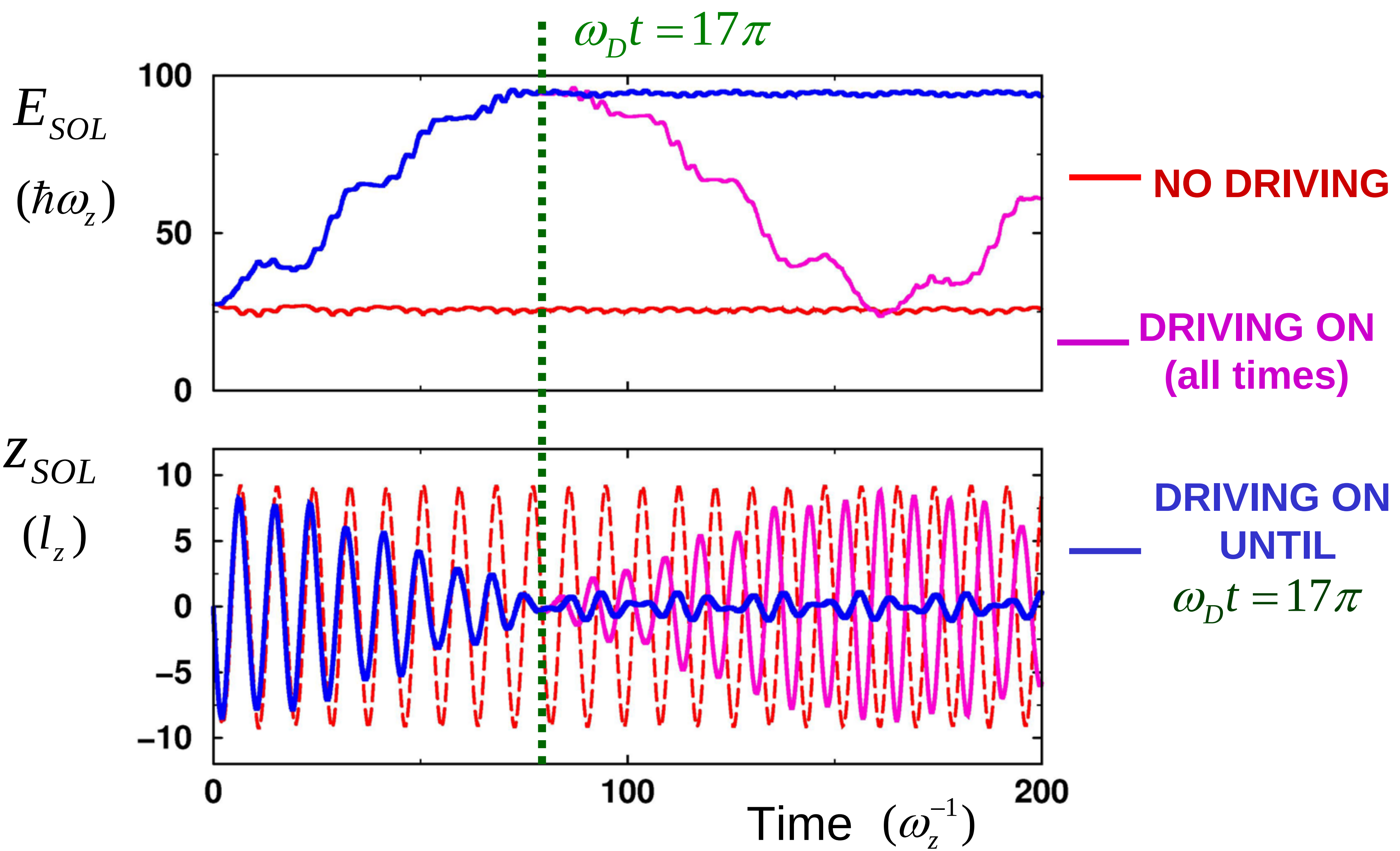
Driving Potential Fixes → Oscillation Frequency of Background Fluid; → Dipole Mode Oscillations



Energy Flow Sensitive to Phase Difference Between Drive & Soliton Oscillations

CREATION OF NEAR-STATIONARY DARK SOLITON

→Energy Transfer Into Soliton Achieved by Removing Drive at Appropriate Time

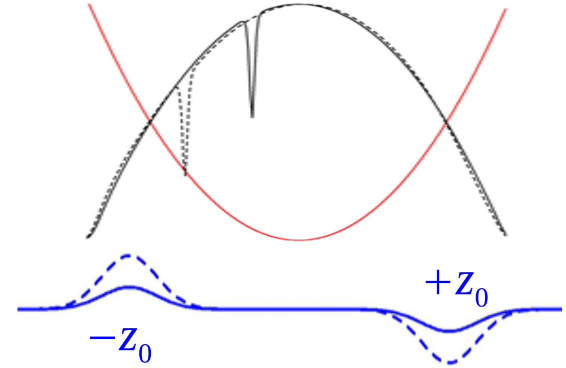


$E \approx E_{SOL}^{MAX}$ → Soliton Becomes 'Stationary' (Black Soliton)

DISSIPATIONLESS REGIME (OPTIMIZATION):

$$V(z) = \frac{1}{2}\omega_z^2 z^2$$

$$\boxed{+\alpha \sin(\omega_D t)\left[e^{-(z+z_0)^2/w^2} - e^{-(z-z_0)^2/w^2}\right]}$$

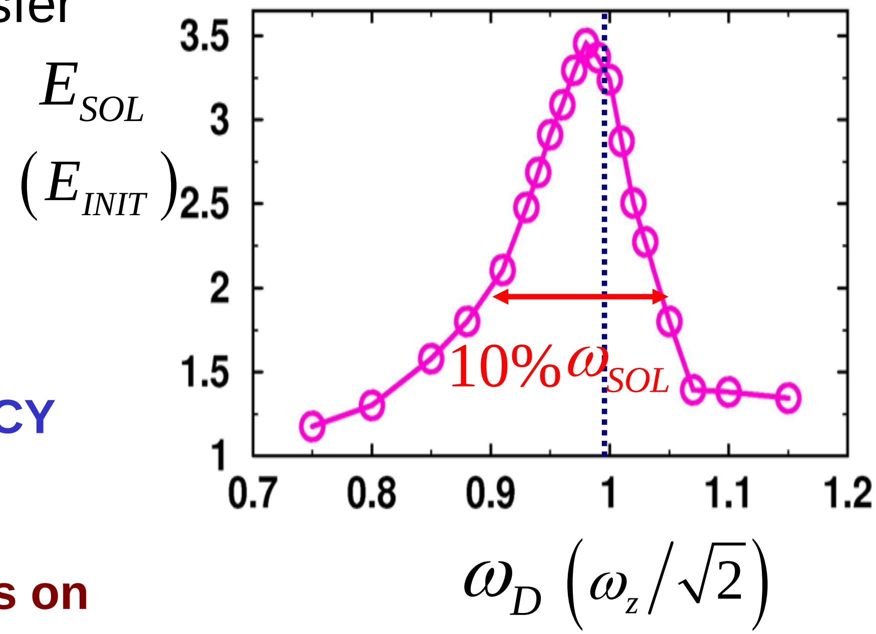


Parameters for Optimizing Energy Transfer

$$\alpha \quad \omega_D \quad z_0 \quad w \quad \& \quad \upsilon$$

many exhibiting resonant behaviour

e.g. RESONANCE IN DRIVE FREQUENCY



Maximum Energy Transfer Depends on Drive Potential Seen by Soliton

Proukakis et al
PRL 93, 130408 (2004)

(II)

EFFECT OF DRIVING:

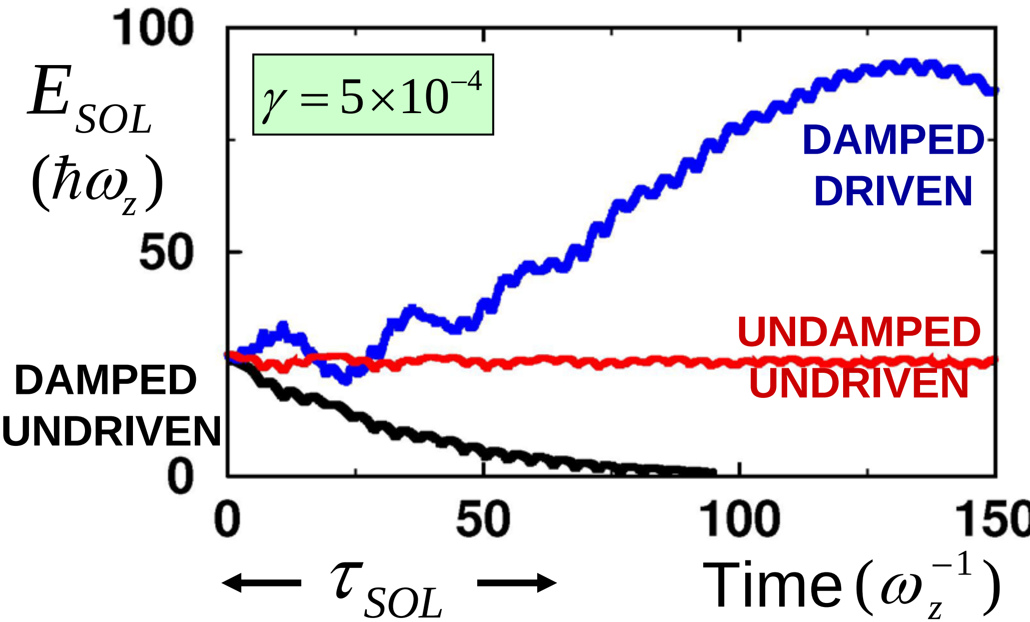
WITH DISSIPATION

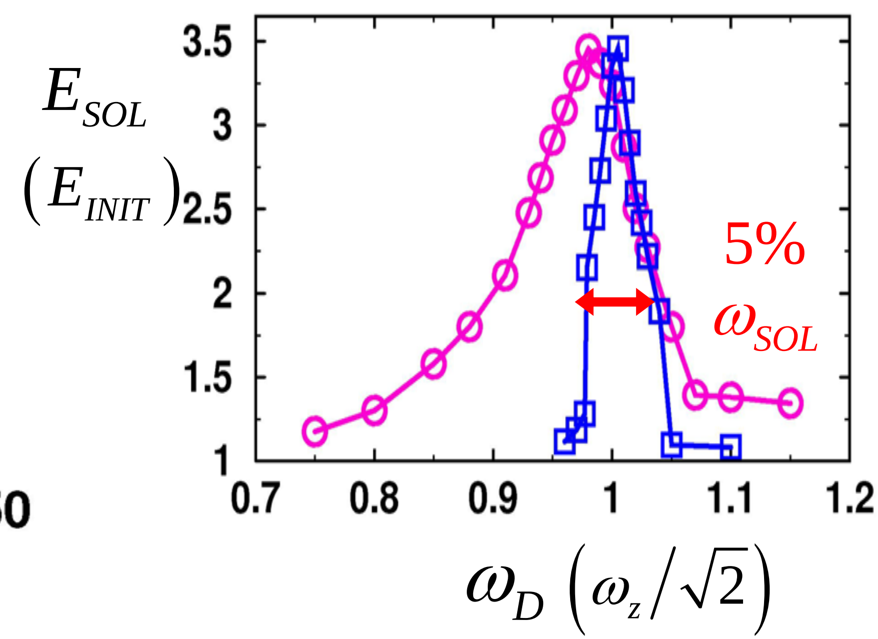
DISSIPATIVE REGIME

→ Introduce Phenomenological Damping

$$i\hbar(1-i\gamma)\frac{\partial\Phi}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+V_{TRAP}+V_{DRIVE}-\mu+g\left|\Phi\right|^2\right]\Phi$$

Choi, Morgan & Burnett PRA 57, 4057 (1998)
Tsubota, Kasamutsu & Ueda PRA 65, 023603 (2003)





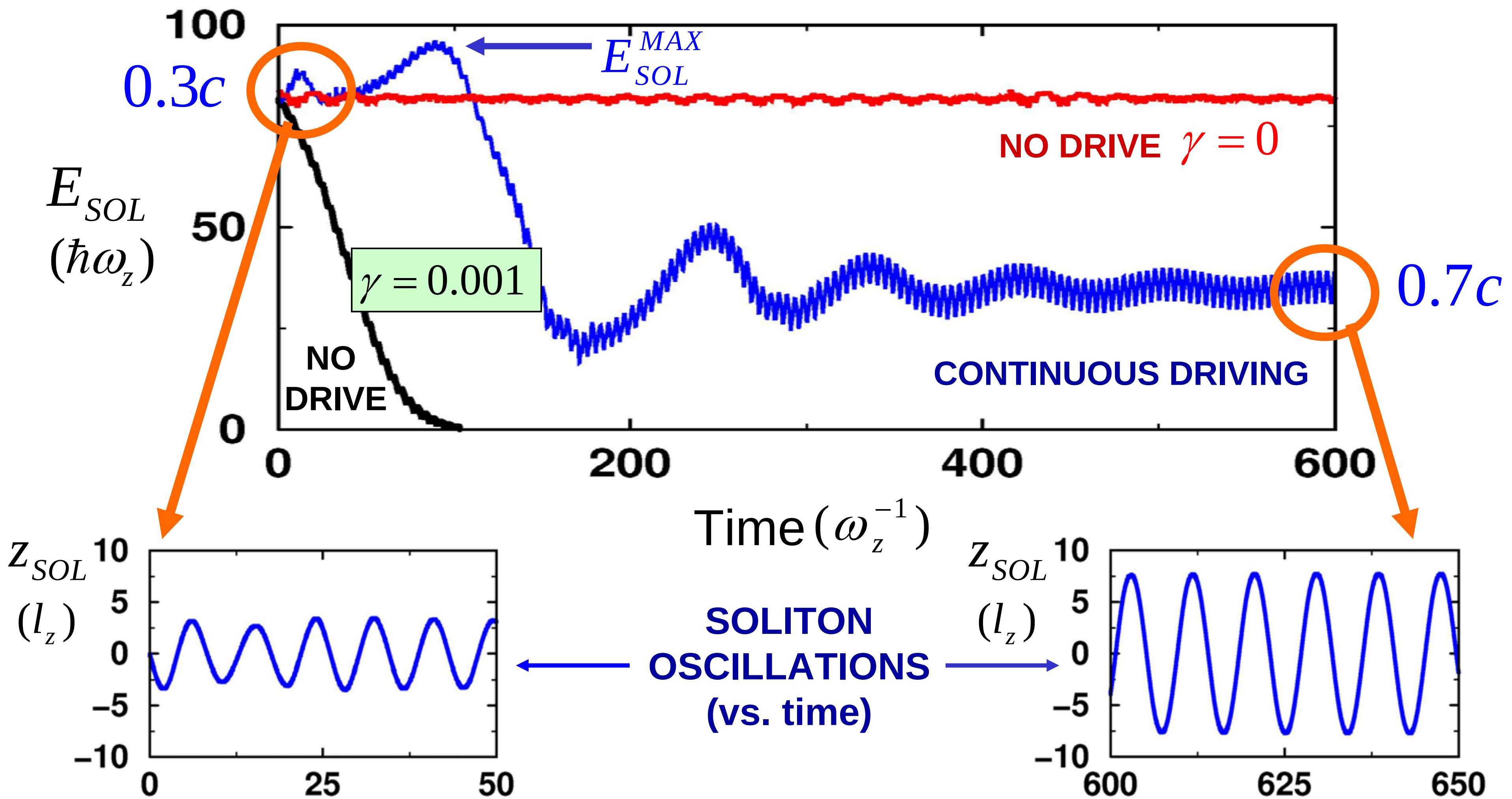
Decay Constant γ Defines Soliton Lifetime (Quasi-1D BEC: $\tau_{SOL} \approx 1s$)

[Muryshev-Fedichev-Shlyapnikov et al. PRA 60, 3220 (1999); PRL 89, 110401 (2002)]

SOLITON STABILIZATION AGAINST DECAY

For optimum stabilization, consider 'slow' soliton $v_0 = 0.3c$

v_0 : Speed of Soliton at Trap Centre (Locally Homogeneous Region)

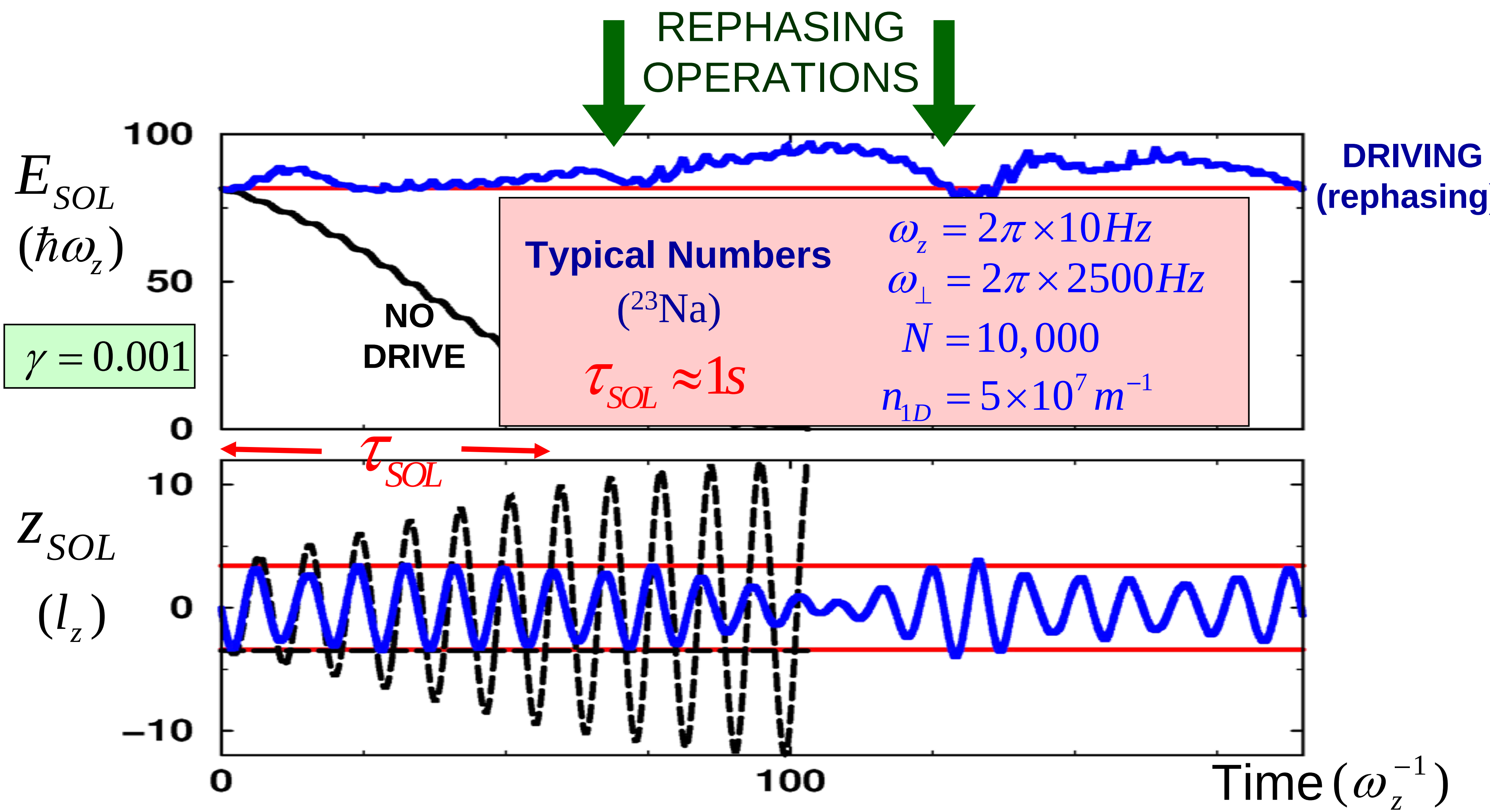


→ **Soliton STABILIZES Against Decay at Reduced Energy**

SOLITON STABILIZATION AT CONSTANT ENERGY

Potential Applications?: Stabilize Solitons at Constant Energy

→ Achieved by Rephasing Drive Relative to Soliton Oscillations



→ Rephasing Operations Can Be Repeated Indefinetely (In Principle)

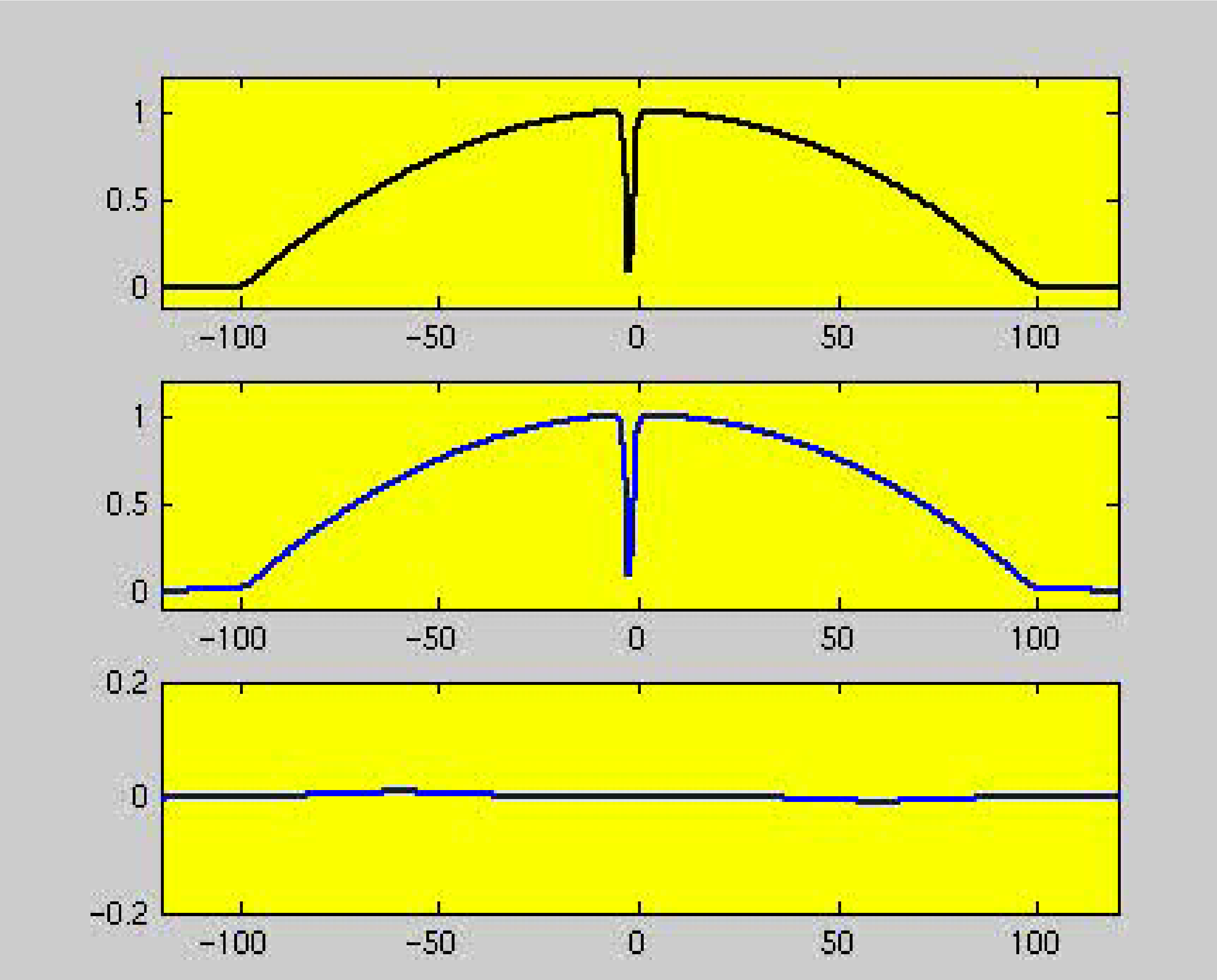
SOLITON STABILIZATION AT CONSTANT ENERGY

SOLITON DYNAMICS IN DISSIPATIVE SYSTEMS $\gamma = 0.001$

NO DRIVING

DRIVING (WITH APPROPRIATE REPHASING)

DRIVE POTENTIAL

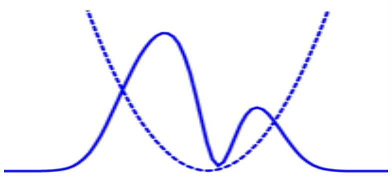


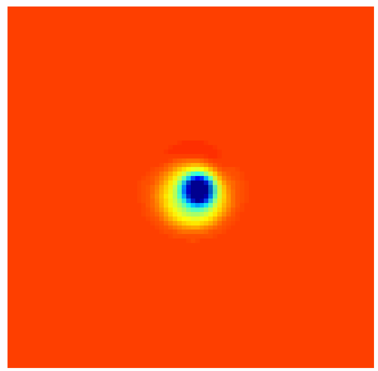
VORTEX – SOUND INTERACTIONS

Parker, Proukakis, Barenghi & Adams, PRL 92, 160403 (2004)

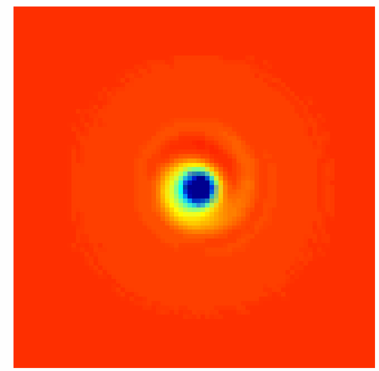
Vortex tends to follow line of constant potential, i.e. precess around trap

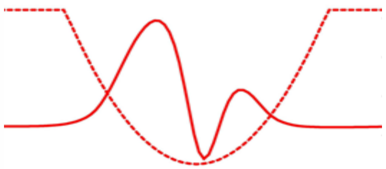
SINGLE TRAP



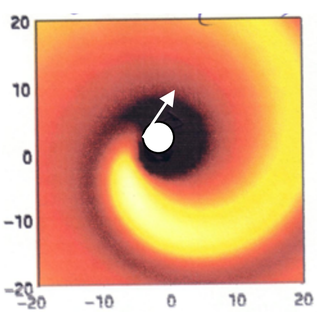


DOUBLE TRAP
(Low Cut-off)



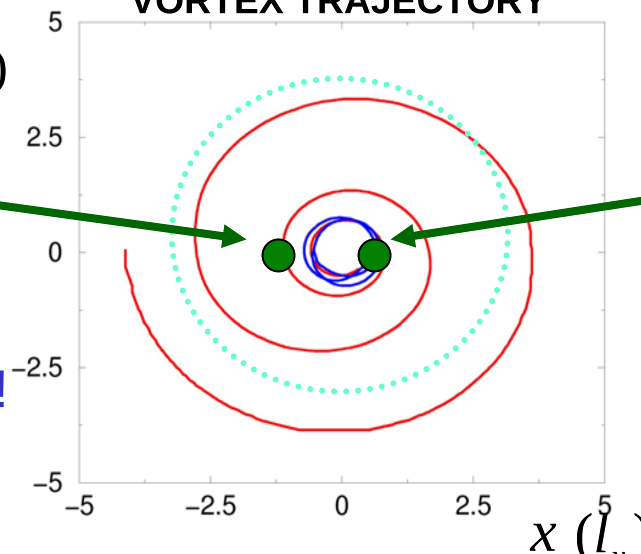


$t = 23\omega^{-1}$

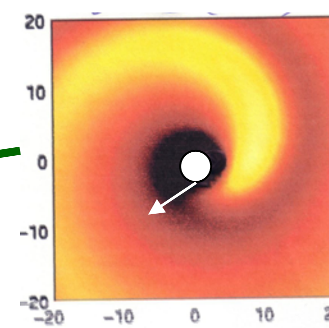


Dipolar Radiation Pattern !!!

Modified by
Vortex Motion



$t = 16\omega^{-1}$



SEE ALSO
Caradoc-Davies et al. PRA (1999)
Kevrekides et al. JPB (2003)

CONCLUSIONS

Dark Soliton Dynamics in Atomic Bose-Einstein Condensates Are Extremely Sensitive to Soliton-Sound Interactions

Interaction Controlled Via:

SYSTEM GEOMETRY

→ **Is Sound Trapped Within System ?**

PRL 90, 220401 (2003)

→ **Is Emitted Sound Field Modified?**

J. Phys. B 37, S175 (2004)
J. Opt. B 6, S380 (2004)

EXTERNAL DRIVING

Create Artificial Sound Field To Pump Energy Into Soliton

PRL 93, 130408 (2004)

Sensitive on Relative Phase Between Drive & Soliton Oscillations

↓

Can Stabilize Soliton Against Dissipation

Similar Studies for Vortices:

PRL 92, 160403 (2004) ; JLTP (In Press, 2004) [cond-mat/0405635 (2004)]

See Also: PRL 86, 1410 (2001) ; PRA 67, 015601 (2003)