Long-lived Dark Solitons
In Atomic Bose-Einstein Condensates

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OUTLINE

1. Intro to Dark Solitons & Decay Mechanisms

2. Quasi-1D Systems ($T \ll T_c$): Soliton Decays via Sound Emission Due to LONGITUDINAL Confinement

KEY POINT: Soliton Dynamics Are Sensitive to Soliton – Sound Interactions

3. How to Control Soliton Dynamics?
   - Manipulate Strength of Soliton-Sound Interactions
   - By Parametric Driving → ‘Long-lived Solitons’
DARK SOLITONS IN ATOMIC BEC

Density Dips Propagating without Dispersion

SOLUTIONS OF HOMOGENEOUS 1D NONLINEAR SCHROEDINGER EQUATION

\[ i \frac{\partial \Phi}{\partial t} = -\frac{1}{2} \frac{\partial^2 \Phi}{\partial z^2} + |\Phi|^2 \Phi \]

T \rightarrow 0 ATOMIC BEC

Gross-Pitaevskii Equation

\[ i \hbar \frac{\partial \Phi}{\partial t} = \left[ -\frac{\hbar^2 \nabla^2}{2m} + \frac{1}{2} m \omega_{\perp}^2 (x^2 + y^2) + \frac{1}{2} m \omega_z^2 z^2 - \mu + g|\Phi|^2 \right] \Phi \]

HARMONIC CONFINEMENT:

Transverse  Longitudinal  Repulsive Interactions

BEC Dark Soliton Experiments:

**NIST:** Science 287, 97 (1999)

**Hannover:** PRL, 83, 5198 (1999)

Rowlands Institute, JILA
DARK SOLITON DECAY MECHANISMS

1. Thermal Fluctuations
   Muryshev et al., PRL 89, 110401 (2002)

2. Quantum Fluctuations
   Feder et al., PRA 62, 053606 (2000)
   Brand & Reinhardt, PRA 65, 043612 (2002)

3. Transverse Excitations
   (3D Systems)
   Feder et al., PRA 62, 053606 (2000)
   Brand & Reinhardt, PRA 65, 043612 (2002)

→ Dark Soliton Decays into Vortex Rings

‘SNAKE INSTABILITY’ OBSERVED EXPERIMENTALLY: Anderson et al., PRL 86, 2926 (2001)

→ Suppressed in Highly-Elongated ‘Quasi-1D’ Systems
DARK SOLITON DECAY MECHANISMS

4. Longitudinal Background Density Gradient (1D Limit, $T << T_c$)

Simplest Case: HOMOGENEOUS 1D SYSTEM

Density $\downarrow$

Potential $\uparrow$

$V_0$

Sound Emission

RENORMALIZED DENSITY PLOTS

$\downarrow t$

$\downarrow Z$

Decreasing $V_0$

Emitted Sound Energy: Up to 50% of Initial Soliton Energy


Experiments: QUASI-1D TRAPPED SYSTEM

Soliton Oscillates In Trap

$\omega_{SOL} \approx \frac{\omega_{TRAP}}{\sqrt{2}}$

$\rightarrow$ Sound Emission

THIS TALK:

Quasi-1D Systems $T << T_c$
CONTROLLING DARK SOLITON DYNAMICS

(A) Manipulate Interactions Between Soliton & Background Sound Field

Soliton Dynamics Controlled By:

(Longitudinal) Confining Geometry
CONTROLLING SOLITON – SOUND INTERACTIONS


‘INFINITE’ TRAP
- Emitted Sound Fully Confined
- Continuous Soliton-Sound Interactions

DOUBLE TRAP WITH LOW CUT-OFF
- Emitted Sound Escapes Inner Trap
- Soliton Decays

‘ON-AXIS’ SOLITON ENERGY

\[ \omega_{\text{beat}} = \omega_{\text{trap}} - \omega_{\text{sol}} \]

\[ \propto (\text{Acceleration})^2 \times (\text{Deformation})^2 \]

RATE OF SOUND EMISSION
EXPERIMENTAL SIGNATURE OF SOUND EMISSION

Observe Change in Soliton Oscillations for Different Cut-offs

<table>
<thead>
<tr>
<th>Cut-off</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIGH</td>
<td>( V_0 = 5\mu )</td>
</tr>
<tr>
<td>LOW</td>
<td>( V_0 = \mu )</td>
</tr>
</tbody>
</table>

SOUND DAMPED OUT

- Soliton Stability Enhanced (Reabsorption of Sound)
- Low Cut-off: Soliton Decays Rapidly (Emitted Sound Escapes)

\[
\tau = 200\, \text{ms}
\]

**TYPICAL NUMBERS:**

- \( \omega_{\text{OUTER}} = 2\pi \times 5\, \text{Hz} \)
- \( \omega_{\text{DIMPLE}} = 2\pi \times 50\, \text{Hz} \)
- \( \omega_{\perp} = 2\pi \times 1250\, \text{Hz} \)

\[\begin{align*}
(\text{^{23}Na}) & \quad N = 18,000 \\
& \quad n_{1D} = 5 \times 10^7 \, m^{-1}
\end{align*}\]
EXPERIMENTAL SIGNATURE OF SOUND EMISSION

Observe Change in Soliton Oscillations for Different Cut-offs

**SOUND DAMPED OUT**

HIGH CUT-OFF \( (V_0 = 5\mu) \)
- Soliton Stability Enhanced
- (Reabsorption of Sound)

LOW CUT-OFF \( (V_0 = \mu) \)
- Soliton Decays Rapidly
- (Emitted Sound Escapes)


Alternatively:

**SOUND ‘DEPHASED’**

Parker, Proukakis, Barenghi, Adams
CONTROLLING DARK SOLITON DYNAMICS

SO FAR:
Manipulate Interactions Between Soliton & Background Sound Field

ALTERNATIVE:
Engineer / Enhance Background Sound Field (Parametric Driving)

Soliton Dynamics Controlled By:
(Longitudinal) Confining Geometry
External Drive
PARAMETRICALLY DRIVEN DARK SOLITON

Dark Solitons → Generally Prone to Dissipative Losses

COMPENSATE AGAINST LOSSES BY Periodically Pumping Energy Into System (Parametric Driving)

e.g. NONLINEAR OPTICS, SHALLOW LIQUIDS, FERROMAGNETS etc.
[ e.g. Barashenkov et al., PRL 90, 054103 (2003) ]

ATOMIC CONDENSATES?

→ Pump Energy Into ‘Soliton’ By Manipulating Sound Excitations
Artificially Create Sound Field which Enhances Soliton Energy

CREATE DENSITY PERTURBATION AT SOLITON FREQUENCY

Proukakis et al.
PRL 93, 130408 (2004)

\[ V(z) = \frac{1}{2} \omega_z^2 z^2 + \alpha \sin(\omega_D t) \left[ e^{-(z+z_0)^2/w^2} - e^{-(z-z_0)^2/w^2} \right] \]

CHOOSE \[ \omega_D = \frac{\omega_z}{\sqrt{2}} \]
(I)

EFFECT OF DRIVING:

NO DISSIPATION
DISSIPATIONLESS REGIME (GPE):

NO DRIVING

CONTINUOUS DRIVING

$E_{SOL}(\hat{h} \omega_z)$

SOLITON ENERGY
(Homogeneous NLSE)

$E_{SOL} = \frac{4}{3} \hat{h} n c \left[ 1 - \left( \frac{v_0}{c} \right)^2 \right]^{3/2}$

$(c \propto \sqrt{n})$

High Energy Soliton = Slow Soliton Speed at Trap Centre
(i.e. Soliton with Reduced Oscillation Amplitude)
DISSIPATIONLESS REGIME (GPE):

- **NO DRIVING**
- **CONTINUOUS DRIVING**

$E_{SOL}(\hbar \omega_z)$

$Z_{SOL}(l_z)$

- **Visualization**: Periodic Cycling Between 2 Soliton States of Different Energy
- **cf.** Cycling of Driven Condensate Between ‘No Vortex’ & ‘Single Vortex’ Configurations

Caradoc-Davies, Ballagh & Burnett

PRL 83, 895 (1999)
DISSIPATIONLESS REGIME (GPE):

Driving Potential Fixes

Oscillation Frequency of Background Fluid

Dipole Mode Oscillations

$E_{SOL}(\hbar \omega_z)$

Energy Flow Sensitive to Phase Difference Between Drive & Soliton Oscillations

Energy Pumping

Energy Loss

Drive Leads Soliton

Drive Lags Soliton

DIPOLE MODE (Rescaled)

Driven Soliton
CREATION OF NEAR-STATIONARY DARK SOLITON

→ Energy Transfer Into Soliton Achieved by Removing Drive at Appropriate Time

\[ E_{SOL} \approx E_{SOL}^{MAX} \]

Soliton Becomes ‘Stationary’ (Black Soliton)
DISSIPATIONLESS REGIME (OPTIMIZATION):

\[ V(z) = \frac{1}{2} \alpha \omega_z^2 z^2 \]

\[ + \alpha \sin(\omega_D t) \left[ e^{-(z+z_0)^2/w^2} - e^{-(z-z_0)^2/w^2} \right] \]

Parameters for Optimizing Energy Transfer:
- \( \alpha \)
- \( \omega_D \)
- \( z_0 \)
- \( w \)
- \( v \)

many exhibiting resonant behaviour

e.g. RESONANCE IN DRIVE FREQUENCY

Maximum Energy Transfer Depends on Drive Potential Seen by Soliton

Proukakis et al
PRL 93, 130408 (2004)
(II)

EFFECT OF DRIVING:

WITH DISSIPATION
DISSIPATIVE REGIME

\[ i\hbar (1 - i\gamma) \frac{\partial \Phi}{\partial t} = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{TRAP}} + V_{\text{DRIVE}} - \mu + g |\Phi|^2 \right] \Phi \]

Choi, Morgan & Burnett  PRA 57, 4057 (1998)

\[ 5\% \]

Decay Constant \( \gamma \) Defines Soliton Lifetime  (Quasi-1D BEC: \( \tau_{\text{SOL}} \approx 1s \))

[ Muryshev-Fedichev-Shlyapnikov et al. PRA 60, 3220 (1999); PRL 89, 110401 (2002) ]
SOLITON STABILIZATION AGAINST DECAY

For optimum stabilization, consider ‘slow’ soliton \( v_0 = 0.3c \)

\( v_0 \): Speed of Soliton at Trap Centre (Locally Homogeneous Region)

\[ E_{SOL} (\hbar \omega_z) \]

\( \gamma = 0.001 \)

\( \gamma = 0 \)

\( 0.3c \)

\( 0.7c \)

\( E_{MAX} \)

NO DRIVE

CONTINUOUS DRIVING

\( z_{SOL} (l_z) \)

SOLITON OSCILLATIONS (vs. time)

\( z_{SOL} (l_z) \)

\( \rightarrow \) Soliton STABILIZES Against Decay at Reduced Energy
SOLITON STABILIZATION AT CONSTANT ENERGY

Potential Applications?: Stabilize Solitons at Constant Energy

→ Achieved by Rephasing Drive Relative to Soliton Oscillations

\[ \tau_{SOL} \approx 1 \text{s} \]

Typical Numbers

\( ^{23}\text{Na} \)

\[ \begin{align*}
\omega_z &= 2\pi \times 10 \text{Hz} \\
\omega_\perp &= 2\pi \times 2500 \text{Hz} \\
N &= 10,000 \\
n_{1D} &= 5 \times 10^7 \text{ m}^{-1}
\end{align*} \]

→ Rephasing Operations Can Be Repeated Indefinitely (In Principle)
SOLITON STABILIZATION AT CONSTANT ENERGY

SOLITON DYNAMICS IN DISSIPATIVE SYSTEMS $\gamma = 0.001$

- **NO DRIVING**
- **DRIVING (WITH APPROPRIATE REPHASING)**
- **DRIVE POTENTIAL**
VORTEX – SOUND INTERACTIONS

Vortex tends to follow line of constant potential, i.e. precess around trap

SINGLE TRAP

DOUBLE TRAP
(Low Cut-off)

Dipolar Radiation Pattern !!!
Modified by Vortex Motion

SEE ALSO
Caradoc-Davies et al. PRA (1999)
Kevrekides et al. JPB (2003)
CONCLUSIONS

Dark Soliton Dynamics in Atomic Bose-Einstein Condensates Are Extremely Sensitive to Soliton-Sound Interactions

Interaction Controlled Via:

SYSTEM GEOMETRY

→ Is Sound Trapped Within System?

PRL 90, 220401 (2003)

→ Is Emitted Sound Field Modified?


EXTERNAL DRIVING

Create Artificial Sound Field To Pump Energy Into Soliton

PRL 93, 130408 (2004)

Sensitive on Relative Phase Between Drive & Soliton Oscillations

Can Stabilize Soliton Against Dissipation

Similar Studies for Vortices:
