BEC in a quasiperiodic optical lattice: a quantum gas at the edge between order and disorder

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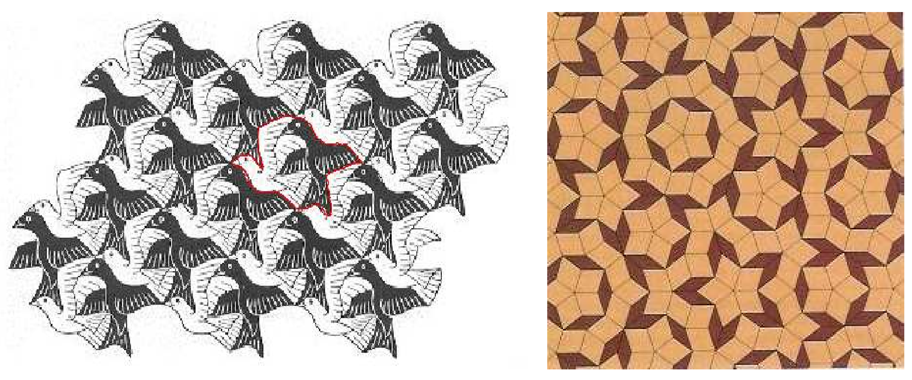
Abstract

We analyze the physics of Bose-Einstein condensates confined in quasi-periodic optical lattices (optical quasicrystals), which offer an intermediate situation between ordered and disordered systems. We discuss in particular the time-of-flight interference picture associated with the extended nature of the wavefunction, as well as the localization effects during the expansion of a Bose-Einstein condensate in an optical quasicrystal. We analyze in detail the crossover between diffusive and localized regimes when the quasi-periodic potential is switched on. We investigate additionally the role of the interatomic interactions in the condensate diffusion.

Long-range order in quasicrystal optical lattices

• Quasicrystals in condensed matter



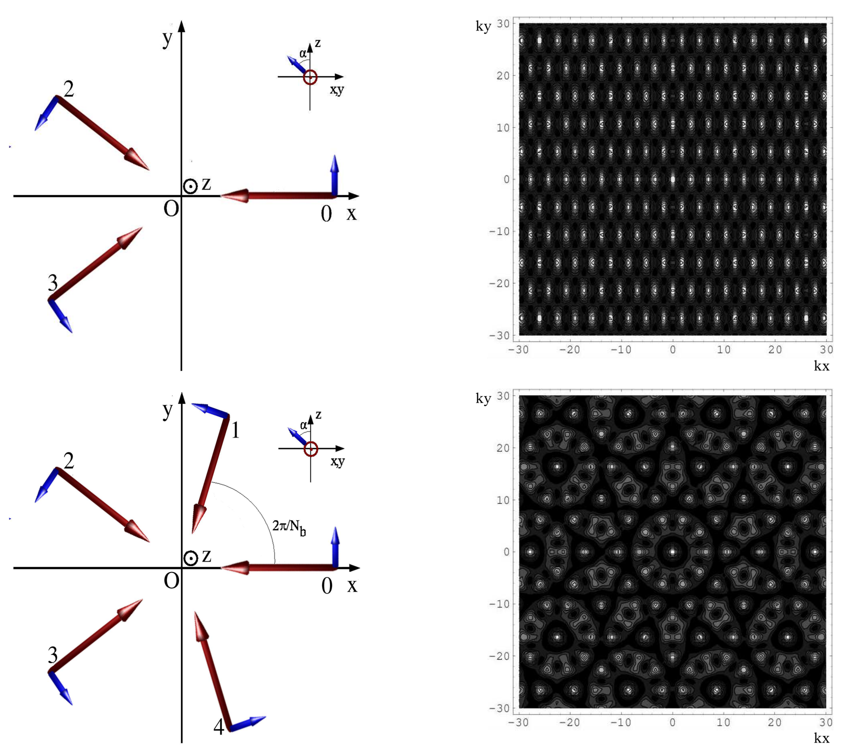
Periodic [left; from M. C. Escher, *Regular Division of the Plane With Birds*, woodcut, (1949)] and quasiperiodic [right; Fivefold symmetric Penrose tiling] tilings of the plan.

Quasicrystals form a new class of solids [D. Shechtman *et al*, Phys. Rev. Lett. **53**, 1951 (1984); D. Levine and P. J. Steinhardt, *ibid.* **53**, 2477 (1984)] intermediate between ordered and disordered systems. They have:

– long-range order (repetitiveness; diffraction peaks)

– no translational invariance (lack of periodicity of position and/or type of the crystal ions)

• Optical periodic and quasiperiodic lattices



Laser beam configuration (left) and lattice potential (right) for a periodic (top) and a quasiperiodic (bottom) optical lattice. In the quasiperiodic case, five laser beams are arranged in the (Oxy) plane with a fivefold rotation symmetry. The periodic configuration corresponds to the same configuration except that lasers 1 and 4 have zero intensity.

The quasiperiodic lattice is long-range ordered but without translational invariance (quasiperiodicity).

• Bose-Einstein condensate in an optical lattice

Consider a Bose-Einstein condensate at zero temperature in a twofold trapping potential composed of

– a harmonic potential $V_{\mathrm{ho}}(\vec{r}) = \frac{M}{2}\left(\omega_\perp^2 \vec{r}_\perp^2 + \omega_z^2 z^2\right)$

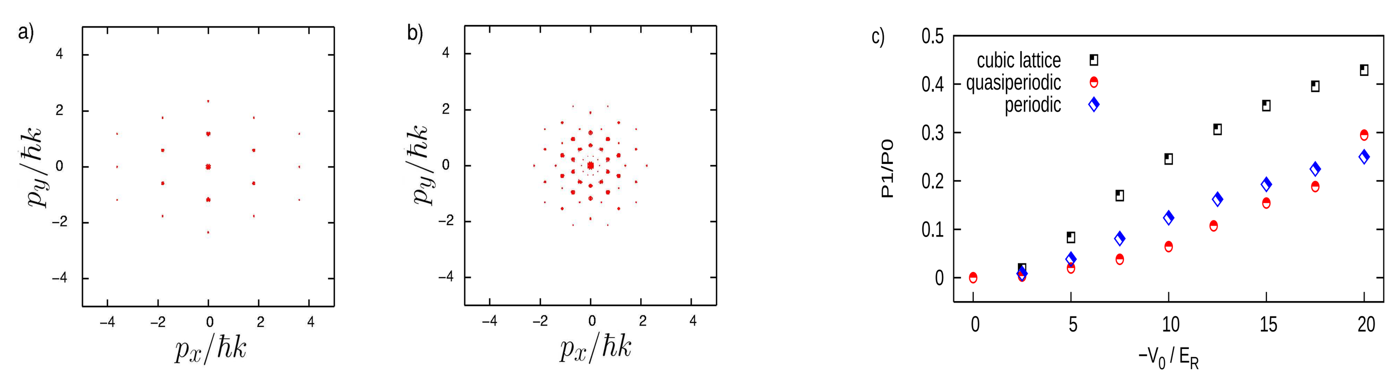
– an optical lattice with periodic or quasiperiodic order (see configurations above)

$$V_{\mathrm{latt}}(\vec{r}_\perp) = \frac{V_0}{|\sum \mathcal{E}_j|^2} \left| \sum_{0\leq j<N_{\mathrm{b}}} \mathcal{E}_j \vec{\epsilon}_j \, e^{-i(\vec{k}_j\cdot\vec{r}_\perp + \varphi_j)} \right|^2 . \tag{1}$$

The harmonic confinement along the axis orthogonal to the lattice plane (z) is assumed to be strong so that transverse degrees of freedom do not support any excitation: the dynamics is 2D.

• Quasiperiodic long-range ordered BEC

The static Gross-Pitaevskii equation $\mu\psi = \left[-\hbar^2\vec{\nabla}^2/2M + V_{\mathrm{ho}}(\vec{r}) + V_{\mathrm{latt}}(\vec{r}_\perp) + g_{\mathrm{2D}}|\psi|^2\right]\psi$ provides us with the static BEC wavefunction. From this, we compute the momentum distribution (which can be experimentally mapped into time-of-flight measurements).



Momentum distributions of BEC's in a) periodic and b) quasiperiodic optical lattices. Also shown in c) is a comparison of resolutions in periodic and quasiperiodic cases.

Our results clearly show that in both cases, the BEC exhibits (periodic or quasiperiodic) long-range order. In the quasiperiodic case, the BEC wavefunction shows a five-fold symmetry similar to the Penrose tiling which is incompatible with any translation invariance. Similar resolutions are obtained for periodic and quasiperiodic lattices.

Coherent expansion of a BEC in periodic and quasiperiodic optical lattices

• Quantum dynamics in optical lattices

We study the coherent expansion of the BEC in a (quasi)periodic optical lattice. The sequence is:

– preparation in harmonic trap and (periodic or quasiperiodic) optical lattice (see above);

– the harmonic trap and eventually the 2-body interaction is/are suddenly switched off at $t = 0$;

– we observe the quantum diffusion of the BEC wavefunction in the (quasi)periodic lattice; this is computed by means of the time-dependent Gross-Pitaevskii equation:

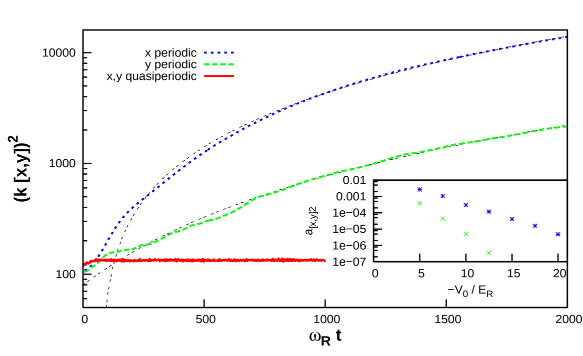
$$i\hbar\partial_t\psi = \left[-\hbar^2\vec{\nabla}^2/2M + V_{\mathrm{latt}}(\vec{r}_\perp) + g_{\mathrm{2D}}|\psi|^2\right]\psi \, . \tag{2}$$

• Diffusion versus Anderson localization

In the absence of interactions, the diffusion corresponds to

– periodic: anisotropic ballistic expansion $\langle\mu^2(t)\rangle = a_{\mu 0} + a_{\mu 1}t + a_{\mu 2}t^2$ with $a_{\mu 2} \propto 1/M^{*2}$ where $M^*/M \sim E_{\mathrm{R}}/J$ is the effective mass

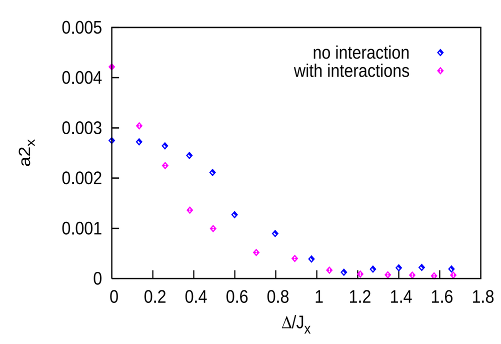
– quasiperiodic: Anderson localization due to quasidisorder



This shows dramatically different transport properties due to the nature of the quantum eigenstates in the periodic (extended wavefunctions) and quasiperiodic (non-extended wavefunctions) lattices. These behavior of the coherent BEC is drastically different to the one obtained with non-degenerate cold atoms where similar diffusions were found for both cases [L. Guidoni *et al*, Phys. Rev. Lett. **79**, 3363 (1997); Phys. Rev. A **60**, R4233 (1999)].

• Disorder-induced phase transition to localization

By controlling the intensity of lasers 1 and 4, one can turn continuously from a periodic to a quasiperiodic lattice and one can correspondingly investigate the transition from a diffusive phase to a localized phase.



Transition from a diffusive to a localized phase in absence or presence of interactions. Here, Δ is the pseudo-disorder parameter (variance of the on-site energy due to weak intensity of lasers 1 and 4).

The phase transition from a diffusive phase to a localized phase in the x direction occurs when the pseudo-disorder is of the order of magnitude of the tunneling J_x.

• Effects of interactions

We finally study the effects of interactions on the coherent diffusion.

– periodic: During the initial preparation of the BEC, the wavefunction results from a competition between interactions and on-site energy (due to harmonic potential). After switching off the harmonic potential, the on-site interaction energy $E_j = cst - V_j$ can depend significantly on the site and consequently make the tunneling non-resonant. More precisely,

∗) if $J \gg |V_j - V_l|$ where j and l are ajacent sites, the ballistic expansion is enhanced due to conversion of interaction energy into kinetic energy

∗) if $J \ll |V_j - V_l|$ the tunneling is non-resonant and this leads to localization.

– quasiperiodic: Due to pseudo-disorder, the interaction energy during expansion (if $\Delta < J$) is converted into kinetic and on-site potential energy. This results in enhancement or reduction of the expansion.