Resonant Transport of Bose Einstein Condensates

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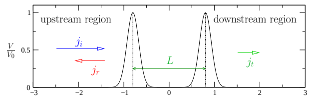
Abstract

The rapid progress of atom chip technology opens the perspective for experiments that probe the transport of Bose-Einstein condensates through mesoscopic waveguides. Particularly interesting in this context is the propagation of the condensate through a double barrier potential created by a sequence of two constrictions in the waveguide, which can serve as a Fabry-Perot interferometer for the condensate. We show that the presence of a finite repulsive interaction between the atoms generally leads to a suppression of resonant transport in such propagation processes. Near-resonant scattering states can nevertheless be populated on a finite time scale by means of a time-dependent control of the external potential [1].

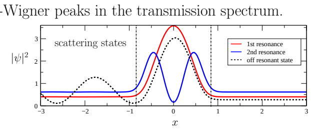
Resonator for Condensates

We consider the propagation of a Bose-Einstein condensate through a quasi one-dimensional waveguide with a longitudinal double-barrier potential $V(x)$ acting as a resonator [2,3].

$$V(x) = V_0 \left(e^{-(x+L/2)^2/\sigma^2} + e^{-(x-L/2)^2/\sigma^2} \right)$$



Noninteracting atoms: resonant scattering states lead to Breit-Wigner peaks in the transmission spectrum.



Basic question:

⟶ Under which conditions can resonant transport be realized for an interacting condensate ?

Existence of resonant states

- Start from effective 1D Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) + g|\psi(x,t)|^2 \right)\psi(x,t)$$

with $g = 2a_s\hbar\omega_\perp$ in quasi-1D waveguides [4].

- Ansatz $\psi(x,t) = A(x)\exp\left[i(\Phi(x) - \mu t)\right]$

$$\Rightarrow \quad \mu A = -\frac{1}{2}\frac{d^2A}{dx^2} + \left(V(x) + \frac{j_t^2}{2A^4} \right) A + UA^2$$

with $j_t = A^2\frac{\partial\Phi}{\partial x}$ independent of x [5].

- Integrate the time-independent equation for $A(x)$ from $x \to +\infty$ to $x \to -\infty$ with the "initial" conditions:

$$A'(\infty) = 0 \quad \text{and} \quad \mu = \frac{mj_t^2}{2A^2(\infty)} + gA^2(\infty)$$

- Calculate the drag F_d that the condensate exerts onto the obstacle [6]:

$$F_d = \int_{-\infty}^{+\infty} dx\, n(x)\, \frac{dV(x)}{dx}$$

→ F_d measures proximity of ψ to a resonant state:

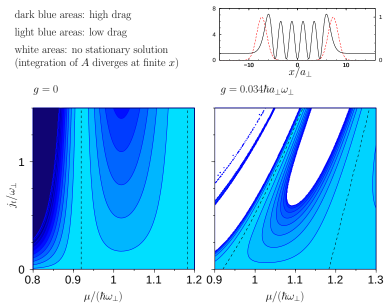
large F_d ⟶ scattering state far away from resonances
small F_d ⟶ scattering state close to a resonant state

Relevant experimental parameters

the bosons: ^{87}Rb atoms
transverse trap frequency: $\omega_\perp = 2\pi 10^3 s^{-1}$
harmonic oscillator length: $a_\perp = \sqrt{\hbar/m\omega_\perp} = 0.34\mu m$
⟹ effective interaction strength: $g \simeq 0.034\hbar\omega_\perp a_\perp$
resonator length: $L = 5\mu m = 14.7a_\perp$
barrier width: $\sigma = 0.5\mu m = 1.47a_\perp$
potential height: $V_0 = \hbar\omega_\perp$
incident current: $j_i = 10^4$ atoms/s

Stationary solutions near 6th resonance

- Calculate the drag F_d as a function of μ and j_t at fixed g



→ chemical potential μ of the resonant state increases with j_t in presence of repulsive atom-atom interaction.

→ For large currents, scattering states exist only in the immediate vicinity of the resonances.

Resonant Transport

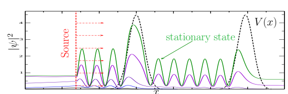
⟶ Can the resonant scattering states be populated in a realistic propagation process?

Numerical simulation

- Expand condensate wavefunction on a finite grid with absorbing boundary conditions [7] at the grid boundaries.
- Integrate time-dependent Gross-Pitaevskii equation in presence of a source term:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) + g|\psi(x,t)|^2 \right)\psi(x,t) + S_0\delta(x - x_0)\exp(-i\mu t/\hbar)$$

- Adiabatic increase of S_0 ⟶ smooth convergence to stationary scattering state with chemical potential μ



- Determine the transmission from the incident current

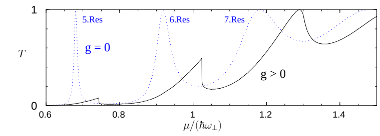
$$j_i = \frac{\hbar|S_0|^2}{mk_0} \quad \text{with} \quad \hbar k_0 = \sqrt{2m\left(\mu - g|S_0|^2/k_0^2\right)}$$

and the total current $j_t = \frac{\hbar}{m}\text{Im}\left[\psi^*\frac{\partial}{\partial x}\psi\right]$:

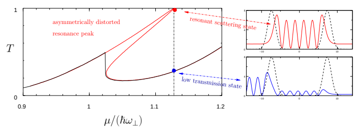
$$T = \frac{j_t}{j_i} = \frac{\text{downstream current in presence of the barrier}}{\text{downstream current in absence of the barrier}}$$

Transmission spectrum

...at fixed incident current $j_i = 10^4$ atoms $/s$



- asymmetric profiles for $g > 0$
- step-like structures in the spectrum
→ bistability phenomenon near the resonance [3]

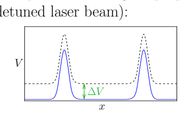


⇒ resonant transport is suppressed for finite interactions

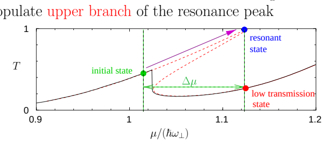
Adiabatic control of resonant transport

→ temporal variation of the external potential during the propagation process (e.g. by red-detuned laser beam):

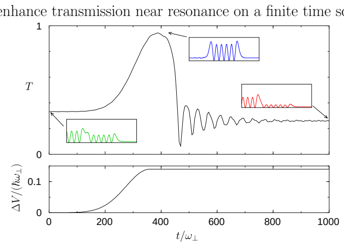
$$V(x) \to V(x) - V(t)$$
$$\Delta V \equiv V(t_f) - V(t_i) = \Delta\mu$$



→ populate upper branch of the resonance peak



→ enhance transmission near resonance on a finite time scale



→ Decay of resonant scattering state within $\tau \sim 10$ ms.

The bottom line

- Resonance peaks are distorted in presence of interactions.
- Straightforward propagation processes do not probe resonant scattering states of the condensate.
- Temporal variations of the potential can be applied to obtain resonant transport on finite time scales.

References

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