Superfluid regimes in degenerate atomic Fermi gases

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Outline

1. Introduction
2. Interaction between particles. BCS-BEC cross-over
3. Molecular BEC regime
4. Strongly interacting regime
5. Ideas for future:

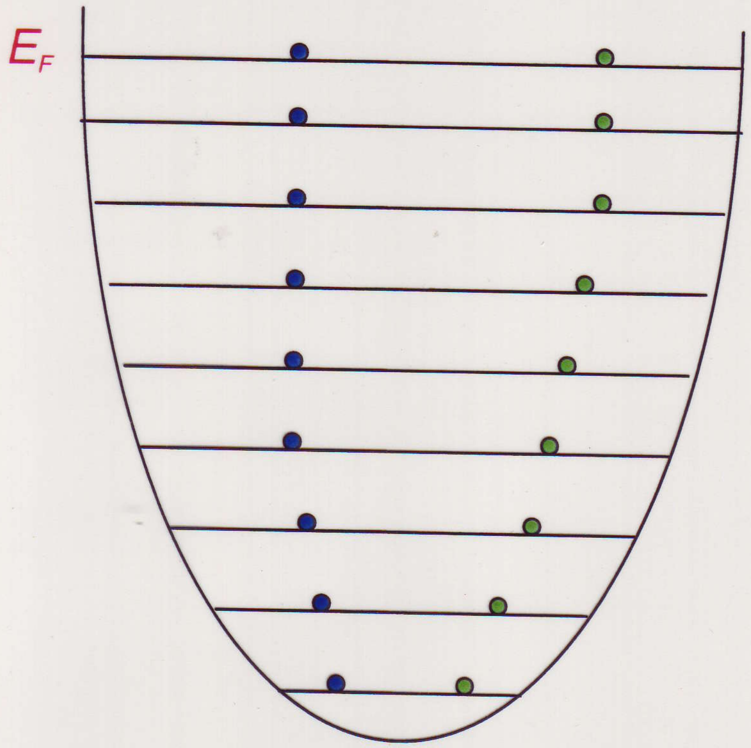
   Towards the BCS regime

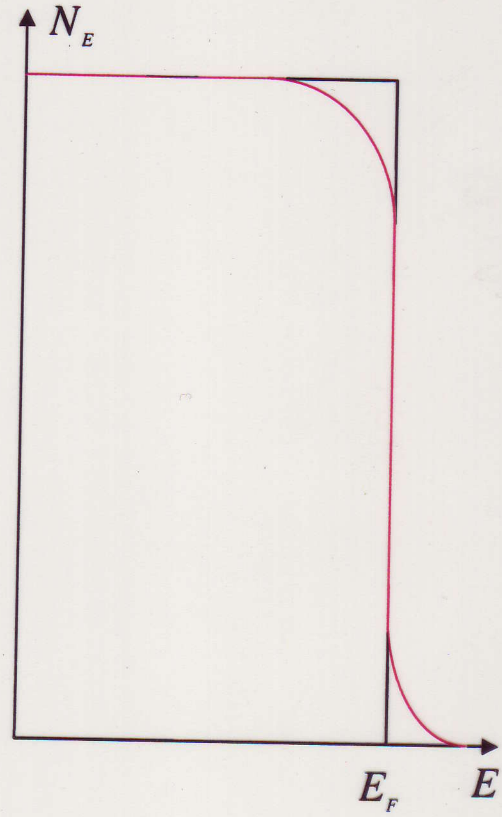
   Mixtures of different fermionic atoms

Collaborations: D.S. Petrov (ITAMP, Harvard)

C. Salomon (ENS, Paris)

Trapped 2-component Fermi gas





$$E_F = \frac{\hbar^2 k^2}{2m}; \quad k_F = (3\pi^2 n)^{1/3}; \quad E_F \sim N^{1/3}\hbar\omega$$

$a<0$ ⟶ Attractive interspecies interaction

Weakly interacting gas $n|a|^3 << 1; \; k_F|a| << 1$

Cooper pairing at very low T

$\vec{k}$ $\qquad -\vec{k}$

Superfluid BCS transition $T_c \sim E_F \exp\{-\pi / 2k_F|a|\}$

$T_c << 0.1E_F$ for ordinary a Very hard to reach

Experiments with Fermi gases ^{40}K ^{6}Li

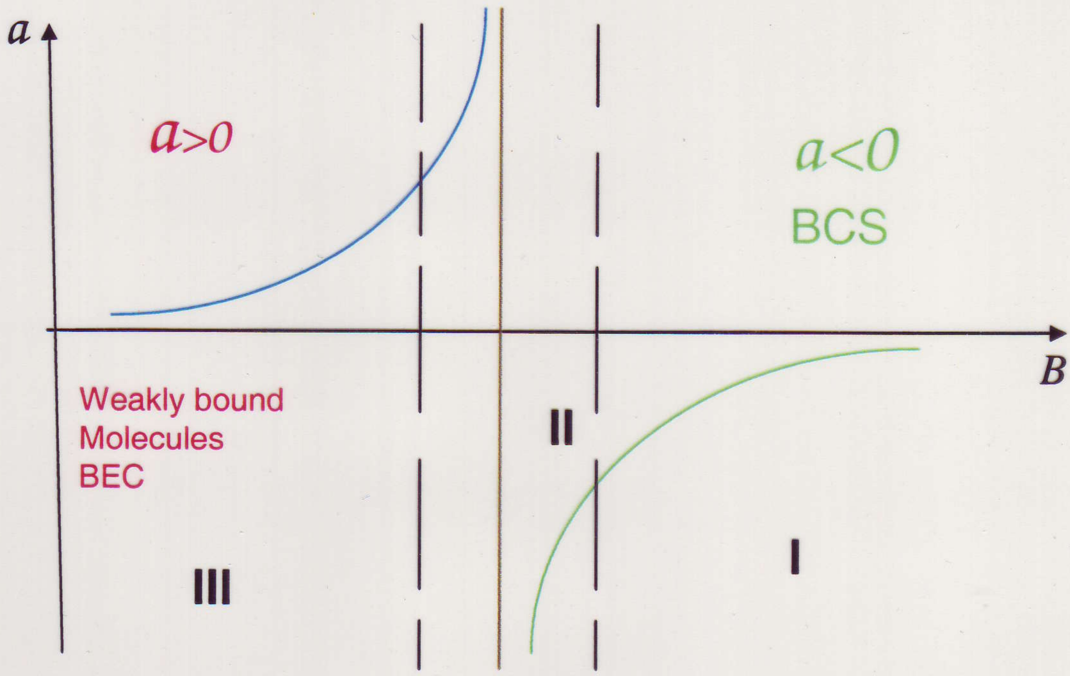
Quantum degeneracy ⟶ JILA 1998 ^{40}K

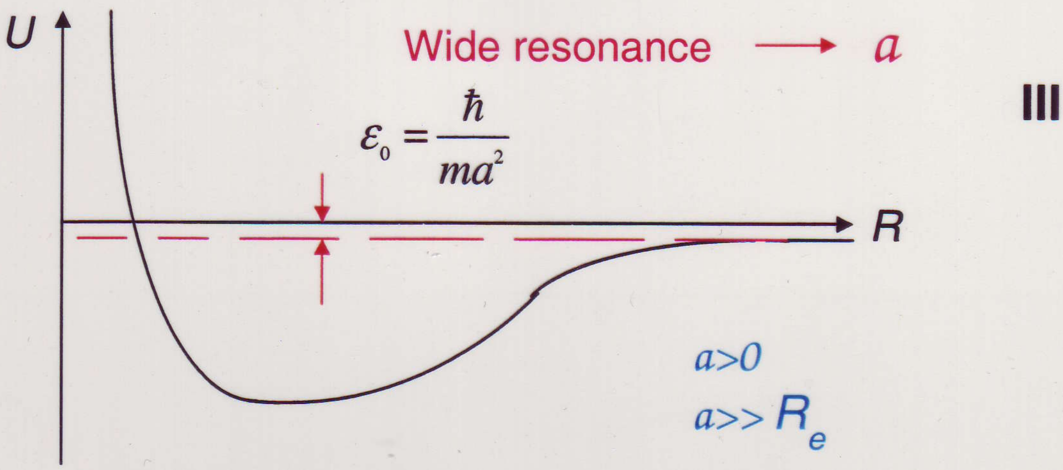
At present $n \sim 10^{13} - 10^{14}\ cm^{-3}$; $T \sim 1\,\mu K$

JILA, LENS K Innsbruck, MIT, ENS, Rice, Duke Li

Dilute limit $nR_e << 1$ Ultracold limit $\Lambda_T >> R_e$

Atom-atom scattering length a Feshbach resonance





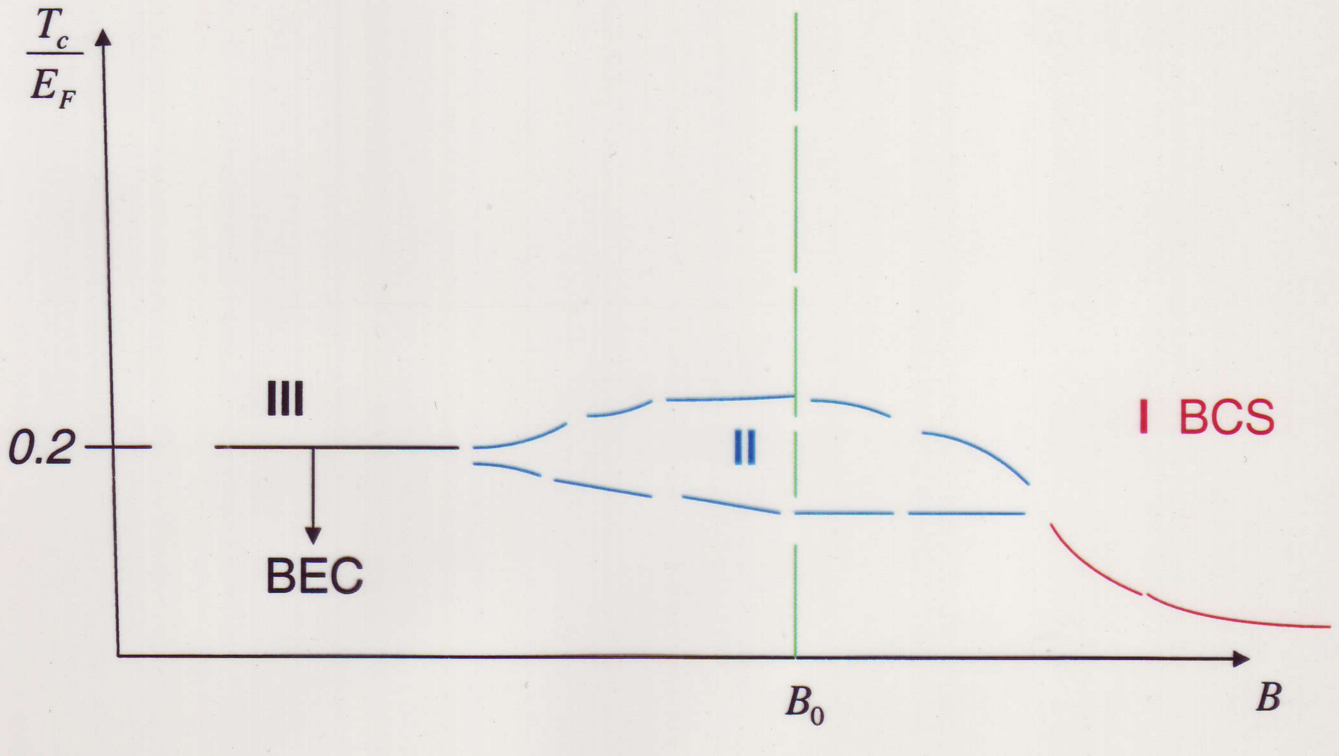
I $k_F |a| << 1$ → BCS

II $k_F |a| << 1$ → strongly interacting gas

III $na^3 << 1$ → Gas of bosonic molecules

$a >> R_e$ → BEC of weakly bound bosonic molecules

$$T_{BEC} \approx \frac{3.31\hbar^2}{2m}(n/2)^{2/3} \rightarrow 0.2\, E_F$$



BCS-BEC cross-over ; Leggett, Nozieres-Schmitt-Rink

I **BCS**

II **resonance superfluidity** (JILA, Toronto)

III **molecular BEC**

III Strongly interacting regime

$T=0 \quad k_F|a| >> 1 \rightarrow$ Only one distance scale $n^{-1/3}$

Only one energy scale $E_F \sim \dfrac{\hbar^2 n^{2/3}}{m}$

Universal thermodynamics J.Ho

Monte Carlo studies $\longrightarrow$ $\mu = \beta E_F$

(Carlson et al, Giorgini/Astracharchik)

Nature of superfluid pairing

Superfluid transition temperature

Excitations etc.

Holland, Timmermans, Griffin, Stringari, Strinati, Tosi, Bulgac, Levine, Ho, Pethick, Bruun, Stoof, Cheng

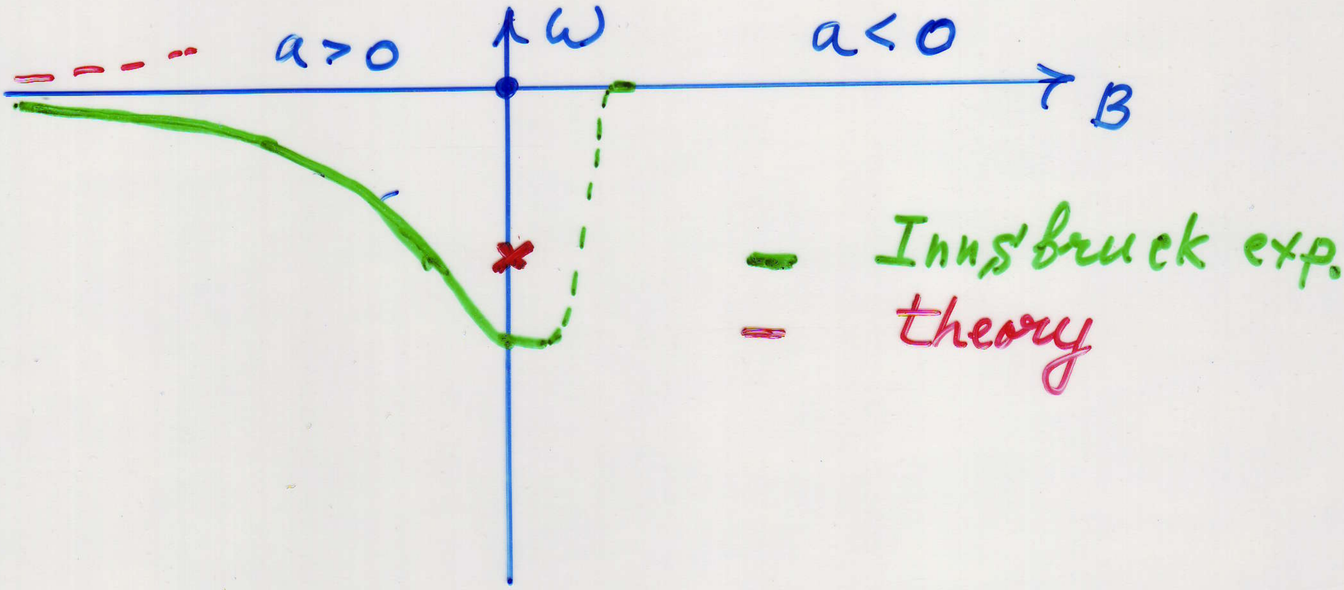
Experiments

1. JILA – MIT $\rightarrow$ BEC-type behavior of fermionic atom pairs
2. "Wrong" behavior of quadrupole excitations $\rightarrow$ Innsbruck, Duke
3. Measurement of the gap (Innsbruck)

II Strongly interacting regime

1. JILA, MIT → BEC-type behavior of fermionic atom pairs

2. Innsbruck, Duke → quadrupole excitation frequencies



3. Measurement of the gap (Innsbruck)

Holland, Timmermans, Griffin, Stringari, Strinati, Tosi, Bulgac, Levine, Ho, Pethick, Bruun ....

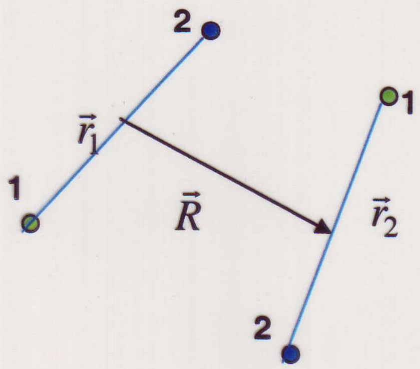
III Weakly interacting gas of bosonic dimers

Elastic interaction — BEC stability

"Old answer" ⟶ $2a$

4-body problem — Exact solution for $a \gg R_e$

Petrov et al, 2003



$\Psi \longrightarrow$ *9 variables*

Zero-range approximation

$$\Psi \longrightarrow f(\vec{r}_2, \vec{R})(1/4\pi r_1 - 1/4\pi a)$$

for $r_1 \to 0$

Integral equation for f

$k \longrightarrow 0$ *s*-wave scattering; *3* variables

$R \to \infty$
$$\Psi = \varphi_0(r_1)\varphi_0(r_2)(1 - a_{dd}/R)$$

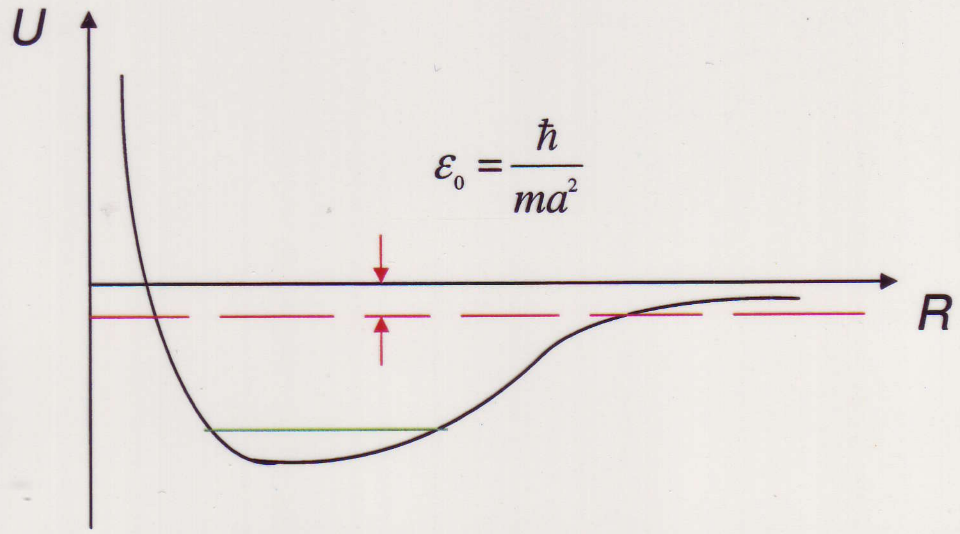
$$\varphi_0(r) = \frac{1}{r\sqrt{2\pi a}} \exp(-r/a)$$

$R \to \infty$
$$f(\vec{r}, \vec{R}) = \frac{2}{rR} \exp(-r/a)\,(1 - a_{dd}/R)$$

$$a_{dd} = 0.6\, a$$

Monte Carlo — Giorgini/Astracharchik *2004*

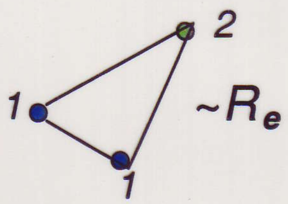
Weakly bound dimers → The highest rovibrational state of the diatomic molecule



Collisional relaxation to deep bound states
(~1ms for Rb_2 at $n \sim 10^{13}$ cm^{-3})

Atom-dimer collisions ($a >> R_e$)

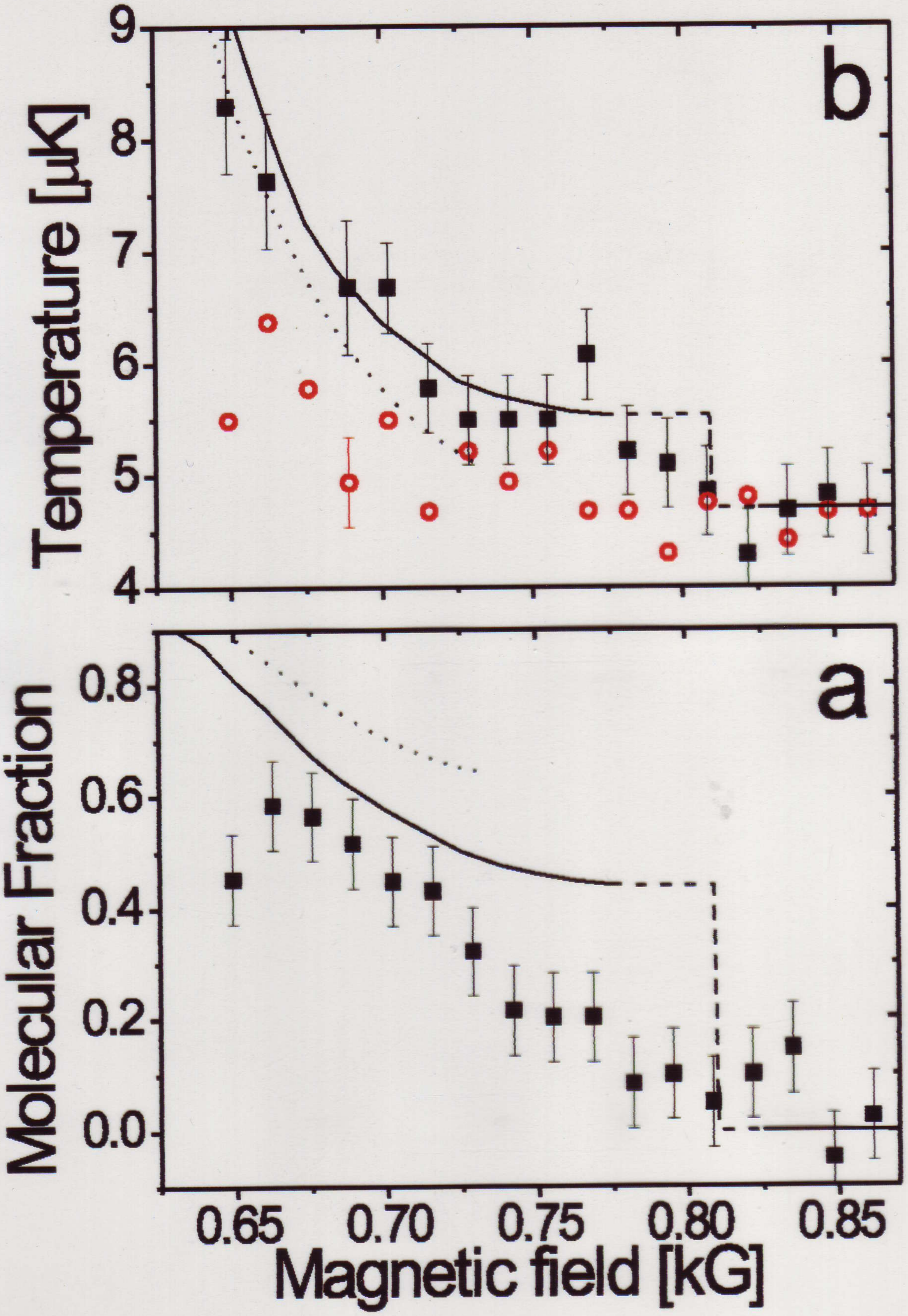
Size → Weakly bound dimer ~ a
Deep bound state ~ R_e(50 Å)<<a



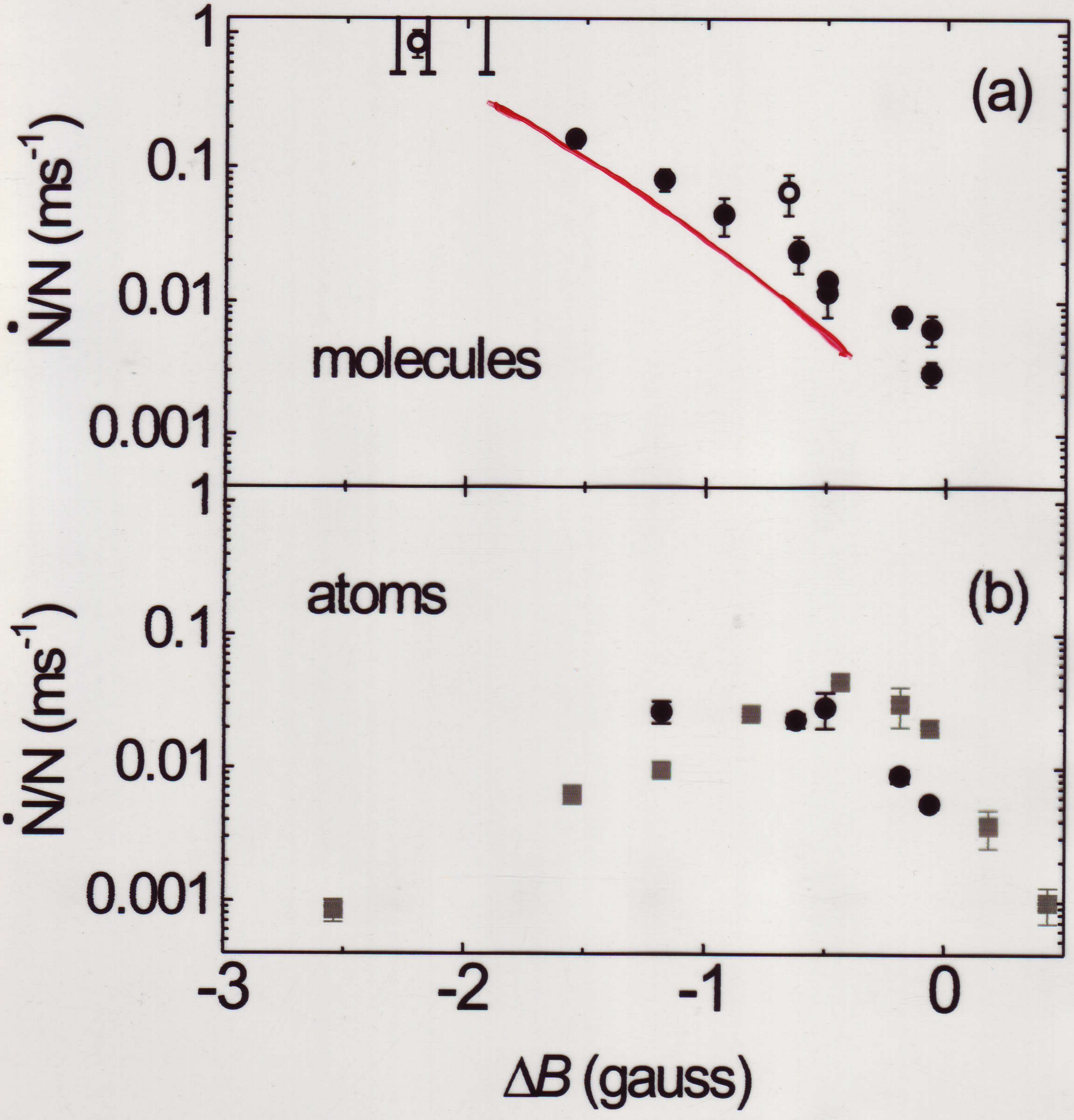
2 particles are identical fermions

Pauli principle

$$\alpha_{rel} \sim (k_{eff} R_e)^{2?} \sim (R_e / a)^{2?}$$



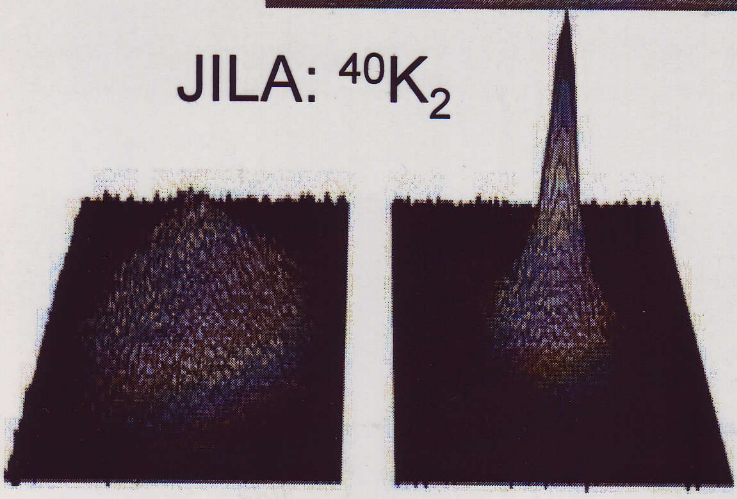
9
8
7
6
5
4
Temperature [μK]
b
0.8
0.6
0.4
0.2
0.0
Molecular Fraction
a
0.65
0.70
0.75
0.80
0.85
Magnetic field [kG]

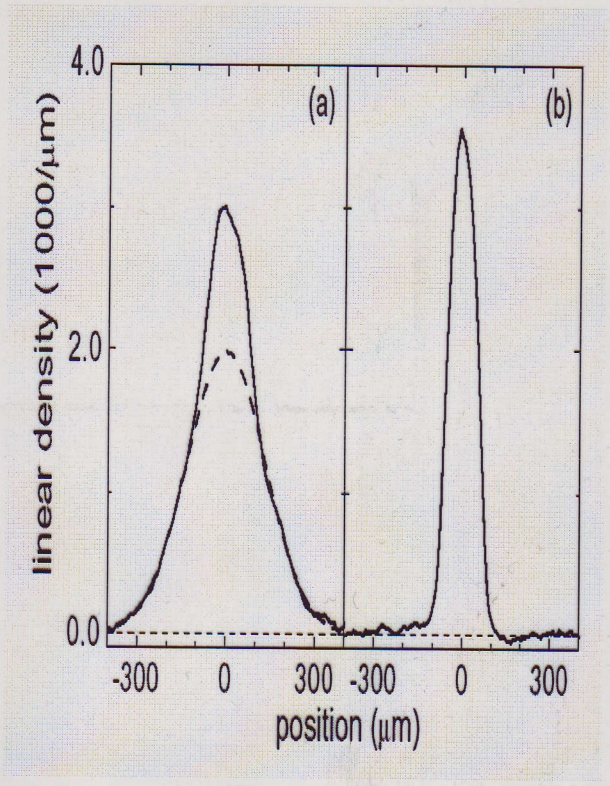


1
0.1
0.01
0.001
Ṅ/N (ms-1)
(a)
molecules
1
0.1
0.01
0.001
Ṅ/N (ms-1)
atoms
(b)
-3
-2
-1
0
ΔB (gauss)

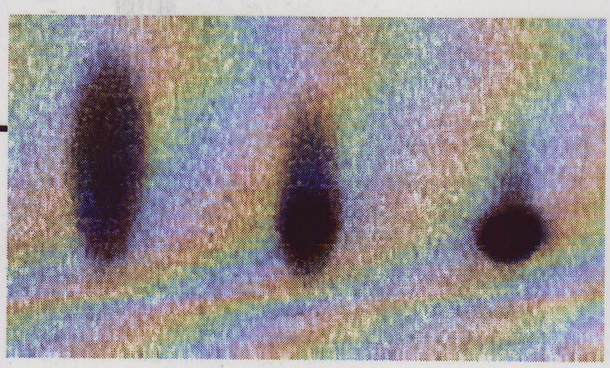
Condensates of molecules

JILA: $^{40}K_2$



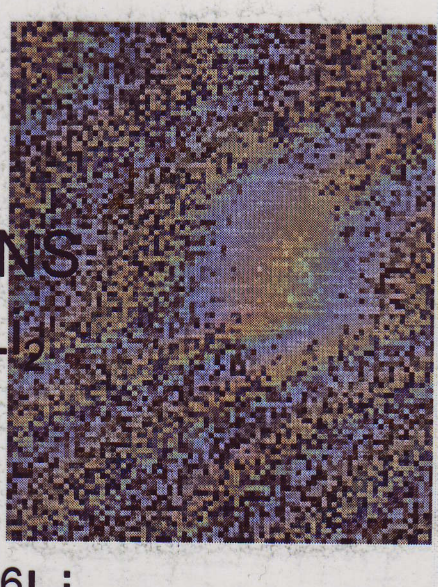


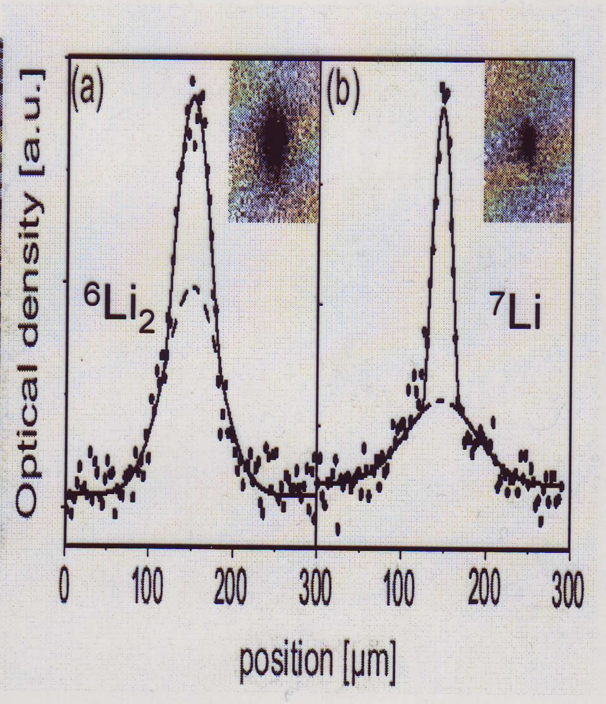
MIT $^{6}Li_2$



$^{6}Li_2$:Innsbruck

ENS $^{6}Li_2$





Also Rice $^{6}Li_2$

Zero-range approximation
($a \gg R_e$)

Petrov et al, 2003

(*) $$\alpha_{rel} = C \cdot \frac{\hbar R_e}{m} \left(\frac{R_e}{a} \right)^{s}$$

$s = 3.33$ atom-dimer

$s = 2.55$ dimer-dimer

Suppressed relaxation

Fast elastic collisions → $a_{dd} = 0.6a$

$Li_2 \Rightarrow \frac{\alpha_{rel}}{\alpha_{el}} \lesssim 10^{-4}$

Efficient evaporative cooling

BEC! JILA, Innsbruck, MIT, ENS, Rice

(*) consistent with JILA, ENS exp.

$T \to 0.3\, T_{BEC}$

Convert back to $a<0$. Cooling!

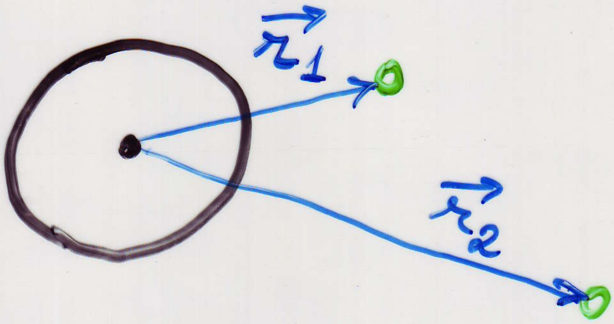
$$S_B = 2N \left(\frac{T_{in}}{T_{BEC}} \right)^3 = S_F \approx N\pi^2 \frac{T_f}{E_F}$$

$T_f \simeq 10^{-2} E_F$; Carr, Castin, G.S. (2003)

Mixture of heavy and light fermionic atoms (^{40}K and ^{6}Li)

Feshbach resonance. Tunable (a).

$a>0$ Weakly bound dimers



Slow relaxation to deep bound states

$$\psi = \hat{A}\, f_0(\vec{r}_1)\,\chi(\vec{r}_2);$$

$$f_0 = \frac{1}{\sqrt{2\pi a}} \cdot \frac{1}{r} \exp\left(-\frac{r}{a}\right)$$

$$\chi(r) = \left(1 - \frac{a}{r}\right)$$

$$\{r_1, r_2\} \ll a \Rightarrow \psi \sim \frac{1}{a^{3/2}}$$

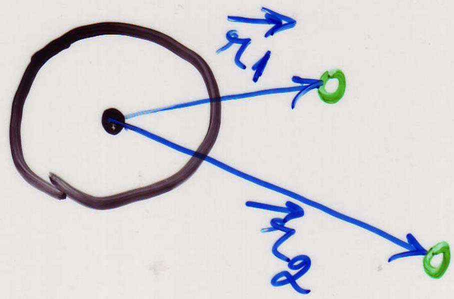
$\frac{1}{a} \rightarrow$ Frank-Condon factor;

$\frac{1}{a^2} \rightarrow$ Pauli principle; $d_{rel} \sim \frac{1}{a^3}$

Long-lived polar molecules !!

Molecules of bosonic atoms

Short-lived !!



$$\psi = \hat{S}\, \varphi_0(r_1)\chi(r_2)$$

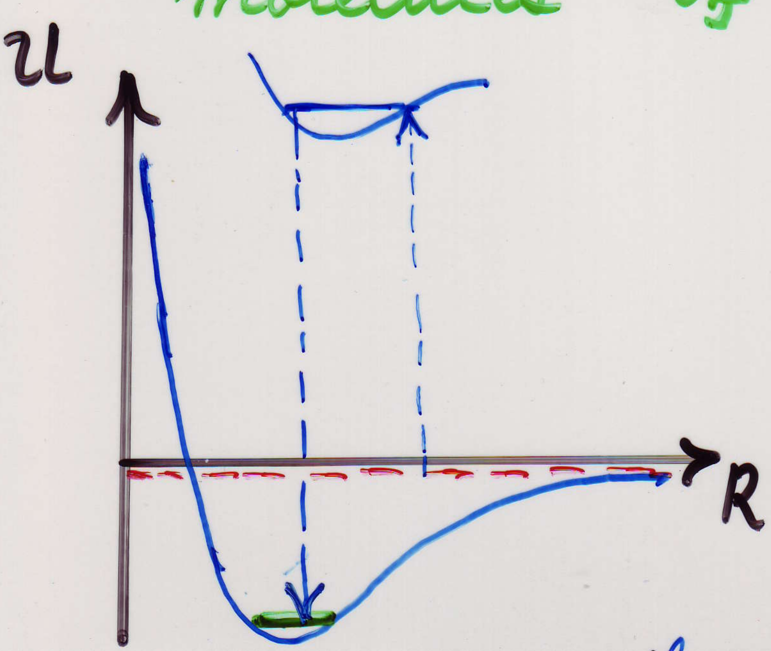
$$\{r_1, r_2\} \ll a \Rightarrow \left.\begin{array}{l} \chi \sim a \\ \varphi_0 \sim \frac{1}{\sqrt{a}} \end{array}\right| \Rightarrow \psi \sim \boxed{\sqrt{a}}$$

$\frac{1}{a} \longrightarrow$ Frank-Condon factor

$a^2 \longrightarrow$ Resonance enhancement

$$\alpha_{rel} \sim a$$

Idea from Yalle studies of molecules of bosonic atoms



Replace bosons by fermions

large τ

large n

Dipolar gas

Conclusions

1. We got remarkable physics with ^{40}K and ^{6}Li Fermi gases
2. More remarkable physics is expected in future, in particular with mixtures of different fermionic atoms