# Density-dependent coupling strength based on two-boson correlations

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## Introduction

We study two-body correlation amplitudes for a system of identical bosons. This allows a large two-body scattering length and a possible renormalization of the coupling constant in order to take into account the density of the system.

Recent years' experiments with Bose-Einstein condensed cold alkali gases have shown many aspects of the quantum nature of dilute boson systems. Later experiments [1, 2] have touched upon the interplay between macroscopic features of a large number of bosons and coupling to degrees of freedom that are hard to understand in terms of a mean-field description of the condensate. A combined description of coherent macroscopic features along with the possibility of freedoms in two- and three-particle subsystems is an attractive goal for theoretical work.

In a hyperspherical study we wrote the many-body wave function as a Faddeev-like sum of two-body amplitudes [3]. It is here applied to systems with arbitrary particle number $N$ and scattering length $a_s$. The details can be found in [4].

## Two-body correlations among identical bosons

The total Hamiltonian for $N$ identical, interacting bosons of mass $m$ trapped in an external harmonic field of angular frequency $\omega$ is

$$\hat{H} = \sum_{i=1}^{N}\left(\frac{\hat{p}_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2\right) + \sum_{i<j}^{N} V(r_{ij}) \, . \qquad (1)$$

where $V(r_{ij})$ is a finite-range two-body interaction term, for example a linear combination of Gaussians.
An initial hyperspherical description [5] was recently developed to deal with correlations [3]. The many-boson system is described by the hyperradius $\rho$, which is the relevant macroscopic length scale. This is given by

$$\rho^2 = \frac{1}{N}\sum_{i<j}^{N} r_{ij}^2 = \sum_{i=1}^{N} r_i^2 - NR^2 \, . \qquad (2)$$

The remaining relative degrees of freedom are described by the $3N-4$ hyperangles $\Omega$. The Hamiltonian then separates into a center of mass part ($\hat{H}_{c.m.}$), a radial part ($\hat{H}_\rho$), and an angular part ($\hat{h}_\Omega$) depending respectively on $\vec{R}$, $\rho$, and $\Omega$:

The adiabatic hyperspherical expansion of the wave function is

$$\Psi(\rho,\Omega) = \rho^{-(3N-4)/2}\sum_{\nu=0}^{\infty} f_\nu(\rho)\Phi_\nu(\rho,\Omega) \, , \qquad (3)$$

Two-body correlations are studied by a Faddeev-like decomposition of the many-body wave function, where all particle pairs are treated equally by s-wave amplitudes.

$$\Phi(\rho,\Omega) = \sum_{i<j}^{N}\phi_{ij}(\rho,\Omega) \approx \sum_{i<j}^{N}\phi(\rho,r_{ij}) \, . \qquad (4)$$

The method allows any strength of the two-body interaction, and therefore any two-body s-wave scattering length $a_s$ can be treated [6].
Resulting effective radial potential:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{d\rho^2} + U(\rho) - E\right] f(\rho) = 0 \, , \qquad (5)$$

$$\frac{2mU(\rho)}{\hbar^2} = \frac{(3N-4)(3N-6)}{4\rho^2} + \frac{\lambda(\rho)}{\rho^2} + \frac{\rho^2}{b_t^4} \, . \qquad (6)$$

Here $b_t = \sqrt{\hbar/(m\omega)}$ is the length unit of a harmonic trapping potential of frequency $\nu = \omega/(2\pi)$. The $\rho$-dependent $\lambda$ is an angular potential (eigenvalue for the eigenfunction $\Phi$.

## Infinitely large scattering length

At the threshold for two-body binding the scattering length diverges:

$$a_s \to \infty \qquad (7)$$

The three-body system: infinitely many bound *Efimov* states [7].
The many-body system: self-bound negative-energy states:

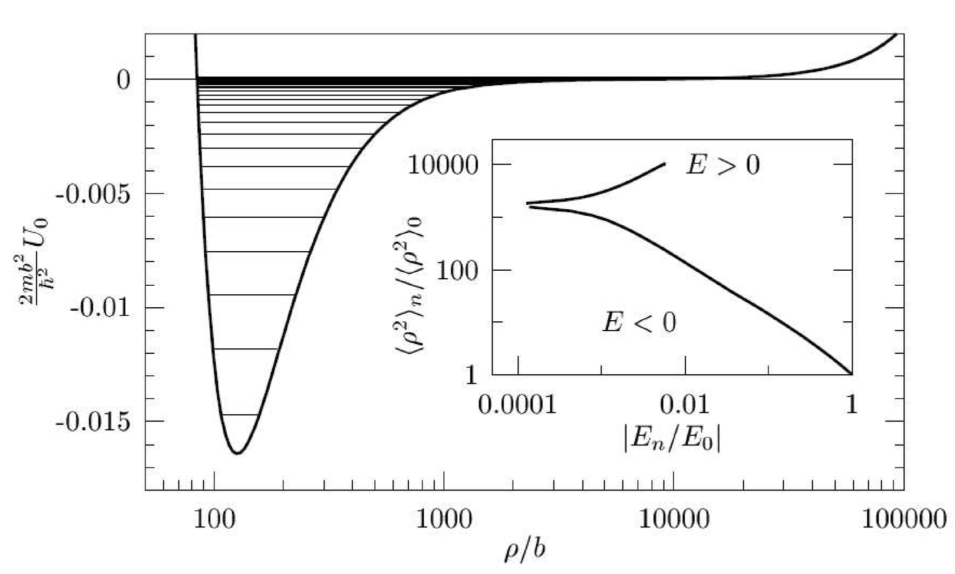


FIGURE 1: Radial potential for $N = 20$, $|a_s| = \infty$, $b_t/b = 1442$.

These excited, non-clustered gaseous states might then be called many-body Efimov states.
However, the true ground state is a crystal.
What is the nature of these excited many-body Efimov states?

## Density-dependent coupling strength

Effective coupling strength as a function of the density $n$

$$V_\delta(\vec{r},n) = g(n)\delta(\vec{r}) \quad , \qquad g(0) = \frac{4\pi\hbar^2}{m}a_s \qquad (8)$$

This can be interpreted in the model with the following density-dependence:

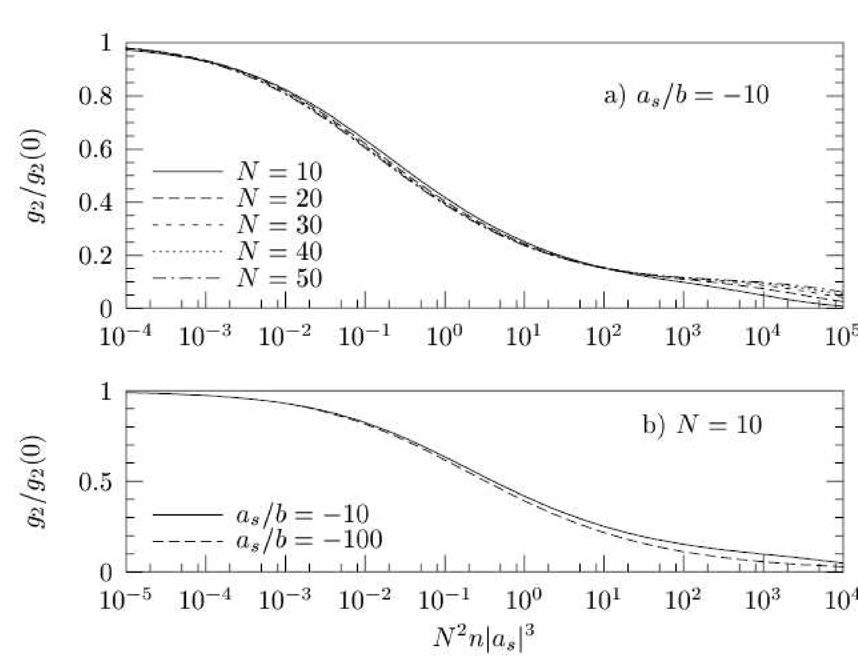


FIGURE 2: Coupling strength in units of zero-density value as a function of the density for ... upper: $a_s/b = -10$ and various particle numbers, ... lower: $N = 10$ and various scattering lengths.

## Critical interaction strength

When the attraction is too strong, that is when the magnitude of the negative scattering length is too large, there is no metastable solution for the system, see for example also [5, 8, 9]. Within this model:

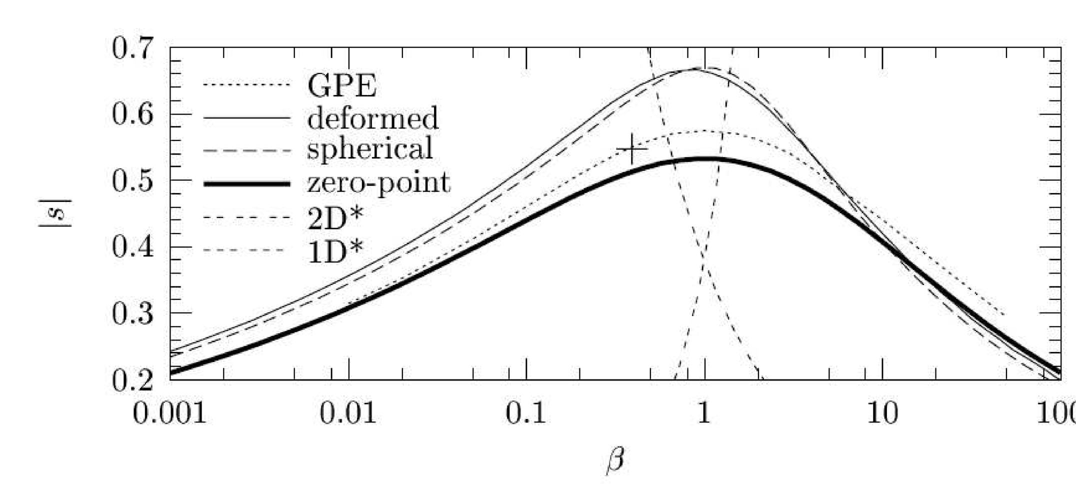


FIGURE 3: The critical strength $|s| = N|a_s|/b_t$ as a function of the deformation $\beta = b_\perp^2/b_z^2$ from the potential (thin solid line), from (dashed line), and from a mean-field Gross-Pitaevskii computation by Gammal *et al.* [10] (dotted line). The thick solid line is obtained by considering the zero-point energy. The plus is the experimentally measured value [11]. Regions below curves are considered stable in the separate treatments. The double- and triple-dashed lines indicate the effective cross-overs to two (2D*) and one (1D*) dimensions [12].

## Effective dimension of a deformed system

An effective 'kinematic' dimension $d$ [13]:

$$\hat{T} = \frac{\hbar^2}{2m}\left[-\frac{d^2}{d\rho^2} - \frac{d(N-1)-1}{\rho}\frac{d}{d\rho}\right] \qquad (9)$$

Depends on $N$ and deformation $\beta = b_\perp^2/b_z^2$ of external trap.

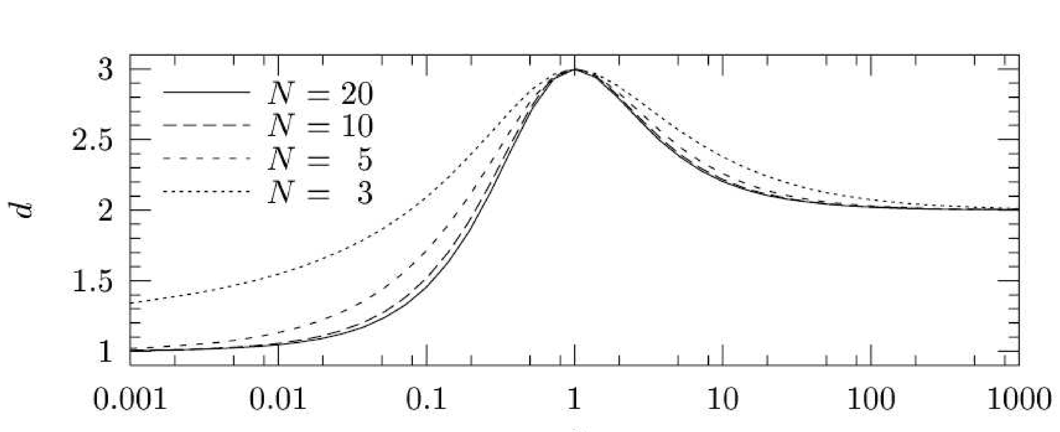


FIGURE 4: The effective dimension $d$ obtained as a function of the deformation parameter $\beta = b_\perp^2/b_z^2$. Curves for larger $N$ are very close to that for $N = 20$.

## Effective interaction strength for a deformed system

An effective deformation-dependent interaction coupling strength:

$$V(\vec{r},d) = g(d)\delta(\vec{r}) \quad , \qquad g(3) = \frac{4\pi\hbar^2}{m}a_s \qquad (10)$$

Here illustrated as the **solid line**: $g(d)/g(3)$

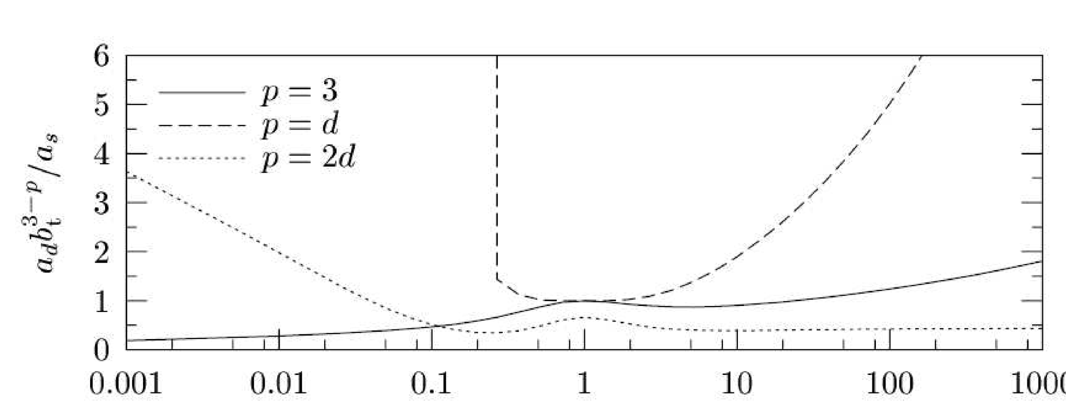


FIGURE 5: The effective interaction strength as a function of the deformation parameter $\beta = b_\perp^2/b_z^2$ in the large-$N$ limit.

## Related applications

- Two-component systems [14, 15]
- Two-dimensional systems [16]
- Fermionic systems (work in progress)
- Rydberg chains (work in progress)

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