Superfluid Fermions in a Slowly Rotating Trap

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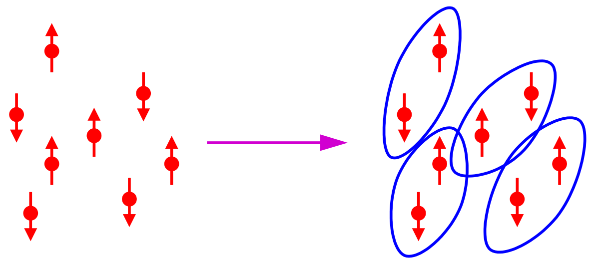
Abstract:

We propose the moment of inertia as a possible observable for the unambigious experimental determination whether a trapped Fermi system has reached the BCS transition or not. The temperature dependence of the moment of inertia below the critical temperature T_c is described in detail. Special care is taken to account for the small size of the system, i.e., for the fact that the level spacing of the trapping potential is of the same order of magnitude as the gap. The usual transport approach, corresponding to the leading order in an expansion in powers of $\hbar$, is not accurate in this case. It turns out that the moment of inertia decreases continuously when the temperature falls below T_c. Qualitatively this behavior can be explained within the two-fluid model, which again corresponds to the leading order in $\hbar$. Quantitatively we find substantial deviations from the two-fluid model due to the small system size.

1. Motivation

- Search for BCS transition in trapped Fermi gases



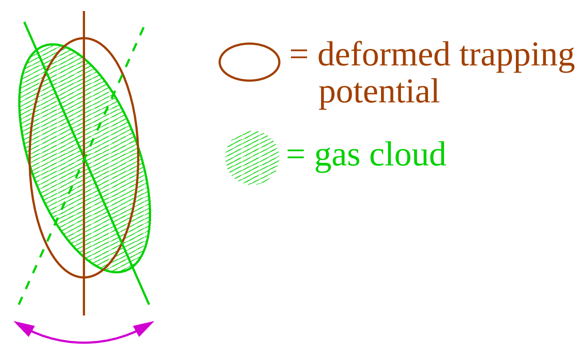
Cooper pairs (binding energy Δ)

$\longrightarrow$ Superfluidity

- Observables?

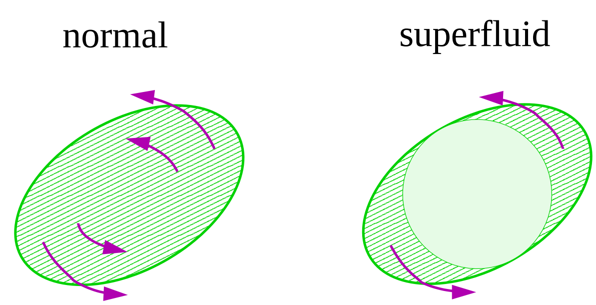
Frequencies of collective modes, e.g. "scissors" mode

$\longrightarrow$ cannot distinguish between superfluid and hydrodynamic regimes

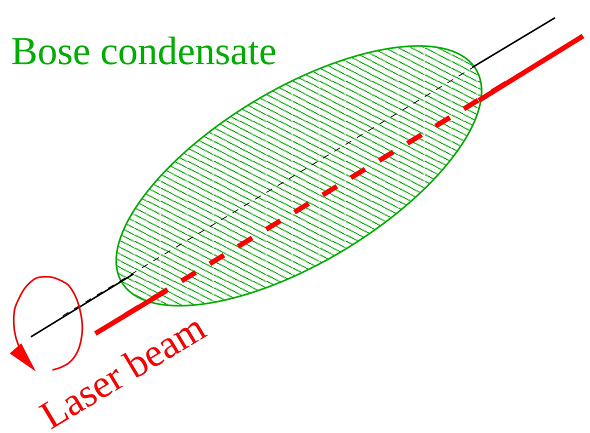


Moment of inertia $\Theta_{normal} \gg \Theta_{superfluid}$

$\longrightarrow$ unambigious identification of superfluid phase



- Experimental feasibility?



Rotation of BEC has been done by using a laser beam as "spoon"
$\longrightarrow$ observation of vortices,...
should be possible with fermions, too

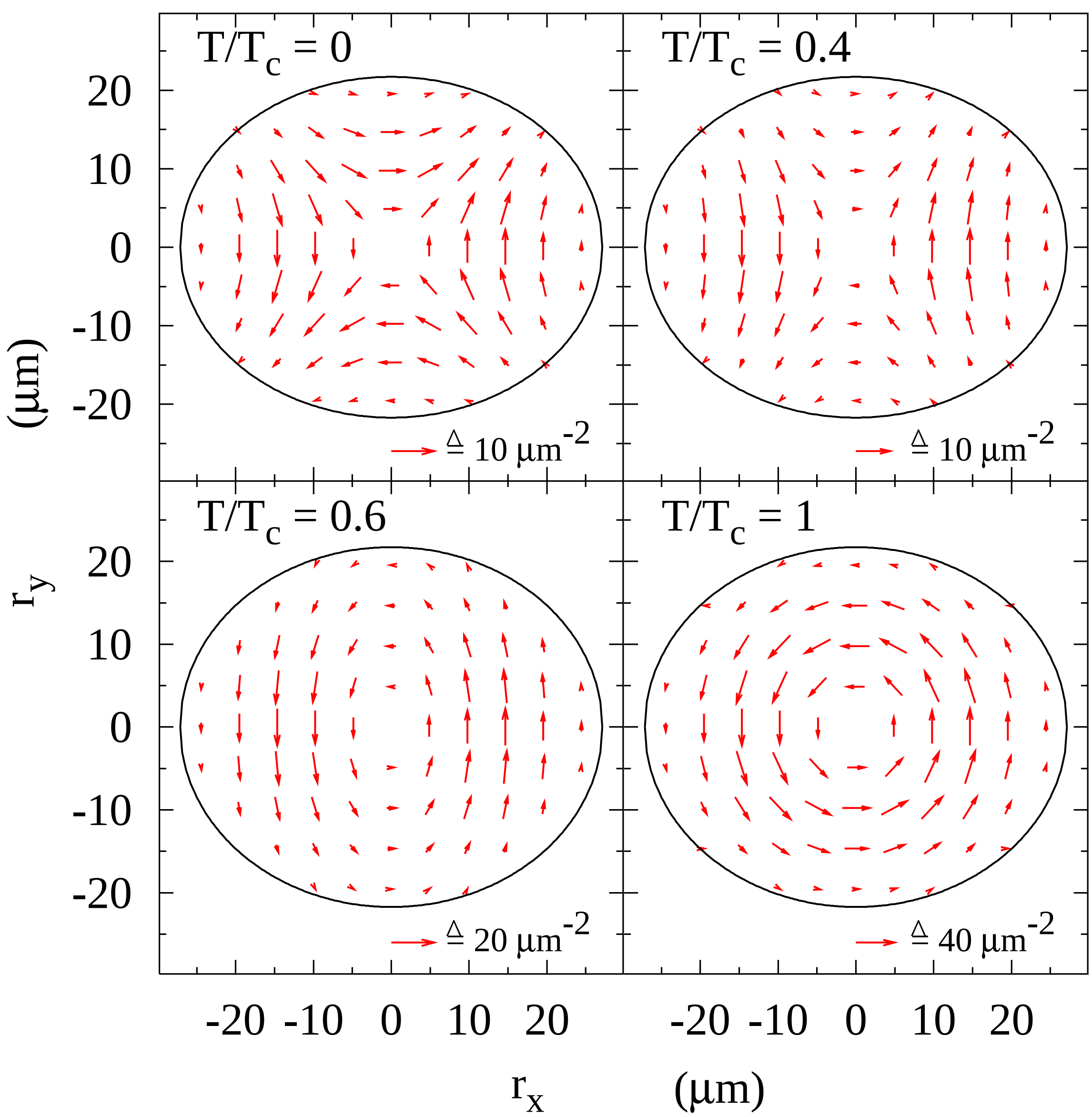
2. Description of the slowly rotating Fermi gas

- Hamiltonian in the rotating reference frame (Ω = rotation frequency):

$$H = \int d^3r \Big[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger(\vec{r}) \Big(\frac{\hat{p}^2}{2m} + V_{trap}(\vec{r}) - \Omega \hat{L}_z \Big) \psi_\sigma(\vec{r}) - g \psi_\downarrow^\dagger(\vec{r}) \psi_\uparrow^\dagger(\vec{r}) \psi_\uparrow(\vec{r}) \psi_\downarrow(\vec{r}) \Big]$$

Trapping potential: $V_{trap}(\vec{r}) = \frac{m}{2}\Big(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\Big)$ ($\omega_x \neq \omega_y$ acts as "spoon")

- BCS approximation $\longrightarrow$ below critical temperature T_c order parameter $\Delta \neq 0$
- Slow rotation $\longrightarrow$ treat $\Omega \hat{L}_z$ term as small perturbation ($\longrightarrow$ quasiparticle RPA)
- Large number of particles $\longrightarrow$ semiclassical phase-space description
- Current distribution: $\vec{j}(\vec{r}) = \rho(\vec{r})\,\vec{v}(\vec{r}) = 2\int \frac{d^3p}{(2\pi\hbar)^3}\,\frac{\vec{p}}{m}\,f(\vec{r},\vec{p})$



3. Interpretation within the two-fluid model

- Two contributions to the current

  Direct change of Wigner function due to ΩL_z term ("normal" current)

  $\longrightarrow$ velocity field of rigid rotation $\qquad \vec{v}_{rigid}(\vec{r}) = \vec{\Omega} \times \vec{r}$

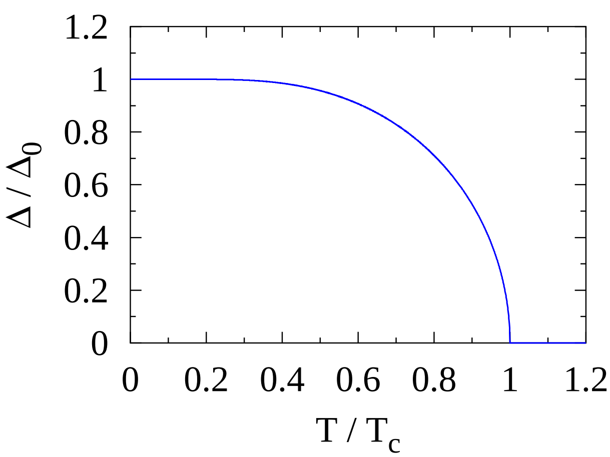
  Order parameter $\Delta(\vec{r})$ acquires a phase $\phi(\vec{r}) \propto xy$ ("superfluid" current)

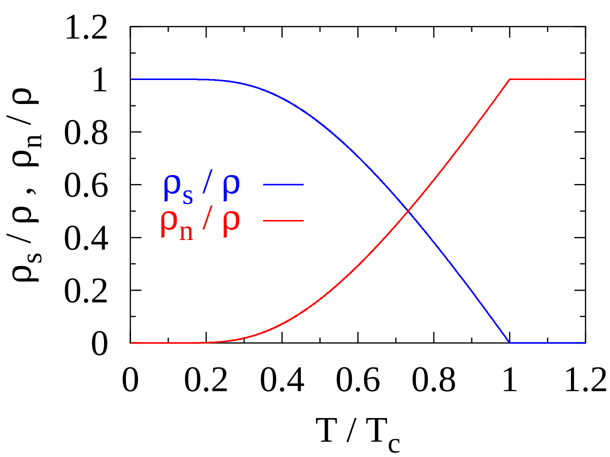
  $\longrightarrow$ irrotational velocity field $\propto \vec{\nabla}\phi(\vec{r})$ $\qquad \vec{v}_{irrot.}(\vec{r}) = \Omega \frac{\omega_y^2 - \omega_x^2}{\omega_y^2 + \omega_x^2} \vec{\nabla}(xy)$

- At finite temperature some Cooper pairs are broken by thermal excitations

  $\longrightarrow$ order parameter Δ decreases

  $\longrightarrow$ normal and superfluid components (two-fluid model)





  $\longrightarrow$ for $0 < T < T_c$ intermediate situation $\qquad \vec{v}(\vec{r}) = \frac{\rho_n}{\rho} \vec{v}_{rigid}(\vec{r}) + \frac{\rho_s}{\rho} \vec{v}_{irrot.}(\vec{r})$

- Qualitative agreement with the current distribution shown above

- However, velocity field not completely irrotational even at $T = 0$ due to quantum corrections of the order $\frac{\hbar\omega}{\Delta}$ (two-fluid model $\hat{=}$ $\hbar \rightarrow 0$ limit)

4. Moment of inertia

- Definition: $\Theta = \frac{\langle L_z \rangle}{\Omega} = \frac{1}{\Omega} \int \frac{d^3r\, d^3p}{(2\pi\hbar)^3} (\vec{r} \times \vec{p})_z\, f(\vec{r}, \vec{p})$

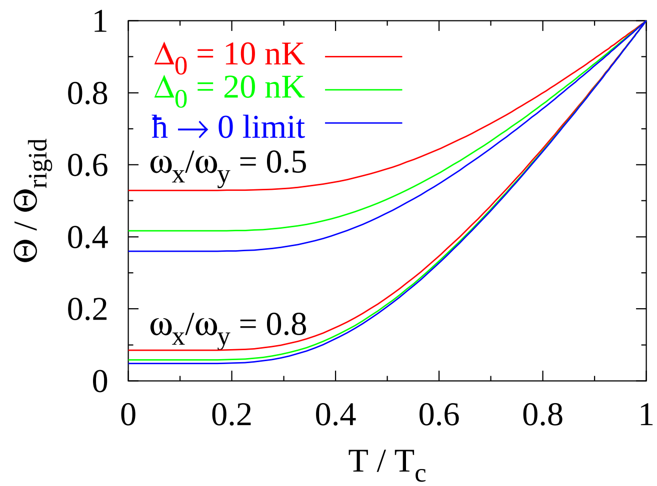
- Expectation from two-fluid model ($\hat{=}\ \hbar \to 0$ limit)

$T > T_c$: normal phase, rigid rotation $\quad \Theta_{rigid} = m\langle(x^2 + y^2)\rangle$

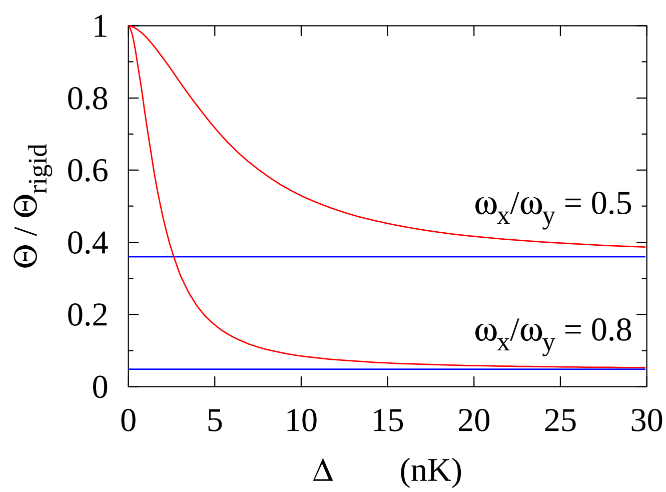
$T = 0$: superfluid phase, irrotational flow $\quad \Theta_{irrot.} = \left(\frac{\omega_y^2 - \omega_x^2}{\omega_y^2 + \omega_x^2}\right)^2 \Theta_{rigid}$

$0 < T < T_c$: intermediate case $\quad \Theta = \frac{\rho_n}{\rho} \Theta_{rigid} + \frac{\rho_s}{\rho} \Theta_{irrot}$

- Temperature dependence of Θ
  for two different deformations ω_x/ω_y
  and two different values for Δ_0
  $\longrightarrow$ qualitative agreement with two-fluid model
  (parameters used for the figures: $\hbar\omega_\perp = \hbar\sqrt{\omega_x \omega_y} = 16.4\,\text{nK}$)



- Deviation of Θ from $\Theta_{irrot.}$ at $T = 0$
  $\longrightarrow$ deviation from two-fluid model
  Θ as function of the gap Δ_0
  for two different deformations



Reference

For more details see M. Urban and P. Schuck, Phys. Rev. A **67**, 033611 (2003).